Auctions with Heterogeneous Entry Costs

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Preliminaries

- Auctions often fail because of insufficient interest by buyers.
  - ▷ Buyers are reluctant to begin an expensive, time-consuming evaluation of an asset unless they are likely to win at a favorable price.
- A buyer's cost of discovering her value may vary significantly across bidders.
  - ▷ In the sale of a firm, prospective buyers may face different regulatory restrictions.
  - ▷ In Internet auctions, a buyer's cost of discovering her value is the opportunity cost of her time.

Preliminaries

We study standard auctions where

- entry is endogenous,
- buyers are privately informed about their entry cost,
- entry (valuation-discovery) costs are heterogeneous, and
- a buyer learns her value upon entry.

The Auction

- A single object is allocated using a standard auction.
- Buyers are risk-neutral, and their values are *i.i.d.* on $[0, \bar{v}]$ according to an increasing *c.d.f.* F with an increasing hazard rate and *p.d.f.* f.
- The seller is also risk-neutral, and his value for the object is zero.

Preliminaries

The Auction

Assuming that bidding strategies form an increasing symmetric equilibrium, seller revenue in any standard auction with a screening value $v \in [0, \bar{v}]$ is

$$\pi(v,n) = n \int_v^{\bar{v}} (yf(y) + F(y) - 1)F^{n-1}(y)dy,$$

the utility of a bidder is

$$u(v,n) = \int_v^{\bar{v}} \left(\int_v^y F(x)^{n-1} dx \right) f(y)dy,$$

and gross social surplus is

$$s(v,n) = \int_v^{\bar{v}} ydF^n(y).$$

Fixed Number of Bidders

Myerson (1981) and Riley and Samuelson (1981)

1. A zero screening value maximizes social surplus.
2. The revenue maximizing screening value $v^F > 0$ is the solution to

   $$v = \frac{1 - F(v)}{f(v)},$$

   independently of the number of bidders – e.g., a first- or second-price auction with a reserve price $r = v^F$ maximizes seller revenue.
3. Bidders capture part of the surplus.
4. Social surplus and seller revenue increase with the number of bidders.

Endogenous entry

- There are N potential buyers who must decide whether to participate in the auction.
- Entry is costly.
- A buyer observes her value upon entry.

Endogenous entry

In a standard auction with a *screening value* v, if each bidder enters with probability p, then seller revenue is

$$\Pi(v,p) = \sum_{n=1}^{N} p_n^N(p)\pi(v,n),$$

and the utility of a buyer to entering the auction is

$$U(v,p) = \sum_{n=0}^{N-1} p_n^{N-1}(p)u(v,n+1),$$

where $p_n^N(p)$ is the binomial probability that exactly n bidders enter. Gross social surplus is

$$S(v,p) = \sum_{n=0}^{N} p_n^N(p)s(v,n).$$

Endogenous entry

- These formulae assume that the screening value v is independent of the realized value of n, which is appropriate when either
  - bidders do not observe the number of entering bidders and so their bidding strategies are independent of n, or
  - the rules of the auction are such that the screening value is the same for every n (e.g., a first or second price auction).
- In the *entry game*, the payoff to a bidder who enters, when every other bidder enters with the same probability p, is $U(v, p)$ minus her entry costs.

Endogenous entry - homogenous costs

- All buyers have the same entry cost $c > 0$.
- Assumption:
$$U(0,0) > c > U(0,1).$$

Endogenous entry - homogenous costs

McAfee and McMillan (1987) – pure strategy equilibria.
(**Levin and Smith (1994)** – symmetric mixed strategy equilibrium.)

1. A screening value of zero maximizes social surplus.
2. A screening value of zero maximizes seller revenue!!
3. Buyers capture no surplus.
4. Social surplus and seller revenue are independent of (decrease with) the number of buyers.

No need to screen buyers by value:
buyer surplus is competed away!

McAfee and McMillan (1987):

Intuition for the result that a screening value of zero maximizes seller revenue:

Key Fact (Prop. 1): *In a standard auction with a screening value of zero, the utility of a bidder is her contribution to social surplus, i.e.,* $u(0,1) = s(0,1)$ *and* $u(0,n) = s(0,n) - s(0,n-1)$ *for* $n > 1$.

Endogenous entry - homogenous costs

If n buyers enter the auction, then the maximum social surplus that can be realized is

$$E(V_{(n)}) - nc = s(0, n) - nc.$$

The contribution to social surplus of the n-th buyer to enter is

$$\begin{aligned} & s(0, n) - nc - [s(0, n-1) - (n-1)c] \\ = \quad & s(0, n) - s(0, n-1) - c \\ = \quad & u(0, n) - c \end{aligned}$$

When buyers enter sequentially, then the n-th buyer enters if

$$u(0, n) - c \geq 0.$$

Hence buyers enter to maximize social surplus. In equilibrium, buyers capture no surplus and thus $v = 0$ maximizes both social surplus and seller revenue.

Endogenous entry - homogenous costs

Levin and Smith (1994):
If each buyer enters with the same probability p, then the maximum social surplus that can be realized is

$$\sum_{n=1}^{N} p_n^N(p)E(V_{(n)}) - Npc = S(0,p) - Npc.$$

Using Proposition 1 we can show

$$\frac{dS(0,p)}{dp} = NU(0,p),$$

i.e., the marginal contribution to gross social surplus of an increase in the probability of entry is proportional to the utility of an entering buyer. Since U is decreasing in p, then

$$\frac{d^2S(0,p)}{dp^2} = N\frac{dU(0,p)}{dp} < 0.$$

Endogenous entry - homogenous costs

Hence the social surplus,

$$S(0, p) - Npc$$

is a concave function of p whose maximum on $[0, 1]$ is attained at the solution to the equation

$$N[U(0, p) - c] = 0.$$

If p^* is a symmetric entry equilibrium, then

$$U(0, p^*) - c = 0;$$

i.e., social surplus is maximized and each buyer captures no surplus. Hence $v = 0$ maximizes (constrained) social surplus and seller revenue.

Endogenous entry - heterogeneous costs

Moreno and Wooders – this paper (finally!)

- Buyers' entry costs are i.i.d. according to a *c.d.f.* H with support $[\underline{c}, \bar{c}]$. (H is increasing, satisfies $H(\underline{c}) = 0$, and has a *p.d.f.* h.)
- We assume that $\underline{c} < U(0,0)$ and $U(0,1) < \bar{c}$.
- A buyer makes entry decisions knowing her entry cost.
- The seller may also charge an admission fee ϕ to enter the auction.

Heterogenous Entry Costs

- A *strategy* for a buyer is a threshold $t \in [\underline{c}, \bar{c}]$ indicating the largest entry cost that enters.
- If all buyers employ the same threshold t, then the number of bidders in the auction is distributed binomial $B(N, p)$, where $p = H(t)$.
- A *symmetric entry equilibrium* is a threshold $t \in [\underline{c}, \bar{c}]$ that satisfies

$$U(v, p) - \phi - z \gtreqless 0 \iff t \gtreqless z.$$

- Write $t^*(v, \phi)$ for the symmetric equilibrium entry threshold when the screening value is v and the admission fee is ϕ.

Heterogenous Entry Costs: Results

Prop. 2. *For each screening value* $v \in [0, \bar{v}]$ *and admission fee* $\phi \in \mathbb{R}$*, there is a unique symmetric entry equilibrium* $t^*(v, \phi) \in [\underline{c}, \bar{c}]$*. The mapping* t^* *is a continuous function. When the equilibrium is interior,* $t^*(v, \phi)$ *solves*

$$U(v, H(t)) = t + \phi,$$

and is decreasing in both v *and* ϕ*.*

Heterogenous Entry Costs: Results

Given a common entry threshold $t \in [\underline{c}, \bar{c}]$, the social surplus generated in a standard auction with a screening value of v is

$$W(v, t) = S(v, H(t)) - Nc(t),$$

where

$$c(t) = \int_{\underline{c}}^{t} z dH(z)$$

is the expected entry cost incurred by each buyer.

Prop. 3. *A screening value and an admission fee both equal to zero maximize social surplus, i.e.,*

$$W(0, t^*(0, 0)) = \max_{(v,t) \in [0,\omega] \times [\underline{c}, \bar{c}]} W(v, t).$$

Heterogenous Entry Costs: Results

Prop. 4. *In an interior entry equilibrium, total buyer surplus is positive and decreasing in both the screening value and the admission fee, and seller revenue is less than the social surplus.*

Heterogenous Entry Costs: Results

Prop. 5. *A revenue maximizing screening value v^* exists, satisfies $0 < v^* < v^F$, and is characterized by the equation*

$$\frac{\partial W}{\partial v} + \frac{\partial W}{\partial t}\frac{\partial t^*}{\partial v} = NH(t^*(v,0))\frac{\partial t^*}{\partial v}.$$

▷ A marginal increase of the screening value above zero has a negligible negative effect on social surplus but a non-negligible negative effect on bidder surplus.

▷ A marginal decrease of the screening value below v^F has a negligible negative effect on seller revenue for fixed number of bidders, but a non-negligible positive effect on entry.

Screening bidders by value raises seller revenue!

Heterogenous Entry Costs: Results

Prop. 6. *If an admission fee is feasible, then the revenue maximizing screening value is zero, i.e., if it is feasible to screen buyers by entry costs, then it is suboptimal to screen bidders by values. Further, a revenue maximizing admission fee* ϕ^* *exists, is positive, and is characterized by the equation*

$$\frac{\partial W(0, t^*(0,\phi))}{\partial t}\frac{\partial t^*}{\partial \phi} = NH(t^*(0,\phi))\frac{\partial t^*}{\partial \phi}.$$

Seller revenue is greater than when an admission fee is not feasible.

$\triangleright$ Reducing the screening value to zero while charging an inspection fee that induces the same entry, increases the social surplus while maintaining bidder surplus unchanged.

When it is possible to screen buyers by entry costs,
it is suboptimal to screen bidders by value.

Entry Caps

Suppose that the seller introduces a cap $\bar{n} < N$; i.e., a cap on the number of bidders.

- Each buyer must decide whether to apply for entry.
- Applying for entry entails a commitment to enter and pay her entry cost and admission fee if admitted.
- If $\bar{n}$ or fewer buyers apply for entry, then each applicant is admitted. If more than $\bar{n}$ buyers apply, then applicants are anonymously rationed so that exactly $\bar{n}$ are admitted.

Entry Caps: Homogeneous Costs

Let n^* be the largest integer such that $u(0, n^*) \geq c$.

Prop. 7. *Assume that all buyers have the same entry cost* $c > 0$. *Then an entry cap* $\bar{n} = n^*$, *and an admission fee* $\phi = u(0, \bar{n}) - c$ *(and a screening value of zero) maximizes seller revenue and social surplus. Moreover, the seller captures the unconstrained maximum social surplus.*

Entry Caps: Heterogeneous Costs

Denote by $n^*(c)$ the largest integer n such that $u(0, n) - c \geq 0$.

Prop. 8. *An entry cap $\bar{n} = n^*(\underline{c})$ combined with a revenue maximizing admission fee and a zero screening value generates more revenue than any admission fee and/or screening value alone.*

Let $v = 0$ and ϕ^* denote the revenue maximizing screening value and admission fee *when there is no entry cap*. Let $t^*(0, \phi^*)$ denote the equilibrium entry threshold.

Why can the seller generate more revenue with a cap on entry?

Entry Caps: Heterogeneous Costs

▷ *Ceteris paribus*, both total buyer surplus and social surplus increase as a result of the entry cap.

▷ Raising the admission fee (from ϕ^*) until the equilibrium threshold for *applying* for entry equals $t^*(0, \phi^*)$, reduces total buyer surplus to below its level without the entry cap.

▷ This entry cap and admission fee increase social surplus and decrease total buyer surplus, thereby increasing seller revenue.

An Example

$$N = 2,\ V_i \sim U[0,1],\ Z_i \sim U[1/4, 1/2]$$

(v, ϕ)	Equilibrium Threshold	Seller Revenue	Total Buyer Surplus	Social Surplus
$(0, 0)$	.3571	.06122 (100.00)	.04592 (100.00)	.10714 (100.00)
$(.0972, 0)$	.3295	.07261 (118.60)	.02529 (55.06)	.09790 (91.37)
$(0, .0750)$	.3250	.07500 (122.50)	.02250 (49.00)	.09750 (91.00)
$(0, .1443)$ $(\bar{n} = 1)$	.3557	.09623 (157.16)	.03522 (76.70)	.13145 (122.68)

The entry cap *raises* the expected number of entrants (from .6 to .66).

Market Thickness: Homogeneous Costs

Levin and Smith show that social surplus (and equivalently seller revenue) are decreasing in the number of buyers.

More competition is *bad* for the seller.

We find that social surplus and seller revenue may either increase or decrease with the number of buyers.

▷ Comparative Statics in N slide next.

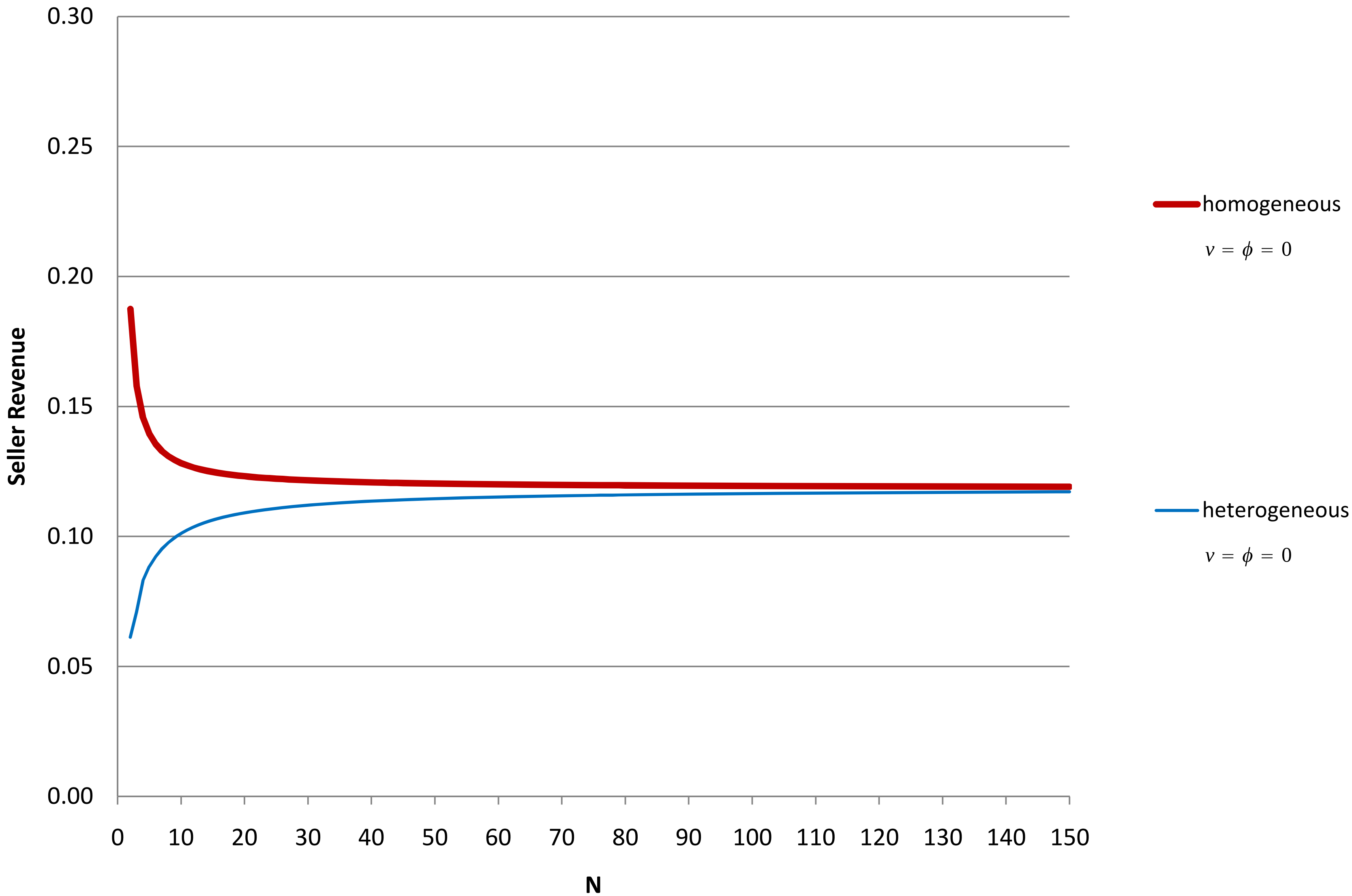

Figure 1: Seller Revenue and the number of bidders

$V_i \sim U[0,1],\ c = 1/4,\ Z_i \sim U[1/4, 1/2]$

Market Thickness: Heterogeneous Costs

Write W_N^* ($\hat{W}_N^*$) for the constrained maximum social surplus when buyers have heterogeneous (homogeneous) entry costs.

Write Π_N^0 ($\hat{\Pi}_N^0$) for seller revenue when $v = \phi = 0$ and buyers have heterogeneous (homogeneous) entry costs.

Prop. 9. *A screening value and an admission fee both equal to zero asymptotically generate the same seller revenue and social surplus whether buyers have homogenous or heterogeneous entry costs, so long as* $c = \underline{c} > 0$; *i.e.,*

$$\lim_{N\to\infty} \Pi_N^0 = \lim_{N\to\infty} W_N^* = \lim_{N\to\infty} \hat{\Pi}_N^0 = \lim_{N\to\infty} \hat{W}_N^* > 0.$$

Hence a screening value and an admission fee equal to zero asymptotically maximize seller revenue when buyers have heterogeneous entry costs.

The prior figure illustrates this result.

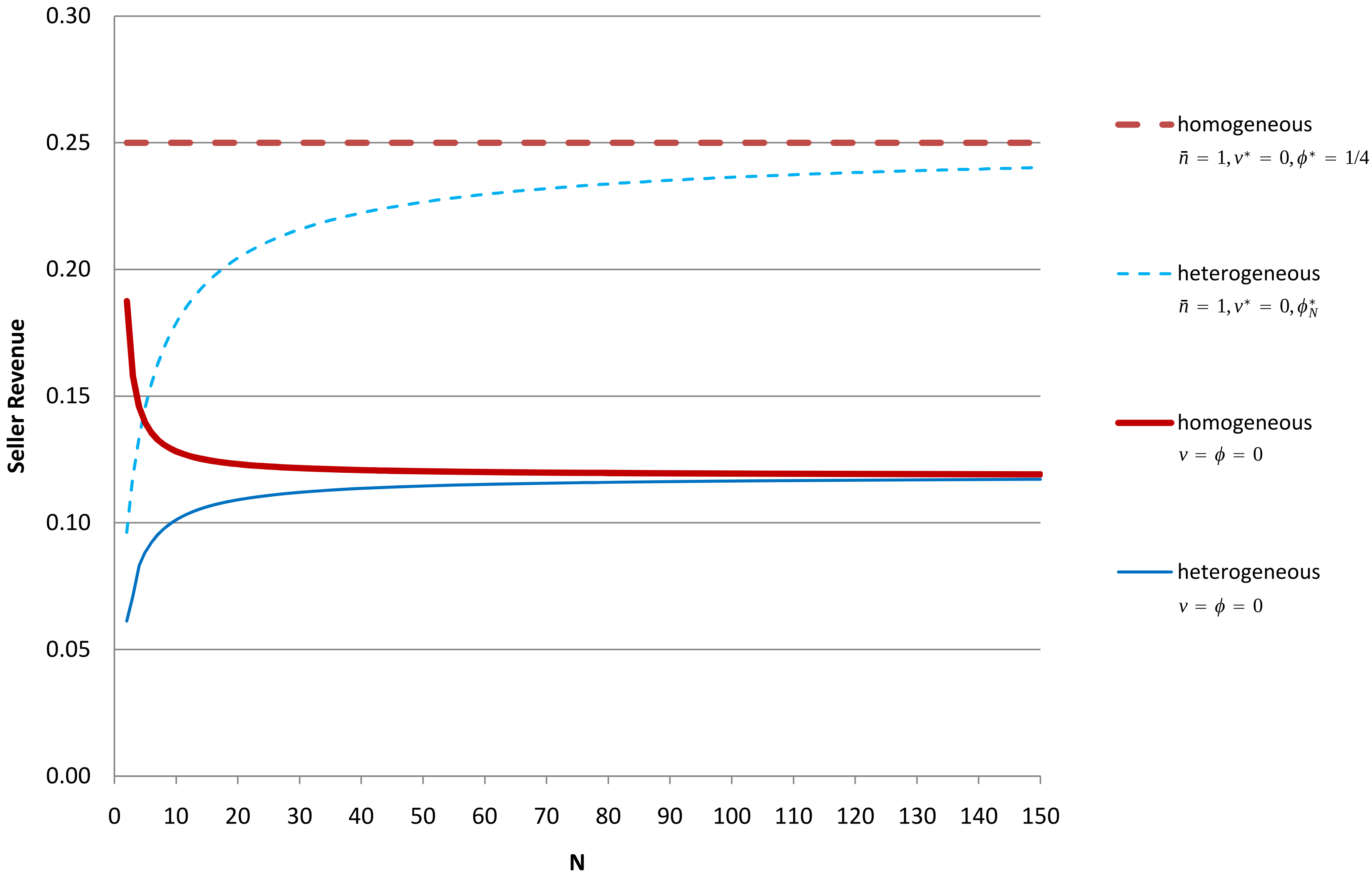

Figure 1: Seller Revenue and the number of bidders

$V_i \sim U[0,1],\ c = 1/4,\ Z_i \sim U[1/4, 1/2]$

Market Thickness: Heterogeneous Costs

Prop. 10. *An entry cap* $\bar{n} = n^* (\underline{c})$ *and a revenue maximizing admission fee and a zero screening value allows the seller to asymptotically capture the unconstrained maximum social surplus,* $s(0, \bar{n}) - \bar{n}\underline{c}$.

Since the unconstrained maximum social surplus exceeds the constrained maximum social, an entry cap is asymptotically advantageous for the seller.

Market Thickness: Heterogeneous Costs

Prop. 11. *If $\underline{c} = 0$ and values are distributed uniformly on $[0, \bar{v}]$, then a screening value and an admission fee both equal to zero asymptotically generate a seller revenue and social surplus equal to $\bar{v}$, i.e.,*

$$\lim_{N \to \infty} \Pi_N^0 = \lim_{N \to \infty} W_N^* = \bar{v}.$$

Hence a screening value and an admission fee equal to zero asymptotically maximize seller revenue.

Summary - Fixed N results

1. A zero screening value maximizes constrained social surplus.
2. The revenue maximizing screening value v^* satisfies $0 < v^* < v^F$.
3. Bidders capture part of the surplus.
4. If an inspection fee is feasible, the optimal admission fee yields a greater seller revenue.
5. An entry cap, combined with an optimal admission fee raises even more revenue for the seller.

Summary - Comparative Statics

1. An increase in the number of buyers may either increase or decrease seller revenue.
2. Heterogeneity of entry costs does "not matter" asymptotically.
3. A zero screening value and a zero admission fee are both asymptotically optimal.
4. With a screening value and admission fee both equal to zero the seller asymptotically captures the entire constrained social surplus.
5. An entry cap the seller can asymptotically captures the entire unconstrained social surplus.
6. An entry cap remains advantageous for the seller even asymptotically.
7. When the lower bound of entry costs is zero and values are uniformly distributed, then a screening value of zero, an admission fee of zero, and no entry cap is asymptotically optimal.