

# Securitized Lending, Asymmetric Information, and Financial Crisis: New Perspectives for Regulation\*

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## Abstract

We develop a model of securitized (Originate, then Distribute) lending in which both publicly observed aggregate shocks to values of securitized loan portfolios, and later asymmetrically observed discernment of the qualities of subsets thereof, play crucial roles, as in the recent paper of Bolton, Santos and Scheinkman (2010). Unlike in their framework, we find that originators and potential buyers of such assets may differ in their preferences over timing of trades, leading to a reduction in the aggregate surplus accruing from securitization. In addition, heterogeneity in agents' selected timing of trades – arising from differences in their ex ante beliefs - coupled with high leverage, may lead to financial crises, implying uncoordinated asset liquidations inconsistent with overall (inter-temporal) market equilibrium. We consider and contrast mitigating regulatory policies, such as leverage restrictions and corresponding ex ante resale price guarantees on securitized asset portfolios. We show that the latter performs strictly better than the former, by ensuring not only bank survival, but also enhancing the social surplus arising from securitized lending, in a better coordinated equilibrium.

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# 1. Introduction

Securitization, via sales of portfolios of long-maturity loans originated by banks to market-based institutions funded using longer maturity liabilities (often aided by implicit governmental support) has been a key part of reality in US as well as other developed financial markets for quite a long time. The presumed benefits arising from such activity are due, in addition to much greater cross-sectional diversification in the resulting portfolios backing securities, to “inter-temporal diversification” owing to which institutions with longer-maturity debt claims are less vulnerable to any (short-term) aggregate shocks impacting on the current market values of assets supporting payoffs on these. Hellwig (1998) was one of the first to emphasize such a role for securitization, in a context of inter-temporal variations in economy-wide interest rates impacting on interim values of long-maturity loan assets, given fixity of originating banks’ short-maturity liability claims, and of the returns (interest rates) on their loans.

However, it was only in the previous decade, of “financial innovation”, that we have witnessed explosive expansion in the securitization of bank-originated lending based on securitization of credit-backed asset portfolios of a far broader quality spectrum, culminating in an even more implosive crash leading to a broad-based financial cum economic crisis, considered to be the worst since the Great Depression of the 1930s. These included credit card debt-based portfolios of varying qualities, and mortgage-backed portfolios with much higher debt to value ratios (also less borrowers’ income information), all subject to potential losses arising from sectoral shocks - plus their “spillovers” into the broader economy - with origins beyond economy-wide interest rate shocks, and their impact on the valuation of payoffs on assets that were largely devoid of default risk, at least in the aggregate. In addition, the financing of various quasi-independent entities providing funding for such securitization was based on quite complex “tranching” of the payoffs arising from the asset/loan portfolios which backed up these liabilities, leading to non-transparency vis-a-vis their risks.<sup>1</sup>

In essence, this phase of rapid expansion of securitization - of at least ostensibly lower risk tranches of portfolios based on bank-originated loans of heterogeneous qualities, and potentially lower average value than at origination - remained still-born, at or just before the near-closure (flow-wise) of these markets in late 2008. As Adrian and Shin (2009a) have noted, the share of Asset Based Securities (ABS) held by intermediaries with high and short-maturity leverage ratios - investment banks, banks and sponsored investment vehicles - was about two-third at the end of 2008, with the remainder held by mutual and pension funds, as well as insurance companies et al.

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<sup>1</sup>When securitized (loan) portfolios, to be sold by their originating agents to others, do contain payoff (default) risks which may be mitigated by better ex ante screening and ex post monitoring by the originators, there is an obvious role for some degree of such tranching of their ex post payoffs. For example, originating agents holding on to their lowest priority (equity) tranches, would serve to better incentivize such screening cum monitoring, while disposing of their higher priority tranches would enable them to divest other risks connected to the future interim market valuations of these assets.

In the process, as securitization markets exploded over 2002-2007 (new issuance sharply slowed over 2007-8, following bad news on some securitized funds), their funding by the investing firms was provided largely via increases in leverage ratios, either directly as with the investment banks, or within “off the book” special purpose entities sponsored by the larger commercial banks, quite often in the form of short-maturity (overnight) Repos.

Subsequently, declines in the market valuations of these underlying asset portfolios, coupled with asymmetric information on their qualities leading to Lemons issues vis-vis mutually acceptable prices, led to collapses in these markets. These in turn led to the possibility - in some cases reality - of Runs on these investing firms, leading to both higher spreads on their Repo rates, as well as enhanced “haircuts”, or margins, on such repo financing. Gorton and Metrick (2009) have documented these crisis-induced phenomena across securities, as well as inter-bank, markets. One of their key findings, elaborated on in Gorton and Metrick (2010), was that post-crisis effects on spreads and haircuts also occurred, albeit to a lesser extent, in securitization markets other than those backed by sub-prime mortgage backed assets, including on credit-card receivables based portfolios. On the other hand, the impact on rates and haircuts was much lower for corporate bonds, which are held largely by investors with either no fixed liabilities, or those of longer maturities. In particular, yield differentials on industrial bonds of differing categories (AAA vs BBB) widened in the financial crisis of 2008-9 to a far lesser extent, than those on banks’ ABS (asset based securities).

These circumstances, and findings, have clearly called for a systematic program of research, on the functioning and potential vulnerabilities of a “market based banking” system, in which banks with specialized expertise originate, package, and distribute portfolios of securities to other financial market participants. In the initial stage of a very rapid expansion of such markets, only a few firms may have had the required expertise to evaluate risks associated with such portfolios, to create tranches of these varying in seniority and risk for sale to the ultimate investors, such as pension funds and insurance firms. During this phase, many securities remained in the portfolios of these specialized entities, investment banks and the sponsored investment vehicles and conduits of large commercial banks. This was associated with large increases in their leverage, often of a short-term nature. The resulting increase in funding for the originated assets was often also associated with increases in the prices of such assets in the short run – Adrian and Shin (2009b) – allowing for easy refinancing of loans made to finance these, so that repayment risks pertinent to their affiliated credit-backed portfolios were difficult to judge (as compared to on corporate bonds), by outside rating agencies as well as by the suppliers of short-term funding to the initial portfolio holders. But, ultimately, when these asset price “bubbles” proved not to be sustainable, the resulting shocks led to values of securities based on loans made to finance such assets collapsing, leading to deleveraging and huge drops in their prices. Shin (2009) provides an outline of such a process of credit expansion and collapse; on pioneering earlier work on this set

of themes, see especially Geanakoplos (2010).

Several recent papers have amplified and elaborated on micro-economic foundations for bank behavior and “systemic risk” - of asset price declines and potential bank failures - in these settings. Acharya, Shin, Yorulmazer (2010), and Stein (2010), have examined this process further, by characterizing banks’ ex ante portfolio choices, over risky long-term loans vs risk-free liquid assets. Liquidity for the purchase of the long-maturity assets of banks, which are sold to service their debts in low return states, is provided by a combination of other banks which have surplus liquidity, as well as by outside investors who are less efficient at realizing value from these assets. Both sets of authors emphasize the externalities on asset prices arising from such inefficient liquidation, that an individual bank may ignore in making its ex ante portfolio choice. Stein focuses on the ostensible liquidity premium (cheaper short-term debt) banks may obtain, with excessive investment in illiquid assets to be sold later at a discount to outside investors in a bad state of nature, whereas Acharya et al emphasize that an originating bank’s full return on long-term assets/loans would not be “pledgeable” to facilitate additional interim financing, to stave off such asset sales in adverse states.

In contrast to these papers, in which an originating institution sells its longer-term assets/loans only in the low individual or aggregate return state, attempting to avert default, Bolton, Santos and Scheikman (2010) develop and analyze another model in which securitization of originated assets to markets is an ongoing, and essential, part of the investment process in longer-maturity and potentially risky assets. The market participants who are potential buyers of these assets *ascribe higher values* to them than their originators do, at least contingent on an aggregate value-reducing shock, which leads the originating institutions to consider selling their assets. Their focus is on endogenizing the timing of these asset sales, by short-run (SR) to long-run (LR) investors, during the time interval following upon such an aggregate shock. Over that period, originators (specialized interim holders) of securitized assets come to know more about their qualities, in terms of prospective future payoffs, of subsets within their holdings. Then, if they had not sold all of their holdings at the start of this stage, their asset market price would come to reflect their incentive to sell only those assets about which they have bad news, or at best no idiosyncratic news beyond the public aggregate shock. Indeed, Bolton et al (hereafter BSS) make a very strong assumption that, for the subset of an SR’s assets on which she has received good news, there is no longer any wedge between their values as perceived by SR vs LR investors. Hence, given that the LR investors face an opportunity cost of holding liquidity to buy such assets, there are no gains to be realized via SR agents trading good assets with LRs.

Building on the last observation, BSS then show that whenever a Delayed trading equilibrium - in which SRs wait until asymmetric information is (thought to be) prevalent, and then sell only their “bad” and “no new information” assets to LRs - does exist, despite a “lemons discount” in its equilibrium market price, it Pareto dominates an Early trading equilibrium, for both SR and

LR agents, in an ex ante sense. It is also associated with relatively higher equilibrium origination of the long-maturity (risky) asset by SR agents, coupled with greater outside liquidity provision by LR investors. Thus, the overall thrust of their conclusions is in sharp contrast with those of Acharya et al (2010), and Stein (2010). In discussing policy implications of their model in a companion paper, BSS (2009), they suggest that when the Delayed trading equilibrium might not exist – owing to the opportunity cost of holding liquid assets for LR agents, coupled with prices reflecting asymmetric information about the qualities of assets to be sold therein - the role of government policy ought to be that of providing a price subsidy to restore its existence, complementing private purchasers.

Despite the richness of its framework, and the elegance of its analysis, these BSS conclusions leave many issues unanswered, and raise other questions. There is, for example, no clear “tipping point” at which a Crisis arises, besides when SR agents discover that there is no delayed trading equilibrium price at which they are willing to trade medium quality assets, about which they have no additional news beyond the initial and public value-reducing aggregate shock.<sup>2</sup> In reality, significant doubts about the sustainability of high and safe (flow) returns on sub-prime mortgage-backed securities arose by mid-2007, while the realization of a financial crisis, with sharply enhanced haircuts and yields, related to credit granted based on such assets, did not materialize until mid-2008. During this long interval, there were also reports of some (investment) banks divesting, or at least curtailing purchases of, mortgage-backed securities, so uniform co-ordination on a (potential) Delayed Trading equilibrium is far from evident. Rather, it suggests to us the possibility of developing differences in opinion among SR agents, about the (medium-term) likelihood of continuation of a benign state for mortgage-backed securities as a whole, leading to their making *differing choices on the timing of trades* in these assets, an outcome infeasible in BSS (2010). Furthermore, the leverage choices made by SR agents who chose not to divest their risky asset portfolios early, plays no role whatsoever in their model.

For these reasons, concerning our beliefs regarding relevant modeling precepts, and our sense that SR agents’ possibly divergent (from 2007 onwards) beliefs, regarding the likelihood of an adverse shock to values of sub-prime mortgage-backed securities as a whole, had an important impact on their choices of timing of trade on the extant holdings thereof, as well as future investments in these, we develop an alternative analysis otherwise in the spirit of the BSS framework. In sharp contrast to them, we assume that the valuation wedge that arises between SR and LR agents, following upon an adverse aggregate shock, applies to *all asset subsets*, irrespective of their heterogeneous qualities as discerned by SRs; Chari et al (2010) assume the same in a

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<sup>2</sup>Indeed, in all of the numerical examples of BSS (2010) in which a Delayed Trading equilibrium does exist - and Pareto dominates the Early trading equilibrium - it is only the LR agents who gain strictly, as a result of incurring lower opportunity costs of providing outside liquidity to SRs. It appears to us to be more than a trifle ironic, to base their theory of financial crises on the *unanticipated* non-existence of the Delayed equilibrium for other parameter values, on the part of SR agents who ostensibly adopt such a trading strategy, despite expecting No strict gains relative to trading earlier!

reputation-based secondary market model.<sup>3</sup> We examine the potential existence of both delayed and early trading equilibria, as in BSS (2010), and agents' preferences over these. We show, in opposition to the BSS conclusions, that LR agents are always worse off in a delayed trading equilibrium whenever it exists, as compared to in the early trading equilibrium for the same exogenous parameters. SR agents, on the other hand, *may be better off* in such a delayed trading equilibrium, but that is the case only if their ex ante prior, regarding the likelihood of the benign aggregate state continuing - the adverse aggregate shock not occurring - is above an interior threshold level. In essence, sufficiently "exuberant" ex ante beliefs are essential for the delayed trading equilibrium to be preferred by (some) SRs. As in BSS (2010), such an SR-preferred delayed trading equilibrium is associated with (weakly) higher investment in the long-term risky asset, and lower (indeed zero) holding of inside liquidity by SRs. However, the overall surplus from asset origination and trading, summed across SRs and LRs, *is strictly lower* in our delayed, as compared to early, trading equilibrium, a result yet again in sharp contrast with the conclusions reached by BSS (2009, 2010).

We then consider, again consistent with our view of empirical reality, a scenario in which a subset of (optimistic/exuberant) agents, who ascribe a lower likelihood to the adverse aggregate shock arising, make their trading and investment choices based on the delayed trading strategy, whereas other SR (as well as LR) agents, who are less optimistic, make their trades immediately, *even before the aggregate shock* has arisen. Such immediate trading plays a key role in our model, unlike in BSS (2010). We use this scenario to sketch a plausible process for a Financial Crisis, in which some "price discovery" from immediate trading by a subset of SR and LR agents serves to provide a basis for Leverage choices of other SR agents, who plan to trade later in a Delayed trading equilibrium, as outlined above. We then show that even small changes in the beliefs of the less optimistic LR agents, hence its impact on their offered immediate trading prices, may lead to (Repo) Runs by the short-term creditors of optimistic SRs, even before an adverse aggregate shock has realized, which is a pre-condition for any type of trading in BSS (2010). The resulting asset sales, by these SR agents who had planned to trade a *proper subset* of their assets in a Delayed equilibrium, leads then to a "market meltdown", prior to a stage in which idiosyncratic asymmetric information about subsets of their held assets has accrued to SRs. The market then collapses, and stays that way. In other words, adverse selection pertinent to delayed trading serves to provide a backdrop for, rather than the immediate triggering mechanism in, a process

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<sup>3</sup>BSS (2010) assume that such a payoff valuation wedge, across SRs and LRs, disappears for subsets of assets discerned (asymmetrically by SR agents) to be of the highest quality. They base this precept on the assumption that the aggregate shock to asset payoffs has absolutely no impact on this subset. To us, this assumption seems more like a notational simplification, rather than a compelling one. As long as even these subsets are subject to some likelihood of paying off less than their maximum levels, conditional on an adverse aggregate shock, outside providers of leveraged financing to SRs who retain such assets would demand equity injections to ensure the safety of their debt, as with asset subsets subject to higher likelihoods of low payoffs. That would, in turn, lower their overall pledgeable value to investors, as in Diamond and Rajan (2000), owing to greater rent extraction by bank (SR) "insiders". Further, under asymmetric information mere retention, chosen by them, *can not* signal quality.

of financial crisis. *Unanticipated non-existence* of an equilibrium plays no role at all.<sup>4</sup>

Our paper is organized as follows. In Section II, we provide an overview of the model in BSS (2010), emphasizing the departure point for our extension of it. Section III deals with our characterization of manifolds of early and delayed trading equilibria in our setting. Section IV develops the implications of mis-coordination - across SRs' trading strategies and leverage choices - for financial crises. In Section V we consider and contrast two key policy interventions: leverage restrictions and guaranteed ex ante resale price supports, both of which can mitigate the impact of such mis-coordination. In Section VI, we conclude, making further comparisons with some recent literature.

## 2. The Model

In this Section we present the “originate and distribute” model, inspired by BSS (2009). In contrast to the model of BSS, where the assets may pay off early, in our model the assets do not pay off until the terminal date. We further demonstrate that this departure from BSS has significant effect on the structure of equilibrium, which has rich implications for understanding the financial crises, as discussed in Section 5.

### 2.1. Outline and motivation for “originate and distribute”

There are four dates,  $t = 0, \dots, 3$ , and two classes of agents, with different investment opportunity sets and intertemporal preferences. Thus, there are potential gains from trade, as outlined in the Introduction and discussed below. The timing and extent of this trade, and the equilibrium consequences on initial portfolio choices and welfare, is the focus of the analysis. Agents place their initial investments at  $t = 0$  and may engage in trade at the early and late interim dates,  $t = 1, 2$ . All assets pay off by  $t = 3$  at the latest.

Short-run (SR) agents are uniquely capable of originating long-maturity risky assets, but ascribe a lower valuation to holding such assets to maturity if the economy is “shocked”<sup>5</sup> than the other set of agents in the model, Long-run (LR) investors. One can think of SR agents as representing banks that are funded with short term liabilities. LR agents can be thought of as pension and other investment funds that are funded with longer-duration liabilities and hence are less concerned with the interim fluctuations of risky assets. As a result, there are

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<sup>4</sup>See also Heider et al (2010) for a model of inter-bank markets, a la Bhattacharya and Gale (1987), which may fail to function due to asymmetric information across banks about the quality of their (collateral) assets. Hellwig (2008) cautions all modellers, of financial crises in a market based banking system, to take into account not just debt and “excessive maturity transformation”, but also other dimensions of what he terms “market malfunctioning”. As an example, he refers to risk-assessment, and ensuing leverage choices, by SR agents predicated on observed price volatility prior to any adverse aggregate shock. Our notion of ex ante leverage choices based on offered - but not taken, by optimistic SR agents - immediate trading prices, is based on the same notion, but amplifies it via linking it to inter-temporal trading strategy choices. That serves to resolve Hellwig’s justified bafflement, regarding the extent of price declines on asset based securities, which defied any reasonable payoff projections.

<sup>5</sup>In the sense of an economy-wide, or non-diversifiable, negative liquidity shock.

potential gains from trade to be had from SRs selling risky assets that they originate on to LRs at one of the interim dates. However, LRs face opportunity costs associated with holding cash, to enable them to buy SR-originated assets. This arises in the form of alternative long-term investments that pay off at  $t = 3$ . These alternative investments have diminishing marginal returns, implying that LRs face increasing marginal costs with respect to holding cash. Trade can also be impaired by adverse selection (Akerlof, 1970) with respect to the quality of SRs assets in a shocked economy. Both sides are aware of the potential trading opportunities that may arise at the interim dates and make their date 0 portfolio choices - over cash and long-term assets - taking these anticipated trades, and the rationally conjectured market equilibrium prices associated with these, into account.

## 2.2. Details and notation

There is a continuum, with measure 1, of each class of agents. All SRs are endowed with one unit of cash, while LRs are endowed with  $K$  units. Cash earns no interest. In addition to holding on to their cash, each agent can invest in a long term asset, depending on their type. The long-term assets available to SRs have uncertain payoffs, while the long-term investments available to LRs have deterministic payoffs. All agents of the same class are symmetric and we focus on symmetric rational expectations equilibria. Denote by  $m \in [0, 1]$  the amount an SR invests in risky assets and by  $M \in [0, K]$  the amount an LR invests in the deterministic long-term asset. Equilibrium levels are denoted by a \* superscript.

*SRs investment opportunity set and preferences:* As shown in the tree depicted in Figure 1, the risky assets available to SRs pay off  $\rho > 1$  with probability  $\lambda$  at  $t = 1$ . Alternatively, the economy is “shocked.” In this case, a risky asset continues until  $t = 2$  whereupon it enters one of three states. In the good (alternatively, bad) state, which occurs with conditional probability of  $q\eta$  (alternatively,  $q - q\eta$ ), the payoff at  $t = 3$  will be  $\rho$  (alternatively, 0). In the neutral state, which thus occurs with conditional probability  $1 - q$ , the payoff at  $t = 3$  is  $\rho$  with conditional probability  $\eta$  or 0 with conditional probability  $1 - \eta$ . The state of an asset held by an SR at  $t = 2$  is her private information. All probabilities are nontrivial:  $\lambda, q, \eta \in (0, 1)$ .

To be clear, at  $t = 1$  all SRs risky assets move in lockstep and the state of the world with respect to these assets is common knowledge. In contrast, if the economy is shocked at  $t = 1$ , risky assets evolve independently of each other at  $t = 2$  and the state of any risky asset held by an SR is then her private information. Since there is a continuum of SRs, there is no aggregate uncertainty. Furthermore, all SRs hold well diversified portfolios of risky assets, meaning that if at  $t = 1$  the economy is shocked then at  $t = 2$  each SR has a deterministic proportion of the risky assets in the good, bad, and neutral states according to the probabilities above. That is, the proportions of good, bad, and neutral assets are given by  $q\eta$ ,  $q - q\eta$ ,  $1 - \eta$ , respectively.

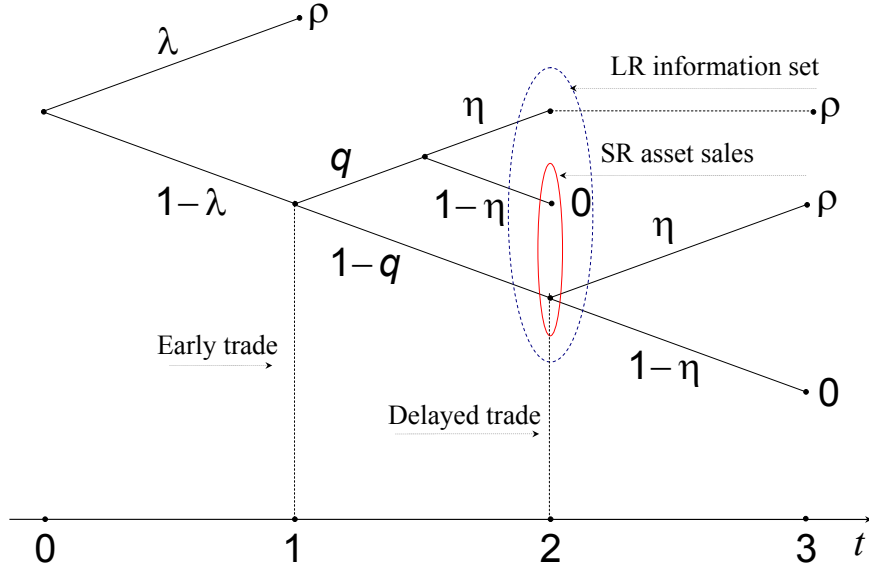


Figure 1: The Time Line of the Events.

SRs seek to maximize

$$\pi_{SR}(C_1, C_2, C_3) = C_1 + C_2 + \delta C_3, \quad (1)$$

where  $C_t$  is an SR's cash flow at date  $t$  and  $\delta \in (0, 1)$ .

*LRs investment opportunity set and preferences:* The long term asset available to LRs has a liquidation value of 0 at  $t = 1, 2$  and a positive payoff at  $t = 3$  determined by  $F(I)$ , where  $I$  is the amount invested. The “production function,”  $F$ , is strictly increasing, strictly concave, and satisfies the Inada conditions. It also has  $F'(K) > 1$ , ensuring that even minute amounts of cash involves an opportunity cost for LRs. In turn, this implies that LRs would only carry cash if they could buy SRs risky assets cheaply (below the actuarially fair value) in some state of the world. LRs seek to maximize

$$\pi_{LR}(C_1, C_2, C_3) = C_1 + C_2 + C_3. \quad (2)$$

*Gains from trade:* The discounting of  $t = 3$  cash flows by SRs, but not LRs, generates potential gains from trade at one of the interim dates.

The actuarially fair value of a unit of the risky asset in the shocked state at  $t = 1$  is  $\eta\rho$ . The model is set up so that this remains the actuarially fair value of the asset in all of the subsequent non-endnodes shown in Figure 1, for example, at  $t = 2$  before information has arrived or if an asset is in the neutral state. However, the value of the risky asset to an SR at any of these nodes is only  $\delta\eta\rho$ .

Note that SRs private information at  $t = 2$  gives rise to a potential adverse selection problem with respect to trading at this date, which could be avoided by trading at  $t = 1$ . The price that will be achieved, though, from trading at either date will have to be determined in equilibrium and will depend on the equilibrium amount of cash carried by LRs.

The (securitization and) selling of the SRs investments in risky assets is central to the model. In particular, it is assumed that

$$\text{A1. } \lambda\rho + (1 - \lambda)\delta\eta\rho < 1.$$

$$\text{A2. } \lambda\rho + (1 - \lambda)\eta\rho > 1.$$

The first assumption (A1) implies that the expected payoff to an SR from holding the risky asset all the way to  $t = 3$  is less than what the SR would get from holding cash. (A2) says that the expected payoff from the risky asset is larger than that of cash, implying that it may be socially optimal for the risky investment to be made (by the assumption that all agents are risk neutral) if they can be transferred to LRs. To generate such trade, it is necessary that LRs opportunity cost of holding cash is not “too large.” The precise condition we assume is stated below [(A3)], after we discuss trading at  $t = 1$  versus  $t = 2$ .

Assumptions (A1) and (A2) that generate the originate and sell (securitize) feature of the model also constrain  $\lambda$  to be in an interval

$$(\lambda_d, \lambda_u) \equiv \left( \frac{1 - \eta\rho}{(1 - \eta)\rho}, \frac{1 - \delta\eta\rho}{(1 - \delta\eta)\rho} \right). \quad (3)$$

*Early versus delayed trade:* Denote the quantity of risky assets and the price per unit an SR sells at  $t = 1$  (early trade) by  $X_e$  and  $P_e$ , respectively. The corresponding notation for trade at  $t = 2$  (delayed trade) is  $X_d$  and  $P_d$ . Given this notation, an SR’s expected payoff can be written

$$\pi_{SR} = m + \lambda(1 - m)\rho + (1 - \lambda)\{X_e P_e + X_d P_d\} + \delta(1 - m - X_e - X_d)E[\tilde{\rho}_3|\Phi], \quad (4)$$

where  $E[\tilde{\rho}_3|\Phi]$  is the per unit expected payoff to the risky assets the SR holds to  $t = 3$  given the expected characteristics of these,  $\Phi$ . Due to the adverse selection problem at time  $t = 2$  the expected characteristics  $\Phi$  of assets traded at time  $t = 2$  depend on second period price  $P_d$ . In particular, if this price is too low then only lemons are traded and hence the expected payoff is zero.

Private information and an associated lemons problem at  $t = 2$  gives rise to the possibility that an SR would hold on to her good assets. If so, (4) becomes

$$\pi_{SR} = m + \lambda(1 - m)\rho + (1 - \lambda)\{X_e P_e + (1 - m - X_e)[(1 - q\eta)P_d + q\eta\delta\rho]\}. \quad (5)$$

In this case, an SR prefers trading early if and only if  $P_e \geq (1 - q\eta)P_d + q\eta\delta\rho$  (all agents are “small,” in the sense that they do not (believe they) influence market prices).

Given a preference for early trading ( $P_d$  is sufficiently low), an SR would invest in the risky asset at  $t = 0$  only if  $P_e(1 - \lambda) + \rho\lambda \geq 1$ . Equality of these terms is required for the SR to hold both cash and the risky asset. Given (5) and a preference for delayed trading ( $P_e$  is sufficiently low), an SR would invest in the risky asset at  $t = 0$  only if  $[P_d(1 - q\eta) + q\eta\delta\rho](1 - \lambda) + \rho\lambda \geq 1$ .

Our analysis in subsequent sections focuses on early versus delayed trading equilibria, where SRs invest in risky assets and, if the economy is shocked, trades either at  $t = 1$  or  $t = 2$  (with  $\delta$  being sufficiently large that trade is subject to adverse selection at  $t = 2$ , i.e., only bad and neutral risky assets would be sold). Since, conditional on a public liquidity shock, there is no aggregate uncertainty and holding cash entails an opportunity cost, it is clear that in equilibrium, if the economy is shocked, all of an LR's cash holdings,  $M$ , will be used to buy risky assets. Thus, in a conjectured early trading equilibrium (where all trade after a shock occurs at  $t = 1$ ),  $X_e = M/P_e$  and so the expected payoff to an LR is:

$$\Pi_{LR} = F(K - M) + \lambda M + (1 - \lambda) \frac{M}{P_e} \eta \rho. \quad (6)$$

The LR optimizes by choosing  $M$  to satisfy the first order condition:

$$F'(K - M_e^*) = \lambda + (1 - \lambda) \frac{\eta \rho}{P_e}. \quad (7)$$

This simply says that the marginal cost to an LR of holding cash must equal the marginal return. The optimal cash holding,  $M^*$ , is strictly positive if  $F'(K)$  is sufficiently small:

$$\text{A3. } F'(K) < \lambda + \frac{(1 - \lambda)^2 \eta \rho}{1 - \lambda \rho}.$$

Assumption (A3) will guarantee the existence of an early trading rational expectations equilibrium.

Similarly, if a delayed trading equilibrium with price  $P_d$  exists, and hence SRs at  $t = 2$  trade not only "lemons" but also neutral assets, the expected payoff of LR agents is given by:

$$\Pi_{LR} = F(K - M) + \lambda M + (1 - \lambda) \frac{1 - q}{1 - q\eta} \frac{M}{P_d} \eta \rho, \quad (8)$$

where  $(1 - q)/(1 - q\eta)$  is the probability of buying a neutral asset, conditional on the fact that both bad and neutral assets are traded at  $t = 2$ . Accordingly, an LR's first order condition in delayed trading equilibrium is given by:

$$F'(K - M_d^*) = \lambda + (1 - \lambda) \frac{(1 - q) \eta \rho}{1 - q\eta} \frac{1}{P_d}. \quad (9)$$

The asset prices are then determined from market clearing conditions that equate the demand and supply of assets at times  $t = 1$  and  $t = 2$ .

### 2.3. Comparison with BSS (2010)

The model captures that SRs (banks) may generate liquidity at an interim date by selling long-term risky assets, but there may be a cost due to adverse selection when they most need liquidity. SRs can potentially avoid adverse selection costs by selling at the early interim date, rather than the late interim date, before asymmetric information develops. However, this may have other costs, since it is costly for LRs to carry cash (by way of an opportunity cost arising from foregone alternative investments in illiquid long-term assets). Since trade at the early interim date may involve a larger portion of SRs risky assets being sold, early trade may be socially inferior to late trade. Thus, there is a potential tradeoff between trading early versus late that relates to a tradeoff between adverse selection and demand-side liquidity costs.

In their setup, BSS show that whenever early and delayed trading equilibria coexist, the delayed trading equilibrium is socially superior. In our setup, this is not the case. Indeed, we will argue below that the delayed trading equilibrium lacks robustness. This dramatic difference in our conclusions and therefore also in our respective interpretations of what constitutes a crisis, and how to respond to it, has its source in our assumption that if the economy is shocked at  $t = 1$ , SRs risky assets do not pay off before  $t = 3$ . In contrast, BSS assume that there is a chance that risky assets can pay off early (i.e. become perfectly liquid). Specifically, they assume that a risky asset pays off  $\rho$  at  $t = 2$  if it is in the good state. In our setup, the payoff of  $\rho$  will not occur immediately, but at  $t = 3$ .

This seemingly minor difference goes directly to the tradeoff between adverse selection versus liquidity costs that is at the heart of the model. In the BSS setup, there is no ex ante adverse selection cost, since the lemons discount to SRs in the neutral state is simply offset by the premium received by SRs in the bad state. Their analysis and results on early versus delayed trading are therefore dominated by LRs' costs of carrying cash.

In contrast, in our setup we allow for the possibility of adverse selection at  $t = 2$  giving rise to a deadweight cost ex ante, namely the loss from delayed cash flows from risky assets that are in the good state at  $t = 2$ . Thus, our setup allows for a benefit from early trading, before adverse selection arises. In our analysis, we will trace out how this affects the results. It turns out that the impact is significant and leads to an alternative view of crises.

## 3. Early vs Delayed Equilibrium: Descriptions and Comparisons

In this Section we proceed to describe both early and delayed trading equilibrium, and characterize the conditions under which one or the other should be expected to arise, depending on agents' preferences over these. Furthermore, we also highlight the differences from the structure of our equilibria with those in BSS (2010) and provide further insights on the key characteristics of equilibria and their robustness. It is in the characterization of delayed trading equilibrium

that the difference between our setup and theirs emerges in a stark way. We show that, unlike in their model, even if a delayed trading equilibrium exists in ours, it is never preferred to the early trading equilibrium by both SR and LR agents, even weakly.

### 3.1. Early Trading Equilibrium

The existence of early trading equilibrium can be demonstrated along the lines of BSS, since the timing of risky assets payoff in the good state at  $t = 2$  does not influence early price  $P_e$ , and since  $P_d$  in the early trading equilibrium is just chosen to guarantee the absence of coincidence of SRs and LRs wanting to delay trading.<sup>6</sup> Consequently, our characterization of early trading equilibria as functions of the probability of good economic state,  $\lambda$ , is essentially the same as in BSS (2010) and is summarized in the following Proposition 1:

**Proposition 1. (Bolton et al).** *For all  $\lambda$  in  $[\lambda_d, \lambda_u)$ , an early trading equilibrium exists, with unit trading prices  $P_e$ , and liquidity holding levels  $\{m, M_e^*\}$  satisfying:*

- (i) *For  $\lambda < \lambda_c$ ,  $m^* > 0$ ,  $P_e(\lambda) = \frac{1 - \lambda\rho}{1 - \lambda}$ ,  $M_e^* = (1 - m^*)P_e$ , satisfying equation (7);*
- (ii) *For  $\lambda_c \leq \lambda < \lambda_u$ ,  $m^* = 0$ , and  $M_e^* = P_e(\lambda)$ , again satisfying equation (7).*

Proposition 1 reveals that there are two types of early trading equilibria: (i) mixed portfolio equilibria, where SRs hold both cash and risky assets, and (ii) corner equilibria, where SRs' cash holdings are 0. This characterization of early trading equilibria involves two segments for probability  $\lambda$ , separated by boundary probability  $\lambda_c$ , in the first of which  $m^* > 0$  for SRs, and in the second of which  $m^* = 0$ , implying  $M_e^* = P_e$ . Interestingly, the early price in Proposition 1 implies that for  $\lambda \in [\lambda_d; \lambda_c]$  SRs expected surplus is  $\pi_{SR} = 1$ . As is clear, in a mixed equilibrium (when probability  $\lambda$  is sufficiently small) all of any strictly positive surplus, resulting from the origination of long-maturity assets by SRs, accrues only to LRs. In contrast, if  $\lambda_c < \lambda < \lambda_u$ , the economy is in a corner equilibrium in which SRs pocket some of the surplus.

Next, we turn to deriving the comparative statics for LRs' early trading equilibrium cash holdings  $M_e^*$  and expected payoffs  $\Pi_{LR}$  as functions of the probability of good economic state,  $\lambda$ . The following Corollary 1 reports the results.

**Corollary 1.** *LR's equilibrium cash holding  $M_e^*(\lambda)$  and expected payoff  $\Pi_{LR}(\lambda)$  are strictly increasing in  $\lambda$  for all  $\lambda \in [\lambda_d, \lambda_c)$ , and strictly decreasing in  $\lambda$  for  $\lambda \in (\lambda_c, \lambda_u)$ .*

**Proof:** see Appendix.

The co-movement of the unit asset prices  $P_e(\lambda)$ , and LR money holdings  $M_e^*(\lambda)$ , across the set of early trading equilibria when  $\lambda$  is in  $[\lambda_d, \lambda_c)$ , may well be thought of as the inverse of “cash in the market pricing” (see Shin (2009) for its exposition) in that unit asset prices, and external

<sup>6</sup>This requires delayed price  $P_d$  to be chosen sufficiently small, so that SRs prefer trading at  $t = 1$ .

(LR) liquidity holdings held in the anticipation of buying these assets following on an aggregate shock to their value, move in opposite directions as a function  $(1 - \lambda)$ , the probability of such a shock. The reason, of course, is that  $m^*$  decreases, hence the quantity of the long-maturity asset supplied by SRs,  $(1 - m^*)$ , increases strictly in  $\lambda$ , i.e., as the probability of the adverse aggregate shock decreases. However, SRs gain nothing from that enhanced surplus!

### 3.2. Delayed Trading Equilibrium

In this Subsection we explore the nature of delayed trading equilibria in our economy and demonstrate that they are substantially different from those in BSS (2010). In contrast to BSS (2010), it turns out that there exists no set of commonly conjectured prices  $\{P_e, P_d\}$  such that both the sellers (SRs) and the buyers (LRs) would prefer delayed over early trading, even weakly. Consequently, we characterize delayed trading equilibria in a setting where SRs decide the timing of trades. Specifically, a delayed trading equilibrium arises when SRs prefer delaying trading, and hence only deliver the asset to the market at their preferred date  $t = 2$  irrespective of LR preferences. Anticipating such a strategy of SRs, LR investor have no other choice but to trade in a delayed equilibrium.

Before we proceed further, we rule out an uninteresting case of pooling delayed trading equilibria, where SRs sell all of their assets regardless of type, by assuming that the discount parameter  $\delta$  is such that:

$$\text{A4. } \delta > \eta.$$

Indeed, on one hand, the delayed equilibrium price  $P_d$  cannot exceed the actuarially fair value of  $\eta\rho$  for LR to be willing to buy. On the other hand, the value of good assets to an SR is  $\delta\rho$ , if he does not sell them. Consequently, assumption (A4) guarantees that  $\delta\rho > P_d$ , and hence SRs strictly prefer not to sell any good assets in equilibrium. Thus, our focus, as in Bolton et al, is on non-trivial delayed equilibria, where both neutral and bad assets are sold. SRs are willing to sell their neutral assets provided

$$P_d \geq \eta\rho\delta. \tag{10}$$

This condition is also necessary to get investment in the risky asset in the first place.

We now demonstrate why BSS delayed equilibria with both SR and LR agents preferring to trade at  $t = 2$  break down in our modification of BSS economy. Let  $P_1$  be the  $t = 1$  price in a delayed equilibrium, so that SRs prefer to trade at  $t = 2$ . SRs' objective function in (5) implies that trading at date  $t = 2$  will be preferred whenever price  $P_1$  is sufficiently low, so that the following inequality is satisfied:

$$P_1 < q\eta\rho\delta + (1 - q\eta)P_d. \tag{11}$$

Similarly, the LR's objective function implies that LR's prefer to trade at  $t = 2$  if their expected return from trading at  $t = 2$ , conditional on both neutral and bad assets being traded at  $t = 2$ ,

exceeds the expected return from an early trade. Similarly to BSS (2010) this leads to the following condition:

$$\frac{(1-q)\eta\rho}{(1-q\eta)P_d} \geq \frac{\eta\rho}{P_1}, \quad (12)$$

where  $(1-q)/(1-q\eta)$  is the conditional probability of buying a neutral asset at  $t = 2$  given that inequality (10) is satisfied, and hence both bad and neutral assets are traded at  $t = 2$ . It can easily be verified that inequalities (10)–(12) cannot hold simultaneously, and hence, there is no delayed equilibrium in which LR agents would prefer to trade at  $t = 2$ . Indeed, the last inequality implies that  $(1-q)P_1 \geq (1-q\eta)P_d$ , which in conjunction with (11) yields  $P_1 < \eta\rho\delta$ . The two latter inequalities  $(1-q)P_1 \geq (1-q\eta)P_d$  and  $P_1 < \eta\rho\delta$  then jointly imply that  $P_d < \eta\rho\delta$ , which contradicts inequality (10) guaranteeing that neutral assets are traded at  $t = 2$ . Thus, we have proven the following Lemma.

**Lemma 1.** *In a delayed trading equilibrium (where  $P_1$  is sufficiently low, so that SRs prefer trading at  $t = 2$ ), an LR would actually prefer trading early as this would earn her a higher rate of return.*

This opposing preferences for the *timing of trades* is a significant departure, in terms of result, from BSS (2010). It is driven by our assumption that after the economy experiences a liquidity shock, assets that turn out to be good yet do not become perfectly liquid in a sense that good assets at  $t = 2$  do not pay off before  $t = 3$ . In contrast, in BSS (2010) there is a range of examples, involving SRs choosing strictly positive money holdings  $m^* > 0$  in both early and delayed trading equilibrium, and thus being indifferent vis-a-vis their payoffs across the two, in which the LR agents strictly prefer to trade late, benefiting from being able to buy a subset of a greater quantity of SR investment in the long-maturity assets in the delayed equilibrium, with lower money holdings  $M_d^*$ .

Given Lemma 1 above, the only case in which a delayed trading equilibrium could arise in our setup is one where SR agents perceive that they will be strictly better off in such an equilibrium, as compared to an early trading equilibrium. As a result, they withhold their supply of the long-maturity asset from its market, until it is common belief that they have asymmetric information about subsets of their portfolio, and would only be selling their average and bad quality assets. In general, such a delayed equilibrium will be supported by a wide range of prices  $P_1$  satisfying inequality (11). However, it is reasonable to consider only refined equilibria where  $P_1$  coincides with an early trading equilibrium price, which reflects SRs belief that the deviation from a delayed equilibrium strategy will result in an early trading equilibrium outcome. The following Lemma allows us to impose further restrictions on the set of plausible delayed trading equilibria.

**Lemma 2.** *SRs would never strictly prefer a Delayed trading equilibrium in which  $m^* > 0$ , over any early trading equilibrium. Such a delayed equilibrium would also make LR agents strictly worse off than in early trading - unlike as in BSS (2010).*

Lemma 2 can easily be established by simply comparing the expected payoffs across the two equilibria. An important implication of this Lemma is that it prompts us to look only for delayed equilibria which entail  $m^* = 0$  for SRs, since otherwise SRs will be better off by switching to early equilibria. For example, consider a set of parameters such that an early trading equilibrium, described in Proposition 1 above, entails money holdings  $m^* > 0$  by SR agents, whereas delayed equilibrium entails  $m^* = 0$  for SRs. As noted in the discussion following Proposition 1, SR agents' payoff in such an early equilibrium would be equal to  $\pi_{SR} = 1$ , and hence be no more than if she had invested only in the liquid asset, setting  $m = 1$ . In contrast, in a delayed equilibrium with  $m^* = 0$ , in which SRs invest all of their endowment in the long-maturity asset, their expected payoff from so doing,  $[\lambda\rho + (1 - \lambda)\{q\eta\delta\rho + (1 - q\eta)P_d\}]$ , must necessarily strictly exceed the unit payoff from just holding the liquid asset, despite gains from trade given up (to the detriment of LR agents' payoffs) by SRs planning not to trade their better quality asset subsets.

Next, we derive necessary and sufficient conditions for the existence of a delayed trading equilibrium with  $m^* = 0$  in which SRs expect to get price  $P_e(\lambda) = (1 - \lambda\rho)/(1 - \lambda)$  (price in early trading equilibrium with  $m^* > 0$ ) if they deviate to an early trade. SRs would strictly prefer to trade in this delayed trading equilibrium, as compared to any early equilibrium involving  $m^* > 0$ .

We proceed in two steps. First, we obtain an economically intuitive necessary condition under which a non-trivial delayed trading equilibrium could conceivably exist. Then, we strengthen this condition by deriving necessary and sufficient conditions for the existence of a delayed trading equilibrium. Furthermore, we provide tractable exogenous bounds on the sets of model parameters under which there exists a delayed trading equilibrium with desired properties.

In any non-trivial delayed equilibrium with  $P_d \geq \delta\eta\rho$ , SRs would only trade a proportion  $(1 - q\eta)$  of their long-maturity assets about which they get either bad or neutral news. To buy these assets at the market clearing price  $P_d$ , LR investors would have to hold  $M_d = (1 - q\eta)P_d$  in liquid assets, on which they obtain the expected return of  $[\lambda + (1 - \lambda)(1 - q)\eta\rho/(1 - q\eta)P_d]$ . From LR's optimization we then obtain the following first order condition for the optimal choice of  $M_d$  in liquid assets:

$$F'(K - M_d) = \lambda + (1 - \lambda) \frac{(1 - q)\eta\rho}{(1 - q\eta)P_d} > 1. \quad (13)$$

Combining the above inequality with the non-triviality condition  $P_d \geq \delta\eta\rho$ , we see that for any  $\lambda$  it must be true that:

$$\delta < \frac{1 - q}{1 - q\eta} < 1. \quad (14)$$

In addition, a consistent equilibrium price  $P_d$  must be such that SR agents strictly prefer to trade in the delayed equilibrium, rather than coordinating on an early one:

$$P_e(\lambda) = \frac{1 - \lambda\rho}{1 - \lambda} \leq q\eta\delta\rho + (1 - q\eta)P_d(\lambda), \quad (15)$$

where we have assumed that  $\lambda < \lambda_c$ , so that the early trading equilibrium entails  $m^* > 0$  (see Proposition 1). Combining the conditions (14) and (15) above, we can derive the following

Lemma which gives a necessary condition for the existence of a delayed trading equilibrium with  $m^* = 0$ :

**Lemma 3.** *Define the “social surplus” per unit of the SR-created long-maturity asset,*

$$S(\lambda) = [\lambda\rho + (1 - \lambda)\eta\rho - 1]. \quad (16)$$

*A necessary condition for the existence of a delayed trading equilibrium with  $m^* = 0$  is*

$$S(\lambda) \geq (1 - \lambda)q^2 \frac{1 - \eta}{1 - q\eta} \eta\rho. \quad (17)$$

**Proof:** see Appendix.

Under the maintained hypothesis that  $\lambda < \lambda_c$ , this necessary condition creates the possibility of a lower bound  $\lambda_*$ ,  $0 < \lambda_* < \lambda_c$ , such that the selected equilibrium would entail early trading for all  $\lambda < \lambda_*$ , and delayed trading for  $\lambda > \lambda_*$ . Such direct dependence of the SR-selected timing of trading, and hence implied equilibrium investment  $(1 - m^*)$  in the SR-originated long-maturity asset, is absent in the BSS (2010) paper. In contrast to our paper, in BSS (2010) framework a delayed trading equilibrium is Pareto preferred by both agent types, albeit weakly by SRs if  $m^* > 0$ . The results of Lemma 3 are further strengthened in Proposition 2 below, which provides both necessary and sufficient conditions for the existence of a delayed trading equilibrium with  $m^* = 0$ , assuming  $\lambda < \lambda_c$  so that SRs expect to trade at a price  $P_e(\lambda) = (1 - \lambda\rho)/(1 - \lambda)$  if they deviate and trade early (see Proposition 1).

**Proposition 2.** *Condition (17) above, together with the condition in inequality (19) below, are necessary and sufficient for the existence of a delayed trading equilibrium in which  $m^*$ , the liquid asset holdings of the selling SR agents, equals zero. Defining:*

$$P_{\min} = \frac{P_e(\lambda)}{1 + q(1 - \eta)}, \quad (18)$$

$$F'(K - (1 - q\eta)P_{\min}) < \left[ \lambda + (1 - \lambda) \frac{(1 - q)\eta\rho}{(1 - q\eta)P_{\min}} \right]. \quad (19)$$

*Moreover, there exist upper and lower bounds on  $\delta$ , given by:*

$$\delta^*(\lambda) = \frac{x}{\eta\rho}, \quad \delta_*(\lambda) = \max\left\{ \frac{x}{\rho}, \frac{P_e(\lambda) - (1 - q\eta)x}{q\eta\rho} \right\}, \quad (20)$$

*where  $x$  solves a nonlinear equation*

$$F'(K - (1 - q\eta)x) = \lambda + (1 - \lambda) \frac{\eta\rho(1 - q)}{(1 - q\eta)x}, \quad (21)$$

*such that for all pairs  $\{\lambda, \delta\} \in \left\{ \{\lambda, \delta\} : \delta_*(\lambda) \leq \delta \leq \delta^*(\lambda) \right\}$  there exists a unique delayed equilibrium with  $m^* = 0$  and price  $P_d = x \geq P_{\min}$  which SRs prefer to an early equilibrium with price  $P_e(\lambda)$ . Furthermore, the length of the equilibrium existence interval on  $\delta$  satisfies the following inequality:*

$$\delta^*(\lambda) - \delta_*(\lambda) < \min\left\{ 1 - \eta, \frac{1 - q}{q} \right\}. \quad (22)$$

**Proof:** See the Appendix.

Proposition 2 establishes necessary and sufficient conditions for the existence of a unique delayed trading equilibrium and provides a tractable characterization of the equilibrium existence regions. It also establishes a lower bound on the equilibrium price  $P_d$ , given by (18), which guarantees that the equilibrium price is high enough to induce SRs to choose to trade late and supply not only the lemons but also average quality assets. The existence region is characterized in terms of upper and lower bounds (20) on the discount parameter  $\delta$ . Intuitively, on one hand, parameter  $\delta$  should be sufficiently high to induce the SRs to trade at  $t = 2$ , so that they get higher expected discounted payoff after receiving good news after  $t = 1$ . On the other hand, it cannot be too high since otherwise  $P_d \geq \delta\eta\rho$  is violated and hence only lemons are traded in the market. Consequently, the equilibrium exists only for  $\delta$  in the medium range, bounded by some  $\delta_*$  and  $\delta^*$ .

The results of Proposition 2 indicate that the bounds on parameter  $\delta$  become tighter as  $\eta$  or  $q$  increases. To understand the intuition we note that as  $\eta$  increases a good outcome becomes more likely in the no-news state at  $t = 2$ . Therefore, for the delayed trade to be an equilibrium outcome, SRs with no news should be more impatient to be willing to sell the asset at time  $t = 2$ . Consequently, the upper bound  $\delta^*$  should decrease leading to the shrinkage of the interval for  $\delta$  supporting the delayed equilibrium. Furthermore, the interval for  $\delta$  shrinks as  $q$  increases. The reason is that higher  $q$  makes the no-news state less likely, increasing the proportion of lemons traded at  $t = 2$ . Consequently, price  $P_d$  decreases, and the no-news SRs should be more impatient (as measured by their  $\delta$ ) to sell assets at  $t = 2$ , and hence  $\delta^*$  should decrease reaching zero in the limit.

From the results of Proposition 2 it can additionally be demonstrated that SRs prefer a delayed equilibrium with  $m_d^* = 0$  to an early one with  $m_e^* = 0$  or  $m_e^* > 0$ , so that  $\pi_d \geq \pi_e$ , where expected payoffs  $\pi_d$  and  $\pi_e$  are given by:

$$\pi_d = \lambda\rho + (1 - \lambda)(q\eta\rho\delta + (1 - q\eta)P_d), \quad (23)$$

$$\pi_e = m_e^* + (1 - m_e^*)(\lambda\rho + (1 - \lambda)P_e). \quad (24)$$

Consequently, the SRs choose to trade late, enforcing the delayed equilibrium.

To facilitate the numerical analysis, from Proposition 1 we observe that the early equilibrium price  $P_e$  (required for the construction of bounds  $\delta_*$  and  $\delta^*$ ) can conveniently be written as follows:

$$P_e(\lambda) = \max\left\{\frac{1 - \lambda\rho}{1 - \lambda}, y\right\}, \quad (25)$$

where  $y$  solves a nonlinear equation:

$$F'(K - y) = \left\{\lambda + (1 - \lambda)\frac{\eta\rho}{y}\right\}. \quad (26)$$

Indeed, it follows from Proposition 1 that:

$$P_e(\lambda) = \begin{cases} \frac{1 - \lambda\rho}{1 - \lambda}, & \text{if } m_e^* > 0, \\ y(\lambda), & \text{if } m_e^* = 0. \end{cases} \quad (27)$$

Moreover, from Proposition 1,  $M_e^* < P_e$  when  $m_e^* > 0$ , and hence from the first order condition (6) and concavity of function  $F(\cdot)$  it follows that  $F'(K - P_e) \geq \lambda + (1 - \lambda)\eta\rho/P_e$ . Consequently, in the early equilibrium with  $m_e^* > 0$  it can easily be demonstrated that  $P_e = (1 - \lambda\rho)/(1 - \lambda) \geq y$ , giving rise to expression (25). Expressions (20) for the bounds  $\delta_*$  and  $\delta^*$  along with expression (25) for the price in the early equilibrium allow for an efficient numerical computation of the existence regions for delayed and early equilibria, which we describe in the next subsection.

### 3.3. Numerical Analysis

In this subsection we numerically explore the existence regions for different equilibria in  $\{\lambda, \delta\}$ -space, aggregate welfare across different equilibria and other relevant economic parameters. In particular, we are interested in the regions where the delayed equilibrium with  $m_d^* = 0$  coexists with early equilibrium with either  $m_e^* > 0$  or  $m_e^* = 0$ . Our construction of these regions is based on the bounds for discount parameter  $\delta$  derived in Proposition 2. In addition to bounds  $\delta_*$  and  $\delta^*$ , we also note that assumption (A1) imposes the following upper bound on  $\delta$ :

$$\delta \leq \bar{\delta}(\lambda) = \frac{1 - \lambda\rho}{1 - \lambda} \frac{1}{\eta\rho}. \quad (28)$$

From the results of Proposition 1 we note that (28) along with assumption (A3) are enough to guarantee the existence of an early equilibrium.

For our numerical analysis we pick the following specification for LR investment technology, satisfying all the conditions in Section 2:

$$F(I) = \frac{K^{1-\alpha} I^\alpha}{\alpha}, \quad (29)$$

where  $\alpha \in (0, 1)$ . Given the concavity of  $F(\cdot)$  it can easily be demonstrated that the nonlinear equations (21) and (26) have unique solutions  $x$  and  $y$  in terms of which the early  $P_e$  and delayed  $P_d$  equilibrium prices are derived. We calculate  $x$  and  $y$  numerically, and by substituting them into expressions (20) obtain the upper and lower bounds for  $\delta$  as functions of  $\lambda$ .

The characterization of the existence regions in Proposition 2 allows us to calculate the lower bound  $\lambda_*$  for the benign state probability  $\lambda$ , such that the delayed equilibrium with  $m^* = 0$  exists (for some  $\delta$ ) whenever  $\lambda \geq \lambda_*$ . The discussion in Proposition 2 implies that  $\lambda_*$  can be obtained as a solution to equation  $\delta_*(\lambda_*) = \delta^*(\lambda_*)$ . Similarly, the expression for the early equilibrium price  $P_e$  in (25) can be employed to characterize the “switching point”  $\lambda_c$ , introduced in Proposition 1, which separates early equilibria with  $m^* > 0$  (when  $\lambda < \lambda_c$ ) and early equilibria with  $m^* = 0$

(when  $\lambda \geq \lambda_c$ ). In particular, it can easily be demonstrated that parameters  $\lambda_*$  and  $\lambda_c$  solve the following equations:

$$\begin{aligned} x(\lambda_*) &= \frac{P_e(\lambda_*)}{1 + q(1 - \eta)}, \\ y(\lambda_c) &= \frac{1 - \lambda_c \rho}{1 - \lambda_c}, \end{aligned} \tag{30}$$

where  $x$  and  $y$  in turn solve equations (21) and (26), and price  $P_e(\lambda)$  is given by (25).

Figure 2 shows the existence regions for delayed and early equilibria in  $\{\lambda, \delta\}$ -space. For the numerical calculations we use the following set of parameters:  $K = 2$ ,  $\rho = 1.2$ ,  $\eta = 1/\rho$ ,  $q = 0.3$ , and  $\alpha = 0.75$ . The existence region for the delayed equilibrium with  $m_d^* = 0$  is the region bounded from above by  $\delta^*(\lambda)$  and  $\bar{\delta}(\lambda)$  and from below by  $\delta_*(\lambda)$ . The early equilibrium exists for all parameters  $\lambda$  and  $\delta$  such that  $\delta \leq \bar{\delta}(\lambda)$ , and  $\lambda_c$  separates the equilibria with  $m_e^* > 0$  (when  $\lambda < \lambda_c$ ) and the equilibria with  $m_e^* = 0$  (when  $\lambda \geq \lambda_c$ ).

The numerical calculations demonstrate that the existence regions for the delayed and early equilibria overlap, and  $\lambda_* < \lambda_c$ . Moreover, bounds  $\delta_*(\lambda)$  and  $\delta^*(\lambda)$  turn out to be decreasing functions of the good state probability  $\lambda$ . To explain this result, we note that the delayed price  $P_d = x$ , where  $x$  solves equation (21), is a decreasing function of  $\lambda$ , which can be established by differentiating equation (21) and showing that  $\partial x / \partial \lambda < 0$ . Intuitively, as probability  $\lambda$  increases, a bad shock at  $t = 1$  becomes less likely. Since the SRs trade only conditional on observing the bad state at  $t = 1$ , ex ante at  $t = 0$  the probability of trade after  $t = 0$  goes down. Therefore, LRs face higher opportunity cost of holding liquidity  $M$ , preferring to invest more in their riskless technology. Consequently, conditional on a bad shock at  $t = 1$ , SRs will face lower demand for their assets both in early and delayed equilibria, and hence, the delayed and early prices are decreasing functions of  $\lambda$ , which translates into decreasing bounds for  $\delta$ .

The delayed equilibrium coexists with the early equilibrium with  $m_e^* > 0$  when  $\lambda \in [\lambda_*, \lambda_c]$  and with the early equilibrium with  $m_e^* = 0$  when  $\lambda \geq \lambda_c$ . The size of  $[\lambda_*, \lambda_c]$  interval depends on the curvature of the technology function,  $-F''(I)I/F'(I) = 1 - \alpha$ , parameterized by  $\alpha$ . To investigate the sensitivity of the size of this region with respect to  $\alpha$  we numerically calculate  $\lambda_*$  and  $\lambda_c$  as functions of  $\alpha$ . Figure 3 presents the results of the calculations and demonstrates that the size of the region decreases as parameter  $\alpha$  goes up.

Figure 2: **Existence Regions for Early and Delayed Equilibria.**

This Figure shows the existence regions for early and delayed trading equilibria for parameters  $K = 2$ ,  $\rho = 1.2$ ,  $\eta = 1/\rho$ ,  $q = 0.3$ , and  $\alpha = 0.75$ . The delayed equilibrium with  $m^* = 0$  exists for all  $\{\lambda, \delta\}$  such that  $\delta_* \leq \delta \leq \delta^*$ ,  $\lambda_* \leq \lambda \leq 1/\rho$ . The early equilibrium with  $m^* > 0$  exists for all  $\{\lambda, \delta\}$  such that  $\delta \leq \delta_e^*$  and  $0 < \lambda \leq \lambda_c$ , and the early equilibrium with  $m^* > 0$  exists for all  $\{\lambda, \delta\}$  such that  $\delta \leq \delta_e^*$  and  $\lambda_c \leq \lambda \leq 1/\rho$ .

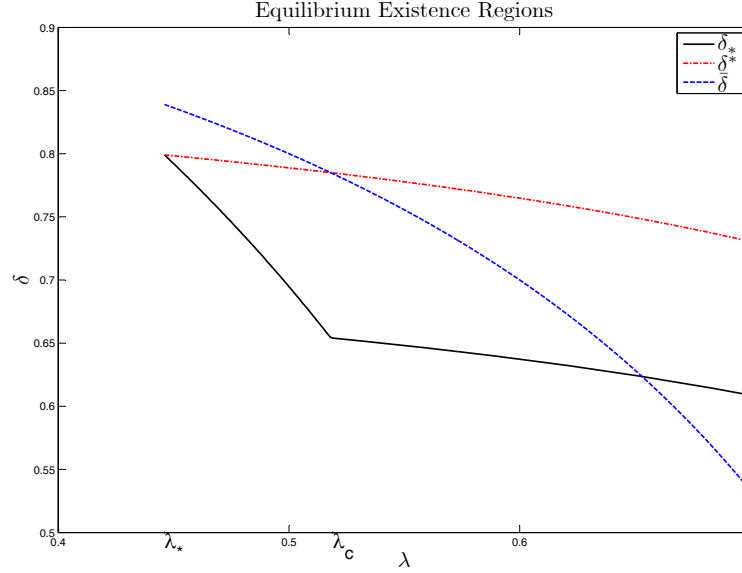
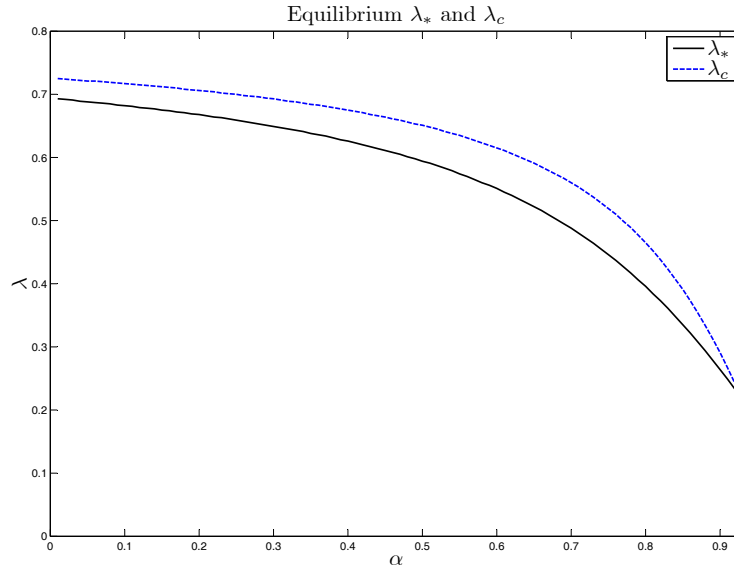


Figure 3: **Equilibrium  $\lambda_*$  and  $\lambda_c$  as Functions of Curvature Parameter  $\alpha$ .**

This Figure plots parameters  $\lambda_*$  and  $\lambda_c$  as functions of curvature parameter  $\alpha$  for parameters  $K = 2$ ,  $\rho = 1.2$ ,  $\eta = 1/\rho$ ,  $q = 0.3$ . Delayed equilibria with  $m^* = 0$  and early equilibria with  $m^* > 0$  coexist if  $\lambda_* < \lambda < \lambda_c$ , while delayed equilibria with  $m^* = 0$  and early equilibria with  $m^* = 0$  coexist if  $\lambda_c \leq \lambda \leq 1/\rho$ .



We now investigate the welfare implications of our analysis. Given the significant overlap of the existence regions it becomes important to compare the aggregate welfare across different equilibria. We quantify the aggregate welfare by an expected total payoff defined as the sum of the expected payoffs of LRs and SRs, denoted by  $\Pi$  and  $\pi$ , respectively. The expected payoffs of SRs are given by expressions (23) and (24) whereas for LRs the expected payoffs in delayed and early equilibria take the following form:

$$\Pi_d = F(K - M_d) + \lambda M_d + (1 - \lambda) \frac{M}{P_d} \frac{1 - q}{1 - q\eta} \eta \rho, \quad (31)$$

$$\Pi_e = F(K - M_e) + \lambda M_e + (1 - \lambda) \frac{M}{P_e} \eta \rho. \quad (32)$$

Figure 3 shows aggregate welfare in delayed and early equilibria for the model parameters  $K = 2$ ,  $\rho = 1.2$ ,  $\eta = 1/\rho$ ,  $q = 0.3$ ,  $\alpha = 0.75$ , and  $\delta = 0.74$ .

The aggregate welfare functions are increasing in probability  $\lambda$ . Moreover, the aggregate welfare in the early equilibrium exceeds that in the delayed equilibrium for each parameter  $\lambda$ . To understand the economic intuition we note that SRs have higher expected payoff in the delayed equilibrium with  $m_d^* = 0$  than in the early equilibrium. However, according to Lemma 2 they do not have strict preference for a delayed equilibrium with  $m_d^* > 0$  over an early equilibrium. Therefore, in the neighborhood of  $\lambda_*$  their welfare will be almost unchanged by switching from a delayed to an early equilibrium. Furthermore, according to Lemma 2 LRs are always strictly better off in the early trading equilibrium, and hence, at least in the neighborhood of  $\lambda_*$  the aggregate welfare should be higher in the early equilibrium.

The fact that the aggregate welfare is higher in the early equilibrium can also rigorously be demonstrated for  $\lambda \geq \lambda_*$ . For the ease of exposition we demonstrate this only for the case when  $m_e^* = 0$ . In this case, from the expressions for SR and LR expected payoffs in (23), (24), (31), and (31), as well as market clearing conditions  $M_e = P_e$  and  $M_d = (1 - q\eta)P_d$  it follows that:

$$\Pi_d + \pi_d = \lambda \rho + (1 - \lambda) \eta \rho (1 - q + q\delta) + F(K - M_d) + M_d, \quad (33)$$

$$\Pi_e + \pi_e = \lambda \rho + (1 - \lambda) \eta \rho + F(K - M_e) + M_e. \quad (34)$$

Comparing the expressions in (33) and (34) we observe that the sufficient condition for the aggregate welfare to be higher in early equilibrium is that  $F(K - M_e) + M_e > F(K - M_d) + M_d$ , or equivalently, given that  $F(K - M) + M$  is an increasing function, the sufficient condition becomes  $M_e > M_d$ . To prove that  $M_e > M_d$  we note that the first order condition for  $M_d$  in (13) along with the market clearing condition  $M_d = (1 - q\eta)P_d$  imply the following inequality:

$$F(K - M_d) < \lambda + (1 - \lambda) \frac{\eta \rho}{M_d}. \quad (35)$$

From the comparison of the above equation with the first order condition for  $M_e$  in (7) and the properties of technology function  $F(\cdot)$  it can easily be demonstrated that  $M_e > M_d$ , and hence the aggregate welfare is higher in an early equilibrium.

Figure 4: **Aggregate Welfare Across Early and Delayed Equilibria.**

This Figure shows the aggregate welfare in delayed and early equilibria for parameters  $K = 2$ ,  $\rho = 1.2$ ,  $\eta = 1/\rho$ ,  $q = 0.3$ , and  $\delta = 0.74$ .  $\Pi_e + \pi_e$  is the aggregate welfare of LR and SRs in early equilibrium while  $\Pi_d + \pi_d$  is the aggregate welfare of LR and SRs in delayed equilibrium.

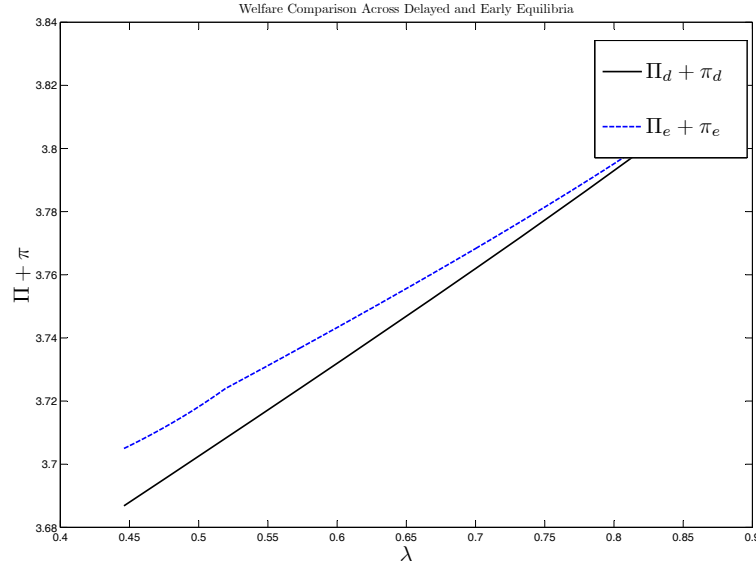
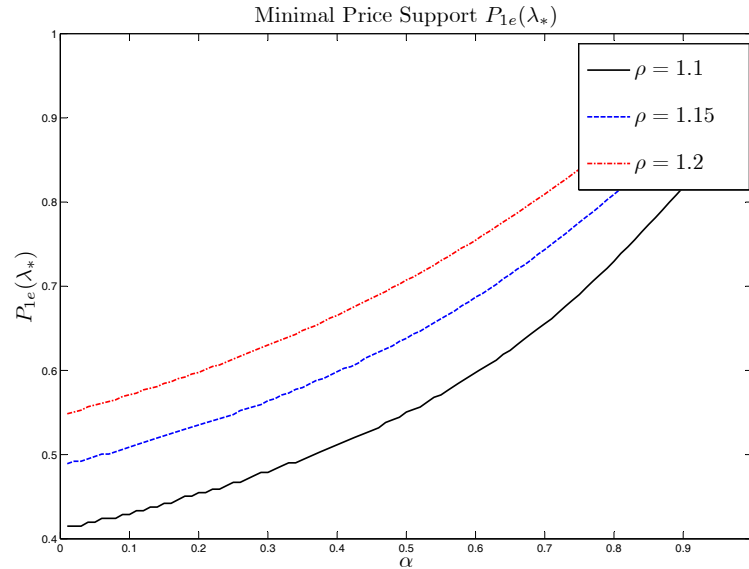


Figure 5: **Price Support  $P_e(\lambda_*)$  as Function of Curvature Parameter  $\alpha$ .**

This Figure shows the price support function  $P_e(\lambda_*)$  for parameters  $K = 2$ ,  $\rho = 1.2$ ,  $\eta = 1/\rho$ ,  $q = 0.3$ .



Finally, Figure 5 shows  $P_e(\lambda_*)$  as a function of the parameter  $\alpha$  for different levels of return  $\rho$ , while the other parameters are as for the previous graphs. It turns out that this function is an increasing function of the parameter  $\alpha$ , as well as the return  $\rho$ . As demonstrated in the subsequent part of the paper,  $P_e(\lambda_*)$  can be thought of as a government intervention price that induces the SRs to switch to early trading equilibrium.

## 4. Strategy-Proofness, Immediate Trading, and Exuberant Pri- ors

We now further explore the concept of delayed equilibrium introduced in Section 3. First, we discuss the real-life evidence in support of the existence of delayed trades, and then address the question of the strategy proofness of delayed trading equilibria. Specifically, we demonstrate that these equilibria are not strategy-proof in a sense that there exist Pareto improving offers by LRs that would induce SRs to switch to an early trade. Next, we reconcile the evidence in favor of the existence of delayed trading with the non-strategy-proofness of delayed trading equilibria.

This reconciliation is achieved by introducing a realistic modification of our model in which agents are additionally allowed to trade immediately at the initial date  $t = 0$ , and SRs can potentially disagree on the probability of benign economic states,  $\lambda$ . We provide an example which demonstrates that such a heterogeneity of beliefs results in market segmentation in which some agents trade immediately at time  $t = 0$  and some at  $t = 2$ , consistently with the anecdotal accounts of recent financial crisis. Finally, we remark that immediate trades play no role in BSS (2010). Consequently, the possibility of immediate trading in conjunction with non-strategy-proofness of delayed trading equilibria even further highlights the difference in economic implications between our model and that of BSS (2010).

### 4.1. Immediate Trading

As we noted in the Introduction, in early 2008, even after some aggregate valuation shocks to the mortgage backed securities market had occurred, in the opinion of the majority of participants, highly levered institutions such as banks and investment banks continued to hold nearly two-thirds in value of these assets on balance sheets. This suggests strongly that the SR agents were not coordinating their planned trading of these assets with LR agents (such as insurance firms and pension funds) on an early trading equilibrium. At the same time, it also appears to be the case that such LR agents had acquired quite significant (nearly one third by value) proportional stakes in such assets, or tranches, from SR originators over the period 2002-2007, before the fully perceived aggregate shocks in housing market were revealed. It was not until mid-2007 that these shocks lead to value declines and risk recognition on mortgage baked securities, culminating in significant lowering of credit ratings on these securities. Since these trades occurred before the

realizations of aggregate shocks, the period of asset acquisitions in 2002-2007 most naturally maps into date  $t = 0$  of our model. Accordingly, we label these acquisitions as *immediate trades*.

The immediate trading plays no role in the BSS (2010) model. Indeed, they note that it is strictly sub-optimal for SRs and LRs to engage in such trades in their setting. Their reasoning is simple: relative to an Early trading equilibrium, immediate trading, at a set of prices satisfying  $\Pi(\lambda) = [\lambda\rho + (1 - \lambda)P_e]$  for SRs to be indifferent between trading at  $t = 0$  and  $t = 1$ , would simply serve to make LR agents worse off, by having to hold a strictly higher amount of liquidity  $M_d(\lambda) > M_e(\lambda)$ . As a result, any immediate trading equilibrium would result in strictly lower origination of the tradable asset by SRs, leading to a (weakly) Pareto inferior outcome. A similar argument applies vis-a-vis comparing a delayed vs an immediate trading equilibria in BSS (2010) model, in which delayed trading equilibrium outcomes Pareto dominate those arising from early trading, when the former exists.

## 4.2. Strategy-Proofness and Exuberant Priors

The latter argument no longer applies, vis-a-vis a delayed trading equilibrium, in the modified setting of our model and, as a result, there is a clear possibility for a role for immediate trading in our setup. However, at least given heterogeneous prior beliefs regarding the likelihood of an (adverse) aggregate valuation shock across agents, not all SR agents would choose to engage in immediate trading either, leading to the possibility of “*segmented markets*”, in which more optimistic SR agents, along with LR agents with higher marginal liquidity holding costs, would wait to trade assets in a delayed trading equilibrium instead. The reason such a possibility arises in our setting is the following. Unlike in the BSS model, in which their delayed trading equilibrium exhausts all feasible gains from trade across SR and LR agents, and hence is Pareto-preferred by them to the early trading equilibrium, in our modified setup LR agents would have strictly preferred trading early instead. Indeed, essentially because of this feature of our analysis, it is easily shown that, being faced with the prospect of engaging in delayed equilibrium trade, the LR agent could make herself and her SR trading partner better off at the margin by making an offer to buy an unit of the latter’s assets early, at time  $t = 1$ , as well as initially at time  $t = 0$ . In other words, our delayed trading equilibrium notion is simply not “*strategy-proof*”. The following Lemma formalizes our intuition.

**Lemma 4.** *Given a delayed trading equilibrium price  $P_d$ , there is always an early trade price offer by an LR of  $P_o > q\eta\delta\rho + (1 - q\eta)P_d$  - that makes both her and her SR trading partner strictly better off, via exchanging an unit of the asset at this price.*

**Proof:** see Appendix.

At first sight, the lack of strategy proofness of our delayed equilibrium may lead to the conclusion that the only valid competitive price-taking equilibrium outcomes in our setup could

be those which are associated with some early trading equilibrium. We take a more eclectic view, by bringing into the consideration the possibility of Immediate trading offers, based on the same idea as in Lemma 4 above. We then show, via an extended example, that if the LR agents' offers are based on a lower estimate of  $\lambda$  than that of a subset of SR agents, then the latter may not find it profitable to sell their asset immediately, as compared to waiting to trade these at their (conjectured) delayed trading equilibrium price  $P_d$ . What this example does not accomplish, however, is the task of full integration of immediate trading based on bilateral offers by some SR and LR agents, with others trading in a delayed, and price-taking equilibrium (with a Lemons discount in prices) later on.

*Example:* Consider a scenario where  $\rho = 1.20$ ,  $\eta\rho = 1$ ,  $\alpha = 0.87$ ,  $\delta = 0.84$ ,  $q = 0.3$ , and  $P_d$  is such that  $q\eta\delta\rho + (1 - q\eta)P_d$  is between 0.892 and 0.9. LR agents, and some SRs as well, believe that the ex ante probability of the benign state continuing is  $\lambda_p = 0.35$ , whereas as other “exuberant” SR agents believe that it is  $\lambda_o = 0.45$ . Both beliefs are consistent with the conjecture that SR agents would prefer to trade in a price-taking Delayed equilibrium over an Early trading one, as  $P_e(\lambda_p) = [1 - 1.2 \times 0.35 / (1 - 0.35)] = 0.892 < 0.9$ . Suppose that LR agents are willing to offer SR agents the equivalent of an early trading price of  $P_o = 0.92$  in their immediate offers, amounting to offers of  $\Pi = 0.35 \times 1.2 + 0.65 \times 0.92 = 1.02$ . The exuberant SR agents would prefer not to sell immediately at this price, as they conjecture that if they wait and then trade in a Delayed equilibrium, at the price  $P_d$ , if and when the aggregate shock would occur, they would obtain the ex ante (at  $t = 0$ ) expected payoff of  $0.55 \times 1.20 + 0.45 \times 0.892 = 1.03 > 1.02$ , their offered immediate trading price. This gives rise to a market segmentation whereby assets are traded at both  $t = 0, 2$ . Indeed, one may think of the post aggregate but pre idiosyncratic private information state  $t = 1$ , as a conceptual rather than a “real time” state, when trading is carried out.

## 5. Implications for Financial Crises

In this Section, building on the insights developed in the previous Sections, we provide a discussion of financial crises and the design of regulatory policies. First, we point out the importance of leverage for the funding of securitization during the recent crisis and discuss supporting evidence. Then, we enrich our tractable example of the role of exuberance in Section 4.2 by incorporating SR agents which are leveraged via repo contracts and choose the level of leverage based on the immediate trading prices at  $t = 0$ . In the context of a simple numerical illustration we discuss how the reassessment of the probabilities of economic shocks by initially exuberant agents may unexpectedly decrease immediate trading prices, leading to a run by repo holders. In such a run all agents rush to trade at  $t = 0$  at sub-optimal prices. This trading at sub-optimal prices we use as a definition of crises and explore potential ways of preventing them via policy regulation.

## 5.1. Excessive Leverage and Crises

It is well known that the explosion of securitization of (potentially) lower quality and riskier (more heterogeneous) loan products, especially over the years of 2002-7, was funded with much higher (compared to that in other banking activities) and short-term uninsured debt, in the form *repo* financing for example. It is also commonly accepted that market doubts about the qualities of the underlying loans, which started accruing from late 2006 onwards, had negative implications for the valuation of even higher grade securities (tranches). Eventually, this accrual of doubts resulted in significant downgrades by credit rating agencies starting in early 2007, after which both the rates paid and “haircuts” (margin requirements) demanded on the repo financing increased, as documented in Gorton and Metrick (2009, 2010). However, this process was slow in the beginning. In particular, while sales of new securities based on newly originated (by now realized to be lower-grade) loans, to be funded via a lower extent of repo finance, essentially ceased by mid-2007, the process of higher haircuts and rates on such repo financing crept upwards from mid-2007 until the first quarter of 2008, before accelerating to basically full-fledged systemic bank/repo runs during the summer of 2008.

In a magisterial review of the consequences and possible causes of the great financial crisis of 2008, Hellwig (2008) suggests that “we must distinguish between the contribution to systemic risk that came from excessive maturity transformation through SIVs and Conduits (used by levered banks to park their holdings of securitized products) and the contribution to systemic risk that came from the interplay of market malfunctioning, fair value accounting, and the insufficiency of bank equity”. What precisely did he mean by “market malfunctioning”? To us, based in part on our reading of Hellwig (2008), “market malfunctioning” might imply at least the two following aspects of agents’ behavior and its consequences for the “unforeseen nature” of some of the market trades. The first, which Hellwig (2010) discusses under the heading of “excessive confidence in quantitative models”, could account for basing leverage choices on currently prevailing levels of the immediate trading prices of the assets, in a time such as that captured in our example above, with a cushion for potential adverse shocks to prices based largely on very recent historical changes (volatilities) in these.

For instance, in the context of our Example above, an “exuberant” SR who intends not to sell her asset immediately, at the offered price of 1.02, may take on a leverage level of 0.95 per unit of the asset, even if she reckons that - contingent on the aggregate shock (fully) realizing - her overall delayed payoff (supportable with a mix of asset sales, and lower leverage on the good subset held on to), is only 0.892. Implicit in such behavior is her belief or hubris that she would have the capability to sell enough of her asset, prior to an aggregate shock fully manifesting itself in its immediate trading price, to reduce her leverage ratio to 0.892, from 0.95. As Rajan (2010) remarks, even Charlie Prince of Citicorp, to whom the by now notorious statement about “keeping on dancing as long as the music is playing” is attributed, expressed a caveat regarding

what might happen “if liquidity dried up” in secondary markets for securitized assets that Citi was holding onto, including those it had carried on the books of its SIVs and Conduits, with implicit promises of Citi supporting their debt liabilities via equity injections, if required. This brings us to what we believe is a second important dimension of market malfunctioning and its interactions with debt.

Suppose that, as say over the last two quarters of 2007, the less exuberant LR agents had lowered their estimated likelihood of the benign aggregate state continuing, from 0.35 to 0.107, so that their maximal price offer for immediate trading now decreased to  $0.107 \times 1.2 + 0.893 \times 0.92 = 0.95$ , in the context of our Example above. As soon as that happens, repo holders of an SR who had taken on the leverage level of 0.95 would start a Run, taking the immediate trading price as the maximal liquidation value of the SR they had funded, as in the model of He and Xiong (2009).<sup>7</sup> If sufficiently many SRs, with similar leverage levels as well as absence of inside liquidity  $m^*$  to finance such withdrawals, then try to sell ALL of their assets immediately, that would lower market prices, ultimately to a level equalling  $\delta\eta\rho$ , at which point the secondary market will collapse, and remain so into period  $t = 2$ , when asymmetric information about qualities of offered assets takes hold. The reason is, of course, that the liquidity available from LR agents for buying these assets equals at most - because some had bought the asset earlier in immediate trading from less exuberant SRs - the level required to support a price level of  $P_d$ , for a volume/measure  $(1 - q\eta)$  units of assets to be sold in Delayed trading. It would thus not suffice even to support the price level of  $P_e(\lambda_*)$  in the Run.

Note that, when one introduces the possibility of endogenous leverage as above, one must specify what one means by the aggregate availability of funds in the SR Sector. One specification is that this amount (normalized to unity above) represents the maximum amount of equity and debt funds this sector can raise in the aggregate, with any overall mix of debt and equity that it chooses. The other interpretation, as noted by Rajan (2010) and others, is one that arises from “global imbalances” leading to much savings being available in economies without sophisticated financial markets, which clamors for (even ostensibly) “safe” investment products, thereby enhancing the debt funding capacity of institutions such as the SRs in our model, which are then constrained - given a pre-specified level of equity capital - only via some acceptable maximum (possibly regulated) leverage ratio, vis-a-vis aggregate funding capacity. In our interpretation, this distinction may be of importance, in terms of regulatory policy.

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<sup>7</sup>If on the other hand the book Leverage Ratio chosen by Optimistic SRs were 0.99, when Offered immediate Price was 1.02 - market debt to equity ratios of 33 were observed in 2007-8 - then relatively Pessimistic LR’s lambda beliefs would only have to decline from 0.35 to 0.25, in order to cause Repo Runs by the short-term debt holders of the more optimistic SRs, as  $0.25 \times 1.2 + 0.75 \times 0.92 = 0.99$  (see He and Xiong (2009)).

## 5.2. Regulatory Policies

Two major regulatory policy interventions that are natural to consider in our setting are *minimum capital*, or equivalently *maximum leverage*, ratio restrictions, as well as minimum asset price Guarantees, possibly coupled with restrictions on Liquidity ratios  $m^*$  ex ante. Let us first consider the former. One may set a maximum leverage ratio on investments in the risky technology to be  $P_e(\lambda_*) = q\eta\delta\rho + (1 - q\eta)P_d(\lambda_*)$ , in which  $\lambda_*$  represents, as before, the switch point above which SR agents would prefer the delayed over the early trading equilibrium. Without much detailed knowledge of the LR agents' opportunity cost function for providing liquidity to the asset market, or  $F(I)$ , this would not be an easy policy to implement: doing so based on the lowest  $\lambda$  that satisfies our necessary condition in Lemma 3 above, will result in a too generous leverage ratio which may result in runs as above. Even if a regulator has the informational capacity to calculate  $\lambda_*$ , there are still two potential difficulties. For  $\lambda > \lambda_*$ ,  $P_d(\lambda)$  in Delayed trading equilibrium with  $m^* = 0$  would be decreasing in  $\lambda$ , as with a set of Early trading equilibrium with  $m^* = 0$ , for  $\lambda > \lambda_c > \lambda_*$ , as described in Proposition 1 and its Corollary above. On the other hand, if the regulatory maximum leverage ratio is set at the level of say  $P_e(\lambda_u)$ , that may be overly restrictive in terms of decreasing the pledgeable value arising from securitizing assets. In our Example above, that would mean decreasing the maximum allowed leverage ratio from say 0.815 to  $[(1 - 0.6 \times 1.2)/(1 - 0.6)] = 0.70$ , which is not required.

In any event, a maximal leverage ratio will not prevent the planned trading strategies of at least a subset of SR agents – all assigning probabilities  $\lambda > \lambda_*$  to the aggregate adverse shock not occurring – being delayed trading coupled with no inside liquidity holding (setting  $m = 0$ ), since there would not be enough external liquidity to absorb so much of risky assets in Immediate or early equilibrium trading. As we have noted above, such delayed trading - in our model, as opposed to that of BSS (2010) - would lead to part of the feasible gains from trade between SR and LR agents being unrealized, decreasing (in our numerical simulations) payoffs aggregated over them.

An alternative regulatory tool, analyzed in the BSS paper, would be for the regulator – with access to fiscal or monetary powers - to provide a minimum price guarantee on the asset. The purpose of such a guarantee in our setting would be the opposite of what it is in the BSS setting, which is to support a delayed trading equilibrium when private liquidity provision by LR agents, in the face of the lemons discount in pricing given adverse selection, is insufficient to support it. Our price guarantee will apply to only Early trading, at a level  $P_e(\lambda_*)$ , or for Immediate trading at an unit price. For  $\lambda < \lambda_*$ , no SR agent would (strictly) prefer to sell to the regulator at these prices. Instead, they would invest and trade as in the BSS Early trading equilibrium, with  $m^* > 0$ , or implement an immediate trading equilibrium with a higher level of  $m^*$ . However, in these states, they would not be constrained by (overly restrictive, see above) leverage regulations. Indeed, now even when LR agents would believe that  $\lambda > \lambda_*$ , they would plan to sell their assets

early, and thus the possibility of Runs of the sort we outlined above would not arise; LR agents would have no reason to make immediate trading offers that are more advantageous to SRs. However, SR agents would invest - given the simple linear-in-payoffs expected utility functions we have assumed - all of their funding capacity in the long-term asset, setting  $m^* = 0$ . As a result, some of the sales of assets originated by SR agents would indeed be to the regulator/government, since private external liquidity provided by LR agents would not suffice to support such a price for this volume of asset sales. If the regulator's marginal cost of providing such liquidity is no lower than that of LR agents at the hypothetical Early equilibrium for these levels of  $\lambda$ , the regulator may seek to couple such a price support with minimum liquidity ratios. A tight one would be at  $m^*(\lambda_*)$  and a looser one would be at  $m^*(\lambda_u)$  provided the latter is indeed non-zero, i.e.,  $\lambda_u < \lambda_c$  in Proposition 1 above. In setting the level of this minimum Liquidity ratio, optimal regulatory policy would thus need to trade off governmental (deadweight or opportunity) costs of providing asset price support, for excessive asset origination at levels of  $\lambda$  in the neighborhood of  $\lambda_*$ , as against constraining such asset origination excessively when  $\lambda$  is closer to  $\lambda_u \gg \lambda_*$ .

Maximum leverage ratios could, however, lead to additional advantageous effects via another, a "general equilibrium" channel, if one assumes that levels of equity capital available to SR institutions is limited, and hence leverage regulations would serve to reduce the overall funding capacity of such agents, relative to that of LR agents. In that case, conditions may be created such that we would have Early trading outcomes in which we would have  $m^* = 0$  for lower levels of  $\lambda_c$ , possibly lower than  $\lambda_*$ , such that SR agents would thereby obtain an interior share of the surplus created by asset origination in Early trading equilibrium as well, for all  $\lambda$  in  $(\lambda_*, \lambda_u]$ . In that event, they would be less likely to be tempted to carry out trading in a Delayed equilibrium, thus eliminating the possibility of excessive asset and leverage creation, and Runs.

Finally, we should note another recent contribution, by Diamond and Rajan (2010), which also has the feature that banks - funded with short-maturity demandable liabilities - may trade their assets late rather than earlier, even if earlier trading would have preserved their solvency. In their model, such a phenomenon arises owing to decisions being made by (in the interest of) the levered equity holders of banks, who realize that - given the pattern of anticipated prices in the secondary market for their assets - they would be unable to fully repay their depositors even after asset sales in future, contingent on a "liquidity shock" leading a fraction of these to withdraw earlier. These choices give rise to the possibility of (avoidable) Runs in the high interim liquidity-demand state. They also discuss alternative regulatory interventions, including their impact on banks' chosen trading strategies.

## 6. Conclusion

In this paper we have developed a model with asymmetrically informed short-run originators and long-run buyers of financial assets, inspired by BSS (2010). In contrast to the latter paper,

we assume different timing of asset payoffs. In particular, while in our model all the payoffs are paid at the terminal date, in BSS (2010) the assets may pay at the intermediate date. We have provided an exhaustive characterization of early and delayed trading equilibria, and derived necessary and sufficient conditions for their existence. We have also argued that the difference in the structure of payments between our paper and BSS (2010) translates into dramatically different economic implications. In particular, in contrast to findings in BSS (2010) we find that the delayed trading equilibria are not robust to Pareto-improving early trades. We have shown that delayed trades can only be rationalized in an extended economy with immediate trades at  $t = 0$  and exuberant investors. Finally, our analysis yields new insights on financial crises and policy regulation. Specifically, we have discussed how a crisis can originate within a setting with SR agents leveraged via repo contracts, when the reassessment of the probability of aggregate shocks by exuberant agents leads to a run resulting in sub-optimal prices.

## Appendix: Proofs

**Proof of Corollary 1.** Consider LR's expected return on money holding  $M$  as a function of  $\lambda$ ,  $R(\lambda)$ :

$$R(\lambda) = \lambda + (1 - \lambda) \frac{\eta\rho}{P_e(\lambda)}, \quad (\text{a.1})$$

which in the segment  $\lambda$  in  $[\lambda_d, \lambda_c)$ , in part (i) of the Proposition, implies that:

$$R(\lambda) = \lambda + \eta\rho \frac{(1 - \lambda)^2}{1 - \lambda\rho}. \quad (\text{a.2})$$

It is straightforward to show, via differentiation, that the right hand side of (a.2) is strictly increasing in  $\lambda$ , which using LR's optimality condition (7) yields the result.<sup>8</sup>

For the second part of the corollary, concerning the region  $[\lambda_c, \lambda_u)$  in which money holdings  $m^*$  of SRs equals zero, so that  $M_e^*(\lambda) = P_e(\lambda)$ , suppose to the contrary that  $M_e^*(\lambda)$ , and thus  $P_e(\lambda)$  are (weakly) increasing in  $\lambda$ . But, then  $R(\lambda)$  would be strictly decreasing in  $\lambda$ , which would contradict LRs' optimality condition, in equation (7).

Finally, the statement that LRs' overall expected payoff is increasing (vs decreasing) in  $\lambda$  whenever  $M_e^*(\lambda)$  is increasing (vs decreasing) in  $\lambda$ , is implied by the axioms of Revealed Preference, applied to LR's objective function, described in equation (2). *Q.E.D.*

**Proof of Lemma 3:** Conditions (15) and  $P_d \geq \delta\eta\rho$ , required for a delayed equilibrium, together imply

$$S \geq (1 - \lambda)[(1 - \delta)\eta\rho - q(1 - \eta)\delta\eta\rho]. \quad (\text{a.3})$$

Which, upon substitution for  $\{\delta, (1 - \delta)\}$  from the inequality (14), implies inequality (17). *Q.E.D.*

**Proof of Lemma 4.** Given a  $P_d$ , the commonly held conjecture of SR and LR agents about Delayed trading equilibrium price, at the margin an LR agents would be indifferent between reducing her planned trade at  $t = 2$  by an unit, and making an offer to use the liquidity freed up to buy  $P_d/P_o$  units of an SR's asset early, at the unit price  $P_o$  equalling;

$$P_o = P_d \frac{(1 - q\eta)}{(1 - q)}. \quad (\text{a.4})$$

The SR agent who is offered this price would be strictly better off by selling early if:

$$P_o > q\eta\delta\rho + (1 - q\eta)P_d, \quad (\text{a.5})$$

which holds provided

$$\eta\delta\rho < P_d \frac{1 - q\eta}{1 - q} \quad (\text{a.6})$$

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<sup>8</sup>It is easily shown that:

$$\frac{dR(\lambda)}{d\lambda} = \left[ 1 - \eta \left\{ \frac{(1 - \lambda\rho)^2 - (1 - \rho)^2}{(1 - \lambda\rho)^2} \right\} \right] > 0.$$

Inequality (a.6) holds for  $P_d > \delta\eta\rho$ , which is true of any non-trivial Delayed trading equilibrium. In contrast, in the BSS model, the analogue of (a.6) would require that:

$$\eta\rho < P_d \frac{(1-q\eta)}{(1-q)} \quad (\text{a.7})$$

which contradicts the LR agent's First Order Condition for optimality, in equation (13). *Q.E.D.*

**Proof of Proposition 2.** We first observe that if there exists a delayed equilibrium with  $m^* = 0$  then the market clearing condition implies that  $M_d = (1 - q\eta)P_d$ . Substituting the expression for liquidity  $M_d$  into LR's first order condition (13) in delayed equilibrium we obtain that the price  $P_d$  in delayed equilibrium is given by  $P_d = (1 - q\eta)x$ , where  $x$  solves a nonlinear equation (21).

By comparing the payoffs from early and delayed trades we obtain the following condition guaranteeing that SRs prefer to trade late:

$$P_e \leq q\eta\rho\delta + (1 - q\eta)P_d, \quad (\text{a.8})$$

where  $P_e$  denotes the early equilibrium price that the SRs expect to see if they switch to early trade. The expression on the right-hand side of (a.8) represents the expected gain from the delayed trade conditional on observing a bad shock at  $t = 1$ .

Moreover, if there exists a delayed equilibrium with  $m^* = 0$  then  $P_d$  should satisfy the following two inequalities:

$$P_d \geq \eta\rho\delta, \quad P_d \leq \rho\delta. \quad (\text{a.9})$$

Indeed, if the first inequality in (a.9) is violated only lemons are traded at time  $t = 2$ , which is not consistent with having a non-trivial delayed equilibrium. The second inequality in (a.9) guarantees that the SRs receiving good news at  $t = 2$  do not trade the assets (as discussed in Section II). Substituting  $P_d = (1 - q\eta)x$  into the inequalities (a.8) and (a.9) and rewriting them as inequalities on  $x$  we obtain the following inequality:

$$\max\left\{\eta\rho\delta, \frac{P_e(\lambda) - q\eta\rho\delta}{1 - q\eta}\right\} \leq x(\lambda) \leq \rho\delta. \quad (\text{a.10})$$

The inequality (a.10) imposes restrictions on  $\{\lambda, \delta\}$  in equilibrium. Resolving the inequality (a.10) with respect to  $\delta$  we obtain an equivalent inequality:

$$\delta_*(\lambda) \leq \delta \leq \delta^*(\lambda), \quad (\text{a.11})$$

where  $\delta_*$  and  $\delta^*$  are given in (20).

So far, the inequality (a.11) has been derived as a necessary condition for the existence of the delayed equilibrium. However, we observe that this inequality is equivalent to inequality (a.10) which also gives a sufficient condition for the existence of a delayed equilibrium. Indeed,  $x$  solves

a nonlinear equation (21) and  $P_d = x$  defines the price in the delayed equilibrium since all the equilibrium conditions are satisfied. In particular, from (a.10) it follows that inequalities (a.8) and (a.9) are satisfied, and hence under the price  $P_d$  the SRs prefer to trade late. Moreover, noting that  $M_d = (1 - q\eta)P_d$  we rewrite the non-linear equation (21) as follows:

$$F'(K - M_d) = \lambda + (1 - \lambda) \frac{\eta\rho(1 - q)}{(1 - q\eta)P_d}, \quad (\text{a.12})$$

which gives the FOC for LRs. Therefore,  $M_d$  defined as  $M_d = (1 - q\theta)P_d$  indeed gives the optimal liquidity level chosen by LRs that anticipate the delayed equilibrium. This completes the proof that inequality (a.11) defines both necessary and sufficient condition for the existence of a delayed equilibrium.

The uniqueness of the delayed equilibrium follows from the properties of production function  $F(\cdot)$ . Since  $F(\cdot)$  is an increasing and concave function the left-hand side of the equation (21) for  $x$  is a monotonically increasing function of  $x$  on the interval  $(0, K)$  and goes to infinity as  $x \rightarrow K$ . On the other hand, the right-hand side of (21) is a monotonically decreasing function  $x$  which becomes infinite when  $x \rightarrow 0$ . Therefore, there exists the unique solution of equation (21) defining the price  $P_d$ .

We now demonstrate that the inequality (19) presents an equivalent way of rewriting the necessary and sufficient condition for the existence of the delayed equilibrium. From the inequality (a.11) on  $\delta$  we observe that the necessary and sufficient condition for the existence of equilibrium is given by the inequality  $\delta^*(\lambda) \geq \delta_*(\lambda)$  which implies that there exists at least one equilibrium pair  $\{\lambda, \delta\}$  satisfying inequality (a.11). By comparing  $\delta^*(\lambda)$  and  $\delta_*(\lambda)$  in (20) we obtain that the inequality  $\delta^*(\lambda) \geq \delta_*(\lambda)$  is equivalent to the following inequality:

$$\frac{x}{\eta\rho} \geq \frac{P_e - (1 - q\eta)x}{q\eta\rho}.$$

Resolving the above inequality with respect to  $x$  we obtain that the necessary and sufficient condition for the existence of the delayed equilibrium is given by  $x \geq P_{\min}$ , where  $P_{\min}$  is defined in Proposition 2. Thus, we obtain an exogenous lower bound  $P_{\min}$  on the delayed equilibrium price. Since  $F'(K - (1 - q\eta)x)$  is an increasing function of  $x$  the inequality  $x \geq P_{\min}$  is equivalent to the inequality (19) in Proposition 2. Therefore, the inequality (19) gives a necessary and sufficient condition for the existence of delayed equilibrium.

We also note that the necessary condition for the existence of a delayed trading equilibrium (17), derived in Lemma 2, is implied by the inequality (19). To demonstrate this, we first observe that  $F'(K - (1 - q\eta)x) > 1$  by assumption. Therefore, the left-hand side of the equation (21) for  $x$  exceeds unity. Consequently, from (21) we obtain the following upper bound on price  $P_d$ :

$$P_d \leq \frac{(1 - q)\eta\rho}{1 - q\eta}. \quad (\text{a.13})$$

Noting that the early equilibrium price satisfies inequality  $P_e \geq (1 - \lambda\rho)/(1 - \lambda)$  allows us to rewrite the inequality  $P_d \geq P_{\min}$  as follows:

$$P_d \geq \frac{P_e}{1 + q(1 - \eta)} \geq \frac{1 - \lambda\rho}{1 - \lambda} \frac{1}{1 + q(1 - \eta)}. \quad (\text{a.14})$$

The inequalities (a.13) and (a.14) imply that:

$$\frac{1 - \lambda\rho}{1 - \lambda} \frac{1}{1 + q(1 - \eta)} \leq \frac{(1 - q)\eta\rho}{1 - q\eta}.$$

After simple algebraic manipulations it can easily be demonstrated that the above inequality is tantamount to the necessary condition (17).

Finally, we prove the inequality (22) that gives an upper bound on the size of the interval for the discount  $\delta$  that supports the delayed trading equilibrium. In particular, from the expressions for the upper and lower bounds for  $\delta$  in (20) we obtain:

$$\delta^* - \delta_* \leq \frac{x}{\eta\rho} - \frac{x}{\rho} = \frac{x}{\rho} \frac{1 - \eta}{\eta}. \quad (\text{a.15})$$

Combining the inequality (a.15) with the inequality (a.13) we obtain:

$$\begin{aligned} \delta^* - \delta_* &\leq \frac{(1 - q)(1 - \eta)}{1 - q\eta} \\ &= \frac{(1 - q)(1 - \eta)}{1 - q + q(1 - \eta)\eta} = \frac{1}{\frac{1}{1 - \eta} + \frac{q}{1 - q}} \\ &\leq \min\left\{1 - \eta, \frac{1 - q}{q}\right\}. \end{aligned}$$

The above inequality demonstrates that the equilibrium existence region shrinks as  $\eta \rightarrow 1$  or  $q \rightarrow 1$ . *Q.E.D.*

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