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UNIVERSITY OF TECHNOLOGY, SYDNEY

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WORKING PAPER SERIES

by BENJAMIN F. HUNT

# A Forecasting Model of Option Pricing Volatility

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We examined the pricing volatility of options written on the Sydney Futures Exchange's 10 Year Bond futures contracts. A forecasting equation designed to provide efficacious forecasts of changes in option volatility was estimated. The estimated forecasting model provided superior forecasts to those of the random walk model. The forecasting model included strong day of the week effects. Further, it indicated that volatility in the underlying futures contract was gradually incorporated into option pricing volatility. The dynamics of the forecasting equation was characterised by negative autoregression. The volatility forecasts were applied to market data using delta, gamma and theta neutral volatility speculative option portfolios.

Pricing volatility is a preferable alternative to option price when quoting and trading options. While at any point in time a particular pricing volatility can be legitimately applied to a range of options with differing strike prices and maturities, an option price is only appropriate for a single option. For this reason option market professionals, who routinely trade a range of options, base their analysis in terms of volatility rather than price. In options markets, pricing volatility, rather than option price, is the natural indicator of market level. It follows that dynamics of option markets is best studied by concentrating on the evolution of volatilities implied by option market prices.

If all relevant information is incorporated in option prices, option pricing volatility ought to follow a random walk. That is,

$$\sigma_t = \sigma_{t-1} + u_t$$

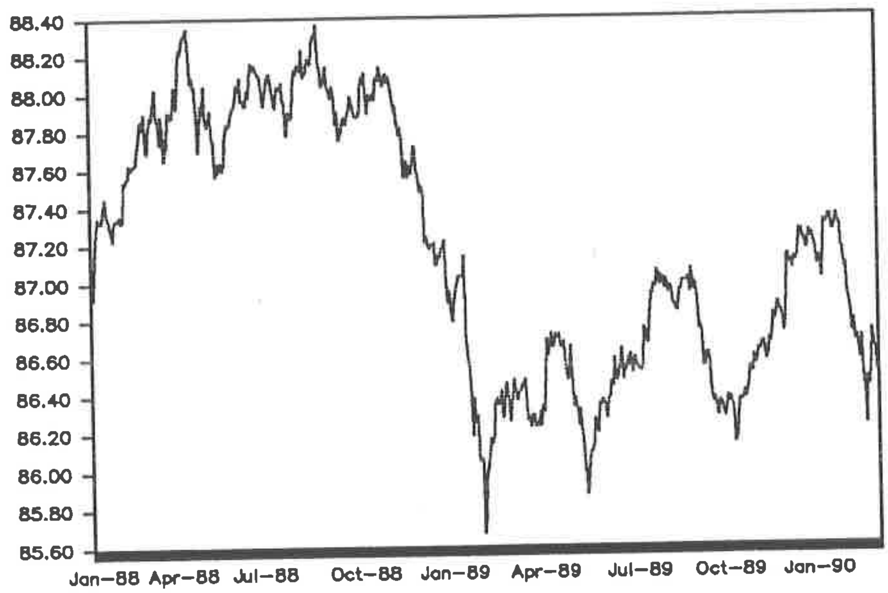
$$\text{or} \quad \Delta\sigma_t = u_t \tag{1}$$

Where $\sigma$ is volatility, t is time and u is an independent random variable. Under the random walk model, the best forecast of future volatility is today's volatility. This study details the search to improve upon this forecast using only data which was available at the time of the forecast.

Whether it is possible to improve upon the random walk forecast is an important issue. The ability to forecast future volatility, and hence future prices, is anathemaic to market efficiency. The construction of a forecasting equation challenges any claims to market efficiency and provides a possible vehicle for profitable market trading.

There have been a number previous option market efficiency studies that have concentrated upon pricing volatility. For example, Latane and Rendleman (1976), Chiras and Manaster (1978) Beckers (1981) and Canina and Figlewski (1991) However these studies focussed on the ability of implied volatility to predict the future volatility of the underlying security. The approach here is different. We are not concerned with the question as to whether current pricing volatilty predicts future asset volatilty but with the question as to whether a number of variables, including current and past pricing volatility can be used to predict future pricing volatility. Apart from Bhar and Hunt (1990), we know of no other studies which have attempted the task of forecasting option pricing volatility.

Figure 1 : Prices of SFE 10 Year Treasury Bond Contracts




This paper makes a specific contribution to the understanding of the way in which market makers perceive and use volatility in pricing options on 10 Year Bonds on the Sydney Futures Exchange. In doing so the paper makes a number of other contributions. It outlines a method whereby a binomial options on futures model can be employed to produce a single at the money index of pricing volatility series from a number of call and put option series. The search for a forecasting equation incidentally highlights the impact of day of the week effects and volatility in the price of the underlying asset have on option pricing volatility. Finally, the paper shows how an option volatility forecasting model can be used in conjunction with a series of delta, gamma and theta hedged option portfolios, to generate speculative trading profits.

The rest of the paper is organised as follows. Section 2 sets out institutional and market detail as describing the method used to produce a pricing volatility series. Section 3 outlines the functional form of the forecasting model. Section 4 discusses the results of estimation. Finally, in section 5 the results of an attempt to use the forecasting model to generate speculative profits are set down.

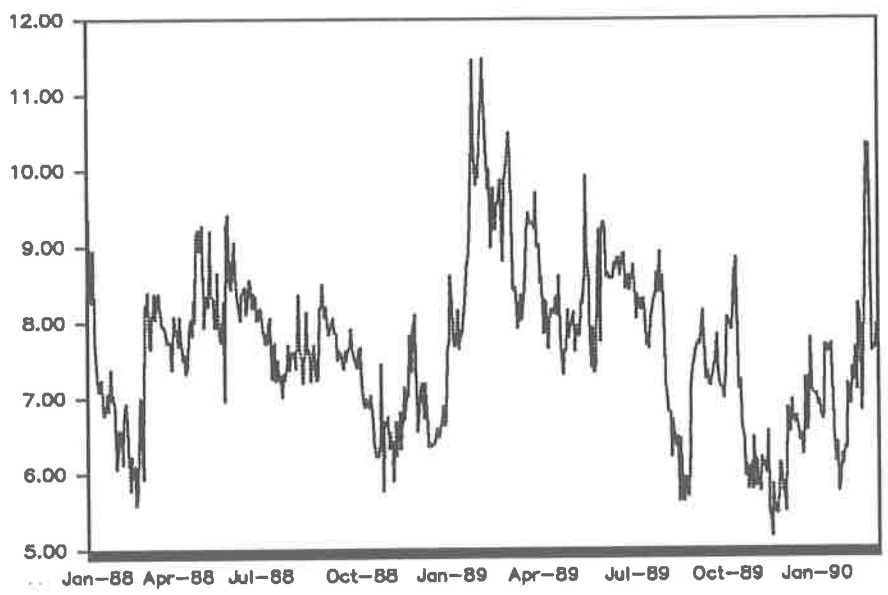
## 2. MARKET DATA

Data on the 10 Year Treasury Bond futures contract and options written on this contract was supplied on disk by the Sydney Futures Exchange (SFE). This study concentrates on the period running from January 4 1988 to March 15 1990. A plot of the price of the near futures contract for this period is presented in Figure 1.

The primary task of this study was to forecast 10 Year Bond Futures options pricing volatility. The first step in accomplishing this task was to create a single historical pricing volatility index from the implied volatilities (standard deviation) of a number of option contracts.

The computation of volatility implied in option market prices raised a number of issues, the resolution of which required the making of a number of assumptions. First, for any particular period, what was current market price? We assumed that market price, an any day was the last traded price provided that this lay within the closing bid/offer spread, otherwise we have assumed that the market price was mid way between the closing bid and offer prices.

Figure 2 : The Pricing Volatility Series for Options on SFE 10 Year Treasury Bond Contract




Another issue concerned the choice of the pricing model to be used in the derivation of implied volatility. We chose to use a zero interest rate binomial options model as it is well suited to the pricing of American futures options [1]. As there is a unique implied volatility for each option contract, we were faced with the choice of a representative implied volatility. We decided that rather than using the implied volatility of a single option to represent pricing volatility we would construct a representative index of 10 Year Bond option pricing volatility from a number of options.

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1 While this model has its limitations( see Lauterbach and Schultz (1990)) it is superior to the most obvious alternative, Black's (1976) model when dealing with American futures options. The binomial model takes account of the possibility of early exercise. The model prices were calculated using 20 time steps. Implied volatilities were calculated to the nearest 0.1% using a secant iterative method,

A single index of pricing volatility was constructed from the implied volatility of four options (two call options and two put options) on the futures contract closest to expiry [2]. The particular call and put option series chosen were such that their strike prices straddled the current futures contract price. Having chosen the option series and computed the implied volatilities, an index of pricing volatility was constructed as a linear weighted average of the four implied volatilities. Within each put/call class, the weights were chosen to produce a "synthetic at the money" implied volatility. The at the money call option implied volatility was then combined, with equal weight, with the at the money put option implied volatility to produce an index of 10 Year Bond Futures pricing volatility. It was to this series that a forecasting model was fitted. The historical pricing volatility series is graphed in Figure 2.

## 3. THE FORECASTING MODEL

The structure of the forecasting model can be summarised as,

$$\Delta \sigma_t \quad = \quad \Phi(L)\, \Delta \sigma \;+\; \beta(L)\, X \;+\; u \qquad (2)$$

Where,

| | |
|---|---|
| $\Phi(L), \beta(L)$ | are coefficient polynomials, |
| L | is a lag operator |
| $\Delta \sigma$ | is the change in volatility, and |
| X | is vector of instrumental variables. |

The formulation of the forecasting model embodies the notion that changes in the pricing volatility, $\Delta \sigma_t$, can be forecast using publically available information. A nonzero $\Phi(L)$ means that past changes in volatility are of use in forecasting future volatility. A nonzero $\beta(L)$ implies that information contained in other data series can be employed to predict future volatility. It ought to be noted that the random walk model (1) is a special case of (2) where both $\Phi(L)$ and $\beta(L)$ vanish.

Three classes of variables were chosen for inclusion in the instrumental variables set, X. [3] These were, (1) day of the week dummies, (2) daily volatility in the underlying futures contract and (3) the number of days to the expiry of the option contract.

Day of the week dummies were included as candidate forecasting instruments on the basis of their pervasive influence in financial markets. There have been numerous studies linking variation in returns to days of the week. More to the point, the studies of both Bhar and Hunt and Harvey and Whaley (1990) had discovered day of the week irregularities in options market pricing volatility series.

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2 The near contract was defined as that futures contract closest to expiry but more than 10 days from expiry. Implied volatility of an option becomes increasingly sensitive to small changes in its market price as it approaches maturity. The switch of contracts at the 10 day mark was designed to prevent the incorporation of spuriously high/low implied volatilities in the pricing volatility index.

3 What constitutes a full instrumental variable set is an open question. In principal we ought to have tested all possible relevant variables, however in the interest of brevity we used only the three variables which were most likely to contribute to the prediction of future volatility.

The rationale for the inclusion of the daily volatility of the underlying asset as an explanatory variable as an instrumental variable is as follows. In the absence of any trend in volatility, and if information is used efficiently, pricing volatility ought to equal the current instantaneous volatility of the underlying asset. This equality should be the outcome of the actions of arbitrageurs attempting to profit from any systematic deviation of pricing volatility from the average volatility of the underlying asset from now until the expiry of the option. While it is recognised that the volatility of assets such as futures contracts, vary considerably over time, these changes are not easily detected. Instantaneous volatility cannot be observed in a security but rather must be inferred from past historical volatility.

The reliance upon historical volatility to estimate future asset volatility, and hence to estimate the appropriate volatility to use in option pricing, admits the possibility that changes in historical volatility is gradually incorporated in pricing volatility. For example, suppose a market maker uses a simple ten day standard deviation of historical returns as a basis for options pricing. In this case a quantum shift in the instantaneous volatility of the underlying asset will be gradually incorporated in the market maker's pricing volatility. The effect of the new volatility will only be fully reflected in the pricing volatility after ten days have elapsed.

We have assumed that the absolute value of the daily return on the underlying asset, in our case 10 Year Bond futures, was a reasonable indicator of historical daily volatility in the underlying asset. [4] This historical volatility variable was included, in a changed form with a number of lags, in the forecasting model.

The number of days to expiry was included in the set of instrumental variables to catch any systematic influence of time upon pricing volatility. The inclusion of this variable is consistent with the findings of Hunt and Bhar and Cania and Figlewski, both of which have identified a negative relationship between time to expiry and implied volatility.

The forecasting model was estimated using the TSP package. The results of estimation and the tests of various restrictions are set out in Table 1

## 4. RESULTS

The results of estimation show that the forecasting model (2) is superior to the random walk model. However, this statistical significant result ought to be kept in perspective by noting that the estimated model enjoys only a 7.5% advantage over the random walk as a predictive tool.

### 4.1 *Influence of Days to Expiry*

Contrary to our *a priori* expectations we could not detect any influence of the time to option expiry on pricing volatility.

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4 Provided expected return is zero, the simplest estimate of a single day's variance of return is the square of the day's return. As we have chosen to characterise volatility as standard deviation, it follows that the best estimate of daily volatility is given by the square root of daily variance i.e the absolute size of daily return.

Table 1 : Results of Model Estimation

Dependent Variable : $\Delta \sigma_t$
Estimation Period Jan 4 1988 to Mar 15 1991

| Variable | Coefficient | Significance Level |
|---|---|---|
| Monday | 0.1379 | 0.003 |
| Tuesday | -0.0465 | 0.301 |
| Wednesday | -0.0022 | 0.961 |
| Thursday | -0.0030 | 0.948 |
| Friday | -0.0700 | 0.139 |
| $\Delta \sigma_{t-1}$ | -0.2053 | 0.000 |
| $\Delta \sigma_{t-2}$ | -0.1257 | 0.005 |
| $\Delta \sigma_{t-3}$ | -0.0482 | 0.279 |
| $\Delta Fvol_{t-1}$ | 0.0077 | 0.033 |
| $\Delta Fvol_{t-2}$ | 0.0083 | 0.047 |
| $\Delta Fvol_{t-3}$ | 0.0036 | 0.074 |

$R^2$ is 0.0749

Where,

| | |
|---|---|
| $\Delta \sigma_t$ | is the change in the pricing volatility index, at time t, |
| Monday etc | are a series of day of the week dummy variables, |
| Fvol | is daily volatility of the 10 Year Bond futures contract. This is defined as the annualised absolute return derived from the holding of a futures contract for a day, and |
| Ldays | is the logarithm of the number of days until the expiry of the option. |

Tests of Exclusion and Restriction

| | |
|---|---|
| All coef. = 0 (i.e. $\sigma$ follows a random walk) | Rejected at 1% level |
| Coefficients on $\Delta \sigma_{t-i} = 0$ , i=1,2,3 | Rejected at 1% level |
| Coefficients on $\Delta Fvol_{t-i} = 0$ , i=1,2,3 | Rejected at 10% level |
| Coefficients on $\Delta Fvol_{t-i}$ are equal, i=1,2,3 | Not rejected at 10% level |
| Coefficients on Ldays = 0 | Not rejected at 10% level |
| Coefficients on day of the week are equal | Rejected at 5% level |

## 4.2 *Day of the Week Effects*

There is evidence of significant day of the week irregularity. In summary, option pricing volatility tended to be high on Monday and low on Friday. This result confirms a similar discovery by Harvey and Whaley in S&P 100 options. That these day of the week effects persist in an equation containing volatility in the underlying asset, would indicate that these irregularities are independent of events occurring in the market of the underlying asset. However an examination of the daily volatility of 10Year Bond futures returns shows the same high Monday/low Friday pattern. The high Monday volatility is at least understandable in terms of the larger close-close interval for Mondays compared with other days in the week.

### 4.3 *Daily Futures Volatilty*

The results of estimation show that past changes in the volatility are significant positive determinants of current pricing volatility. In addition, it was not possible to reject the hypothesis that the coefficents on each of the first three lagged futures volatility terms are of equal magnitude. This finding is consistent with the observed practice of market makers to employ standard deviation of daily returns to estimate pricing volatility.

### 4.4 *Autoregressive Terms*

Each of the first three lagged dependent terms enters the forecasting equation with a negative coefficient. This result is an indication of reversion in the time structure of option volatility. The negative autoregressive structure of the forecasting equation is consistent with Stein's (1988) findings of mean reversion and overreaction in the S&P 100 index options. [5]

## 5. APPLICATION OF THE FORECASTING EQUATION

The forecasting model was applied to market data in an attempt to generate trading profits in the following manner. Starting in July 1988, for a particular day, data for the previous six months was used to estimate a forecasting model. This estimated forecasting model was then employed to produce a forecast of the next day's change in volatility. A market position was then taken on the basis of the volatility forecast. For example, if it was forecast that volatility was to drop, we would enter into a position in options that would profit from the expected fall in option pricing volatility. The next day the position would be subsequently reversed with the realisation of a profit or loss on the round transactions. Having completed this process for a particular date we would advance one day and start the process again.

The nature of the trading positions entered into on the basis of forecast changes in volatility bear some discussion. There were two principles used in constructing these option portfolios. First, we decided on principle that each of these daily positions ought to be equally sensitive to a change in volatility. That is, the change in the value of each daily position in response to a unit change in volatility ( ie the vega of each of the daily positions) ought to be constant.

Secondly, it was necessary to insulate the options position taken from other principal option influential variables namely, time and price changes in the the underlying asset. This was accomplished by constructing each day a speculative position which was delta, gamma, and theta neutral. [6]

The desired vega constant, theta, delta, gamma neutral speculative portfolios were created daily from four options contracts, two calls and two puts. These options were those described earlier in the section dealing with the construction of the option volatility index. Note that the daily option position must satisfy the set of equations (3),

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5 While our motivation for model construction and estimation differs from that of Stein, the functional used here is similar to Stein's mean reverting process. His model can be viewed as a restricted version of equation (2) where all coefficients save a constant and that on the first lagged dependent variables are zero.

6 If we let V represent the value of an option position, P the price of the underlying assset, Vol the volatility of an option and t time, then delta = $\delta V/\delta P$, gamma = $\delta \text{delta}/\delta P$, theta = $\delta V/\delta t$, and vega = $\delta \text{Vol}/\delta t$.

$$\begin{matrix} v_1 & v_2 & v_3 & v_4 \\ d_1 & d_2 & d_3 & d_4 \\ g_1 & g_2 & g_3 & g_4 \\ t_1 & t_2 & t_3 & t_4 \end{matrix} \quad \begin{matrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{matrix} \quad \begin{matrix} = \\ = \\ = \\ = \end{matrix} \quad \begin{matrix} +/-k \\ 0 \\ 0 \\ 0 \end{matrix}$$

or,

$$A * w = b \tag{3}$$

where,

| | |
|---|---|
| v | is vega, |
| d | is delta, |
| g | is gamma, |
| t | is theta, |
| w | is the amount of options held, |
| k | is an arbitrary constant, negative k is used to generate short volatility positions, positive k for long volatility positions and |
| 1,2,3,4 | index numbers identifing particular options, |

The speculative position entered into on a given day was identified through a solution of (3), that is,

$$w = A^{-1} b \tag{4}$$

The results of speculation on the basis of forecasts provided by the estimated pricing volatility equations are set out in Table 2. The profit resulting from this volatility speculation is placed in perspective by the perfect foresight benchmark. This bench mark is the profit that would have been generated from volatility speculation if we were able to predict, without error, the future direction of pricing volatility.

Table 2 : Speculative Trading Profits

| Forecast Basis | Average Daily Profit |
|---|---|
| Perfect Foresight | \$71.74 |
| Forecasting Model | \$34.90 |

The results in the body of the table show the profit arising from volatility speculation using a four option portfolio, constructed to be delta, gamma, and theta neutral and possessing a vega = $+/-10^5$, over the period July 1988 to March 1990.

Clearly, we were able to use the forecasting model to generate trading profits. However these speculative profits were much less than the maximum possible, which is given by the perfect foresight profit figure. In interpreting these results two things ought to be borne in mind. First, we have not attempted to account for transaction costs such as commission or the bid/ask spread. Secondly, we assumed, for the purpose of calculating speculative profit and loss, that we could enter and exit buy and sell the options at the end of day market prices.

## 6. CONCLUSION

We constructed an *at the money* pricing volatility series from call and put options on the SFE's 10 Year Bond futures contract. Subsequently, we estimated a forecasting equation designed to furnish efficacious forecasts of changes in option volatility . The estimated forecasting model was statistically significant improvement on the random walk forecast model. The estimation process confirmed the presence of day of the week irregularities in option volatility. Further, the forecasting equation indicated that volatility in the underlying futures contract was gradually incorporated into option pricing volatility. The dynamics of the forecasting equation was characterised by negative autoregression. The application of the model via delta, gamma and theta neutral volatility speculative option portfolios produced positive trading profits.

## References

Beckers, S. (1981) "Standard Deviations Implied in Option Prices as Predictors of Future Stock Price Variability", *Journal of Banking and Finance*,5 , pp363-381.

Black, F.,(1976) "The Pricing of Commodity Contracts", *Journal of Financial Economics*, 3, 167-178.

Canina, L. and Figlewski, S. (1991) "The Informational Content of Implied Volatility", New York University Salamon Centre Working Paper S-91-20

Chiras D. and Manaster, S. (1978), "The Information Content of Option Prices and a Test of Market Efficiency", *Journal of Financial Economics*, 6, 213-234

Harvey, C.R. and Whaley, R.E., (1990) "Market Volatility Estimation and the Pricing of S&P options", Duke University Fuqua School of Business

Laterbach, B. and Schultz, P., ( 1990) "Pricing Warrants: an Empirical Study of the Black Scholes Model and its Alternatives", *Journal of Finance*, 45, 1181-1209.

Latane, H.A. and Rendleman, R.J. (1976) "Standard Deviations of Stock Price Ratios Implied in Option Prices", *Journal of Finance*, 31 369-381.

Stein, J., (1989) "Overrreaction in the Options Market", *Journal of Finance*, 44, 1011-1023.