Real-Financial Interaction: A Reconsideration of the Blanchard Model with a State-of-Market Dependent Reaction Coefficient

Carl Chiarella
Peter Flaschel
Willi Semmler

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Carl Chiarella  
School of Finance and Economics, University of Technology, Sydney, Australia

Peter Flaschel  
Faculty of Economics, University of Bielefeld, Bielefeld, Germany

Willi Semmler  
Faculty of Economics, University of Bielefeld, Bielefeld, Germany

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Abstract

We reformulate and extend the Blanchard model of output dynamics, the stock market and interest rates that studies Keynesian IS-LM analysis from the perspective of a richer array of financial assets, namely equities and long-term bonds, than the usual presence of money and short-term bonds. Thus investment demand now depends on Tobin’s average $q$ in the place of the real rate of interest and as a result share price dynamics feed back into the real sector, thereby creating the link for the real-financial interaction studied by Blanchard. We reconsider the results achieved by Blanchard without use of logarithms and other simplifications in the expression for the short-term rate of interest. Our main contribution is to extend the model to allow imperfect asset substitutability and imperfect forecasts of capital gains in the place of Blanchard’s limit case of perfect substitutes and myopic perfect foresight. Our more general framework, and in particular the assumption of a state-of-the-market dependent speed of reaction to expected asset return differentials, allows us to develop a mode of dynamic analysis that provides an alternative to the conventional jump variable technique of the perfect limit cases. We show how as a consequence the stock market dynamics can display periods of bull and bear markets having both activated and tranquil phases that give rise to a variety of adjustment patterns.

JEL classification E12, ...

Keywords: Real-financial interaction, stability, jump variable technique, expectations, phase diagram switching, persistent fluctuations, reaction coefficient, return differential.
1 Introduction

In this paper we reconsider the model of Blanchard (1981) which reformulates Keynesian IS-LM analysis from the perspective of a richer array of financial assets, namely long-term bonds and equities besides the usual presence of money and short-term bonds of (textbook presentations of) the IS-LM model.\(^1\) The basic change in Blanchard’s extension of the IS-LM approach is that investment demand (and consumption demand) now depend on Tobin’s average \(q\) in the place of the real rate of interest which implies that share price dynamics (but not yet long-term bond price dynamics) feed back into the real sector, thereby creating one of the links for the real-financial interaction. The conventional LM schedule or money market equilibrium of this approach, which as usual is dependent on real output and income in turn provides the channel back from real to financial markets, in determining the share price dynamics by assuming a situation of perfect substitution and myopic perfect foresight with respect to the interest bearing assets.

The real-financial interaction is of course at the very core of macroeconomic theorizing. Starting from Keynes (1936, Ch.18) one may first of all assume the one sided relationship between the real and the financial sector shown in Figure A. In this figure asset markets bring forth a rate of interest and returns structure by themselves without any interference from the market for goods and labor. The markets for financial assets are thus at the top of the market hierarchy of macroeconomic theorizing and conceived as being basically independent of what happens in the real markets. Their outcome however determines real investment behavior based on the structure of financial returns compared

\(^1\)Or long-term bonds in the place of short-term ones if the Baumol-Tobin inventory approach to money holdings is replaced by a Keynesian approach to speculative money holdings as in the Tobin portfolio choice model, see Crouch (1972) for details.
to the rate of profit or the state of confidence that characterizes the sector of firms. This investment
in turn determines via the multiplier process (which may be formulated in static, in dynamic or even
in Metzlerian terms) the outcome on the market for goods and thus the production level that firms
will realize. Depending on the type of production function that is assumed this will finally determine
employment and thus the rate of unemployment on the market for labor.

Starting from a situation of high unemployment (and depressed goods markets as well) it was already
conceded by Keynes (1936, Ch.19) himself that there may exist a fundamental feedback from the
real markets to the financial markets that may improve the situation on the real markets by lowering
interest rates and by subsequently improving the investment climate (and also consumption demand
for durable consumption goods in particular). This favorable effect for economic stability and a
return to a position of full employment has been named the Keynes-effect in the literature (and is
often also combined with the Pigou real balances or wealth effect which works in the same direction).
The working of this well-known Keynes-effect is summarized in Figure B.

The Keynes Effect:

![Diagram of the Keynes Effect]

Figure B:
The Keynes-effect, the basic stabilizing feedback from the real markets.

We can see from this figure that the feedback chain from falling wages and prices to falling interest
rates and then to increasing goods demand and output of firms as well as higher employment is a long
(and maybe also uncertain) one which in no way supports the argument that labor market problems
can be remedied by a policy that concentrates solely on the labor market. Unemployment is here due
to the interdependence of real and financial markets and can thus be cured only by improving this
interdependent situation through an appropriate fiscal or monetary policy, perhaps in conjunction
with appropriate labor market and wage management policies. Hicks (1937) formulated on this basis
the now traditional IS-LM model of the interaction of goods and financial markets by concentrating
on the interaction between output and the rate of interest as described by goods and money market equilibrium curves.

To this traditional IS-LM approach to the interaction of real and financial markets, Blanchard (1981) adds basically the dynamics of the stock market as described above coupled with a simple and stable dynamic multiplier description of the output dynamics on the market for goods. Figure C describes these stock market dynamics in isolation by showing that there is a positive feedback mechanism between expected capital gains, share price dynamics and actual capital gains, feeding back on expected capital gains.

![The Share or Equity Market](image)

Figure C: Destabilizing stock market dynamics.

Rising capital gain expectations on equities drive up equity prices and thus actual capital gains which in turn lead to further increases of expected capital gains and so on, until something in this mechanism changes to counterbalance the cumulative instability of this share price adjustment process. In Blanchard (1981) that something is based on myopic perfect foresight (so that there is no distinction between expected and actual capital gains) and application of the so-called jump variable technique (JVT). According to this technique agents' expectations adjust (via a jump) so as to place the dynamics onto a stable manifold of an otherwise unstable saddle path dynamics. The economic scenario behind such jumps is rarely made explicit but relies very heavily on the rational expectations viewpoint that agents have full knowledge of the model and hence perfect foresight of the dynamics.

Further rates of return, i.e., in the case of equities the dividend rate of return plus capital gains, are all equalized in Blanchard's approach by way of his assumption that all interest bearing assets are
perfect substitutes. In our view this makes the above feedback mechanism even more powerful, by creating a law of motion that - in contrast to the dynamic multiplier - is characterized by centrifugal (in the place of centripetal) forces around the asset markets equilibrium curve. Figures 1 and 2 in the following section illustrate this point.

There is a similar situation on the market for long-term bonds as Figure D shows, which again can be interpreted in terms of a fast adjustment mechanism or the perfect substitute / myopic perfect foresight limit case of the Blanchard model. Since investment (and consumption or in sum aggregate demand) do not react in Blanchard's model to what happens on the market for long-term bonds this destabilizing feedback chain is however left in the background in Blanchard (1981), as it will also be in our reformulation of his model in the present paper.

The Market for Long-Term Bonds

![Diagram of market dynamics for long-term bonds]

**Figure D:**
Destabilizing dynamics on the market for long-term bonds.

In this paper we reconsider and generalize Blanchard's description of the real-financial interaction with two objectives in mind. Our first objective concerns the formulation of the model. We seek to make more transparent the role of Tobin's q in the investment demand. We give a formulation that avoids some variables being expressed in their naturally occurring units (eg. output) and others being expressed as logarithms of their naturally occurring units (eg. money and price). Our formulation is also made consistent with the broader macroeconomic framework developed by the authors in Chiarella, Flaschel, Groh and Semmler (2000) thus making transparent how financial market dynamics may be incorporated into that broader framework. Our second and more significant objective is to reconsider the dynamics of real-financial interactions in a way which does not rely on the poorly motivated jump variable technique to resolve the dilemma of unstable saddle path dynamics around the steady state.
In section 2 we outline the model of real-financial interaction, focusing on the role of Tobin’s average \( q \) in the investment function and on the role of the differential between expected equity returns and the short term rate of interest in driving the stock market dynamics. Our framework is also more general than that of Blanchard in that we assume less than infinite speed of adjustment to this return differential, and we also assume less than perfect foresight of expectations of the change in Tobin’s \( q \). In section 3 we analyse the dynamics of the model along the lines of Blanchard, in particular assuming infinite adjustment of the share market to the return differential and perfect foresight expectations. We also discuss how the jump-variable technique is used to "close" the dynamics of the variant of the model analysed in this section and what we perceive as some of the shortcomings of this approach. In section 4 we consider the more general version of the model with less than infinite adjustment speed to return differentials in the stock market and less than perfect foresight expectations. We also introduce a mechanism by which the speed of adjustment of the stock market to the return differential changes according to the state of the market. We show how to analyse the qualitative dynamic behaviour of this model by the technique of phase diagram switching which makes clear how the dynamics can alternate between periods of bull and bear markets having both tranquil and activated phases. In section 5 we draw some conclusions.

2 The Model

In this section we introduce the notation and set out the static and dynamic economic relationships that define the macroeconomy with real-financial interaction that will be the object of our study.

2.1 Notation

The following subsections provide the variables, the parameters and the mathematical notation we shall use in our reformulation of the Blanchard (1981) model of real-financial interaction, exhibiting the market for goods and the four asset markets (money, short-term bonds, long-term bonds and equities). Note that we use extensive form variables (and thus not logarithms of money and the price level as in Blanchard (1981)) throughout and that the money demand function, though here completely linear, represents a generalization of the form that was used by Blanchard (1981) in his equation (2). We start from linear behavioral equations in all markets of the model, in order to study the role of intrinsic or unavoidable nonlinearities first, before we make any attempt to allow for nonlinear reaction patterns, investment functions and the like. For simplicity, Tobin’s \( q \) is only used (in average form) in the investment function which is based on the assumption of fixed proportions in production. Adding a dependence of consumption on Tobin’s \( q \) would not alter the conclusions obtained in this paper. We focus on the case of fixed proportions in production in order to make more transparent the dynamic structure of the economic processes under study. This follows the same style of treatment as in Chiarella and Flaschel (2000a) who show that the case of smooth substitution in production does not alter the basic qualitative dynamic features of the type of dynamic macroeconomic models under study here.

2.1.1 Variables

The variables of the model are:

\[
Y \quad \text{Output}
\]
\[ Y^d \] Aggregate Demand
\[ p_e \] Share price
\[ q = p_e E/(pK) \] Tobin's average \( q \)
\[ m^d \] Demand for real balances
\[ r \] Short-term interest rate
\[ p_b \] Bond price
\[ \Pi \] Real profits
\[ \rho = \frac{\Pi}{K} \] Rate of profit
\[ L^d \] Labor demand
\[ p^e(\hat{p}^e) \] Expected value of (change in) the price of equities
\[ q^e(\hat{q}^e) \] Expected value of (change in) Tobin's \( q \)

2.1.2 Parameters

The parameters of the model are the following:\(^2\)

\[ c \] Marginal propensity to consume
\[ i \] Marginal propensity to invest
\[ T \] Real taxes
\[ G \] Real government consumption
\[ M \] Money supply
\[ p(=1!) \] Price level
\[ w \] Nominal wages
\[ k \] Money demand parameter
\[ h_0 \] Money demand parameter
\[ h_1 \] Money demand parameter
\[ m \] Real balances
\[ B \] Number of long-term bonds
\[ E \] Number of equities
\[ K \] Real capital stock
\[ \delta \] Depreciation rate of the capital stock
\[ x = Y/L^d \] Labor productivity
\[ u = wL^d/(pY) = w/(px) \] Wage share
\[ l \] Autonomous component of investment
\[ C \] Autonomous component of consumption
\[ \beta_x \] Adjustment speed of a variable \( x \)

2.1.3 Mathematical Notation

We shall make use of the following mathematical notation:

\[ \dot{x} \] Time derivative of \( x \)
\[ \ddot{x} = \dot{x}/x \] Growth rate of \( x \)

\(^2\)All parameters are assumed to be positive unless otherwise stated.
2.2 The equations of the model

The equations of the model are subdivided into algebraic (=instantaneous) ones and dynamic ones, which together determine the statically and dynamically endogenous variables of the model.

2.2.1 Algebraic equations

This block of the model describes a simple aggregate demand function \( Y^d \) (equation 1) which assumes that this demand depends on output (via consumption) and Tobin’s average \( q \) (via investment). The influence of \( Y \) and \( q \) on \( Y^d \) could however be based on consumption (a wealth effect besides influences from temporary income changes) as well as investment behavior (a stock adjustment principle besides the valuation of firms on the stock market). The demand for real balances (equation 2) is a traditional one depending positively on output and negatively on the short term interest rate. Next we make use of standard equations for money market equilibrium (equation 3), which will be used in inverted form in the following. Finally, we have an obvious equation for real profits (equation 4) which depend on economic activity \( Y \) solely. Thus we have:

\[
\begin{align*}
Y^d &= C + c(Y - \delta K - T) + I + i(q - 1) + \delta K + G, \quad c \in (0, 1), \\
m^d &= M^d/p = kY + h_o - h_1r, \\
m &= m^d, \\
\Pi &= (1 - u)Y - \delta K. 
\end{align*}
\]

2.2.2 Dynamic equations

The dynamic variables are output, share and bond prices. Output adjustment is based on the simple textbook dynamic multiplier story as indicated in equation (5). Short-term bonds, long-term bonds and shares are assumed to be perfect substitutes and there is perfect foresight on capital gains. Thus equation (6) and (7) indicate that the dividend rate of return on equities and the interest rate of return on bonds have both to be augmented by a corresponding expression for capital gains when comparing their full return with the return on short-term fixed price bonds. Thus we write

\[
\begin{align*}
\dot{Y} &= \beta_y(Y^d - Y), \\
r &= \frac{p\Pi + \dot{p}_eE}{p_eE} = \frac{p\Pi}{p_eE} + \dot{p}_e = \frac{\Pi/K}{q} + \dot{q}, \\
r &= \frac{B + \dot{p}_bB}{p_bB} = \frac{\Pi}{K} + \dot{q}. 
\end{align*}
\]

Note that the bond price dynamics do not feedback into the rest of the system, since there are no wealth effects nor interest rate effects in the aggregate demand function on the market for goods.
We generalise Blanchard’s treatment of share price dynamics in the following way:

\[
\begin{align*}
\dot{q} & = \beta_q \left( \frac{\Pi/K}{q} + z - r \right) \\
\dot{z} & = \beta_z (\dot{q} - z), \quad z \equiv \ddot{p}_e = \dot{q}_e
\end{align*}
\] (8) (9)

This extension assumes (equation (8)) that share prices \( p^e \) (as reflected by Tobin’s \( q \)) react with less than infinite speed to the differential between expected equity returns (dividend return plus expected capital gains) and the short term rate of interest. Equation (9) states that expectations of capital gains adjust with some lag to share price changes. The original Blanchard model assumes that both \( \beta_q \) and \( \beta_z \) are equal to infinity and thus represents a limit case (not necessarily a continuous one) of equations (8) and (9) with possibly high, but still finite adjustment speeds.

Note that we are assuming a background of a stationary economy in which capital stock is not growing and no new shares are issued. Furthermore the assumption of constant nominal and real money stock will result in no inflation and so in steady state there is no increase in equity price.

2.2.3 Fixed proportions and real profits

The assumption of a standard fixed proportions technology with only labor as variable input, and with capital depreciation, gives rise to the following expressions for real profits (\( \Pi \)) and rate of profit (\( \rho \)) where \( u \) denotes the given wage share of the model:

\[
\begin{align*}
\Pi & = Y - \delta K - w/pL^d, \quad L^d = Y/x, \quad x = \text{const.} \\
& = Y(1 - w/(px)) - \delta K = Y(1 - u) - \delta K, \quad u = w/(px) = \text{const.} \\
\rho & = \Pi/K = (1 - u)Y/K - \delta.
\end{align*}
\]

We shall generally assume in the following that the depreciation rate is positive, i.e., that there is a negative constant in the profit function of firms. These profits are distributed to share holders and thus fully enter the dividend rate of return component of the rate of return of share holders. Firms thus do not have any income, if one neglects the fact that the dynamic multiplier implies unintended inventory changes of firms that are not treated explicitly here; see Chiarella, Flaschel and Semmler (2000c) in this regard.

3 Analysis of the model

In this section we analyse the dynamics of the model in a similar fashion to Blanchard in that we consider the limit case of \( \beta_q \) and \( \beta_z \) equal to infinity in equations (8) and (9). In other words we assume instantaneous adjustment of share prices to the expected return differential and perfect foresight of expected share price changes.
3.1 Reduced form dynamics

The model of this section can be reduced to a 2D autonomous differential equation system in the variables output $Y$ and Tobin's $q$, since the law of motion for the price of long-term bonds $p_b$, the third differential equation discussed below, does not feedback into these dynamics.

The inverted money market equilibrium from equations (2) and (3) reads

$$ r = \frac{kY + h_o - m}{h_1}, \quad m = M/p $$

The use of this equation and the assumption that $\beta_q$ and $\beta_z$ are infinite reduces equations (5) - (9) to the following nonlinear autonomous differential equations system of dimension 2:

$$
\begin{align*}
\dot{Y} &= \beta_y (C + c(Y - \delta K - T) + I + i(q - 1) + \delta K + G - Y) \\
&= \beta_y ((1 - c)Y + i(q - 1) + A, \quad A \equiv C + I + \delta K + G - c(T + \delta K), \\
\dot{q} &= \frac{kY + h_o - m}{h_1} q - ((1 - u)Y/(K - \delta)) = rq - \rho
\end{align*}
$$

The laws of motion for $Y$ and $q$ are interdependent and thus interacting solely with each other, since there are no interest or wealth effects in aggregate demand as far as the holding of long-term bonds is concerned. The law of motion for $p_b$ given by equation (7) is thus an appended one. It exhibits a positive derivative with respect to its state variable $p_b$: $dp_b/dp_b = r$, which means that the jump variable technique (or other solution methods) used for the treatment of the value of the stock $q$ must be applied to this law of motion as well.

Note that the second law of motion, equation (12), is nonlinear due to the multiplicative $Yq$ term and similarly for the third appended law of motion because of the $Yp_b$ term. There are thus natural or intrinsic nonlinearities in the interaction of real and financial markets (if fully interdependent) that give the dynamics a structure with some similarities to the famous Lorenz equations, see Strogatz (1994, p.301). However typically these intrinsic nonlinearities are not sufficient to bound the dynamics in situations of instability of the steady-state.

3.2 The IS-curve: $\dot{Y} = 0$

Setting the first law of motion equal to zero and solving for $q$ gives the following goods market equilibrium representation or $\dot{q} = 0$-isocline of the model. This IS-curve,

$$
\dot{Y} = 0 : \quad Y = \frac{1}{1 - c} [A + i(q - 1)],
$$

3This should be contrasted with Blanchard's equation (2) which uses the logarithm of $m$ in place of our $m$.
4The dynamics of long-term bond prices should however be allowed to feed back into the rest of the dynamics in future extensions of the model.
5For a detailed dynamic analysis of integrated macrodynamic models exhibiting these kinds of intrinsic nonlinearities we refer the reader to Chiarella and Flaschel (2000b).
is attracting, since the marginal propensity to spend has been assumed to be less than one. All positions off the IS-curve are therefore attracted by this curve, as indicated by the horizontal arrows in Figure 1. We assume in the following that \( A - i > 0 \) holds true, which is plausible given the usual empirical magnitudes of the parameters involved.

![Graph](image)

**Figure 1: The IS-curve (representing goods market equilibrium).**

### 3.3 The 'LM'-curve: \( \dot{q} = 0 \)

Similarly, by setting the second law of motion to rest (\( \dot{q} = 0 \)), gives us the LM-curve or asset markets equilibrium curve of the model:

\[
\dot{q} = 0 : \quad q = \frac{(1 - u)Y/K - \delta)h_1}{kY + h_o - m} \equiv p/r. \tag{13}
\]

We point out that this isocline is a hyperbola with two branches. Furthermore it is important to distinguish two cases; the first in which \( q'(Y) < 0, q''(Y) > 0 \) that corresponds to Blanchard's bad news case; the second in which \( q'(Y) < 0, q''(Y) > 0 \) that corresponds to Blanchard's good news case. These two cases are displayed in Figure 2.

Along the \( \dot{q} = 0 \) isocline we have that the money market is (as always) in equilibrium and the stock market as well, in the sense that share prices are no longer changing then and there is a static equality between the dividend rate of return and the rate of interest on short-term bonds. Note that positions off the LM-Curve tend away from this curve as indicated by the vertical arrows in Figures 2a and 2b, i.e., share prices are subject to centrifugal forces off the \( \dot{q} = 0 \)-isocline. Depending on the slope of the LM-curve the dynamics may therefore be characterized by saddlepath dynamics as will be shown below.
Figure 2a:
The 'LM'-curve (representing money market and equity market equilibrium):
Blanchard's Bad News Case.

Figure 2b:
The 'LM'-curve (representing money market and equity market equilibrium):
Blanchard's Good News Case.
In the following discussion it is convenient to write the isocline equation (13) as

\[ q(Y) = \frac{h_1[(1-u)Y/K - \delta]}{kY + h_0 - m} = \frac{h_1[(1-u)Y/K - \delta]}{D(Y)}. \]

For the slope of this function we obtain:

\[ q'(Y) = \frac{h_1(1-u)(h_0 - m)/K + \delta k}{D(Y)^2} \leq 0. \]

We have denoted by \( Y \) in Figure 2 the solution of \( D = D(Y) = kY + h_0 - m = 0 \), i.e., the unique value of \( Y \) where money market equilibrium displays a zero rate of interest. To the left of \( Y \) interest is negative and to the right positive, i.e., only the latter output levels are economically meaningful for the following discussion (see Figure 2b above for a further restriction on the economic phase space of the dynamics).

Lemma 1:

For the slope of the function \( q(Y) \) to the right of the value \( Y \) for \( Y \) we have:

\[ q'(Y) \leq 0 \iff -(1-u)Y - \delta K \geq 0 \iff h_0 - m \geq 0 \quad \text{if} \quad \delta = 0 \quad \text{holds true}. \]

Proof: The definition \( Y = \frac{m-h_0}{k} \) implies

\[(1-u)Y - \delta K = (1-u)(m-h_0)/k - \delta K = -(1-u)(h_0 - m) + \delta K]/k\]

which implies the assertion.

This lemma states that the numerator of the function \( q(Y) \) is positive (negative) at the zero of the denominator if and only if the function \( q(Y) \) is strictly decreasing (increasing), starting from \( +\infty \) (\( -\infty \)) at the lower bound \( Y \) of output \( Y \); see Figures 2a,b. We note that the border case \((1-u)Y - \delta K = 0\) gives an LM-curve with slope zero throughout. This exceptional case will be ignored in the following investigations.

In the case \( q'(Y) > 0 \) we thus know that \( Y \) must always be positive, while we can have \( Y \geq 0 \) in the case \( q'(Y) < 0 \). Note that we have neglected the case of a negative \( Y \) in the Figures 2a,b. If we assume \( \delta = 0 \) in addition, we get as special case

\[ q'(Y) \leq 0 \iff -(1-u)Y \geq 0 \iff h_0 \geq m \iff Y \geq 0. \]

We thus can have Blanchard’s good news and bad news cases in the present formulation of the model even if profits are positive for all positive output levels, see Chiarella, Flaschel and Semmler (2000a) for an opposite case.

Lemma 2:

There holds \( q'(Y) \geq 0 \iff -q''(Y) \geq 0 \).
Proof: The second derivative of the function $q(Y)$ is given by

$$q''(Y) = -h_1 \frac{[(1-u)(h_o - m)/K + \delta k]2k}{D(Y)^3}$$

which proves the proposition, since $D(Y) > 0$ holds to the right of $\mathcal{Y}$.

This lemma states that the $q(Y)$ curve is strictly concave (convex) if and only if $q(Y)$ is strictly increasing (decreasing), as shown in Figures 2a,b.

3.4 The positive steady states of the model and local stability issues

We now investigate the steady state positions of the dynamics of equations (11) – (12) by means of the Figures 1, 2a and 2b.

Proposition 1:

Assume $q' < 0$. There exists exactly one positive steady state $(Y_0, q_0) > 0$ of the considered dynamics fulfilling $Y_0 > \mathcal{Y}$ (and thus $r_0 > 0$).

Proof: See Figure 3a, the construction of which is justified by Lemmas 1 and 2.

In the case of an LM-curve that is strictly decreasing we therefore always have a unique steady state solution that is interior to the economically viable domain. This result is obvious from the inspection of Figures 1 and 2a, due to the shapes of the IS and the LM curves.

Proposition 2:

Assume $q' > 0$. There may be two, one or no positive steady states $(Y_0, q_0) > 0$ of the considered dynamics with a positive rate of interest $r_0$.

Proof: See Figure 3b.

Note here that the slope and the (always negative) intersection of the IS-curve with the vertical axis can be chosen independently of the parameters that determine the LM-curve. There is therefore obviously scope for two or no steady state solutions in the interior of the economic domain. Note also that there is the unimportant limit case of a single steady state solution if the IS-curve is tangent to the LM-curve. Note finally, that in the situation where there are two steady states exhibiting positive output levels, one or both of them may be economically meaningless since the implied share price may not be positive then. We conclude that the case of a positively sloped LM-curve needs further specification in order to allow for meaningful and unambiguous economic conclusions under
Figure 3a: A uniquely determined Steady State in Blanchard's Bad News Case.

Figure 3b: Two or No Steady States in Blanchard's Good News Case.

all circumstances, see the extension of the model of this paper in Chiarella, Flaschel and Semmler (2000b).

The Jacobian $J^{2D}$ of the dynamics (11) – (12) reads at a steady state:

$$J^{2D} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} -(1-c)\beta_y & i\beta_y \\ kq_0/h_1 - (1-u)/K & r_0 \end{pmatrix}$$  \hspace{1cm} (14)

It is related to the slopes of the IS- and the LM-curve as follows:

Slope IS: \quad \frac{1-c}{\bar{i}} = \frac{-J_{11}}{J_{12}}

Slope LM: \quad \frac{(1-u)/K - kq_0/h_1}{r_0} = \frac{-J_{21}}{J_{22}}

These last results follow from direct inspection, or alternatively from the fact that $J_{11}dY + J_{12}dq = 0$ along the IS-schedule whilst $J_{21}dY + J_{22}dq = 0$ along the LM-schedule.
Proposition 3:

We consider steady state solutions that are economically meaningful in the variables \( Y, q, r \), i.e., that lie in the positive domain of the phase space and above \( Y \).

1. Slope IS > Slope LM (at the steady state) \( \iff \det J^{2D} < 0 \), i.e., such steady states are surrounded by saddle path dynamics.
2. Slope IS < Slope LM (at the steady state) \( \implies \det J^{2D} > 0 \), and trace \( J^{2D} < 0 \) if \( 0 < r_0 < (1 - c) \beta_y \) holds, i.e., these steady states then represent stable nodes or foci. They are unstable foci or nodes otherwise.
3. In the case of the foci or nodes that may arise at \( E_2 \) in proposition 2, we have that (according to conventional wisdom) expansionary changes in monetary or fiscal policies have indeed a depressing effect on the steady state output level \( Y_0 \), contrary to what is expected for such policy actions.

Proof: See the above calculations of the slopes of IS- and LM-curves and Figure 4, for the good news case and Figure 3a for the bad news case.

\[ \square \]

\[ \begin{align*}
q \\
\text{LM-curve} \\
\text{IS-curve}
\end{align*} \]

\[ \begin{align*}
Y
\end{align*} \]

Figure 4: The Good News Case: Saddles and Stable Nodes of Foci.

Note that Blanchard (1981) only investigates the situation where the IS-curve is steeper than the LM-curve, i.e., only the steady states \( E_1 \) of Figures 3a and 4, while he states in a footnote that \( E_2 \) has undesirable comparative static and stability properties. This however is not so obvious since the instability of the adjustment rule for share prices \( q \) is there overcome by a strongly stabilizing multiplier effect that indeed make the dynamics converge to \( E_2 \).
We have shown in this section that the simple extension and modification of the IS-LM model which adds a stock market and makes investment dependent on the value of stocks relative to the replacement value of the capital stock can produce some complications for steady state determination as well as the stability of its steady state solutions. This is in particular true for the cases where the IS curve is steeper than the LM curve where saddlepath dynamics is generated from the assumptions of perfect substitutes and perfect foresight. We now proceed to discuss the solution technique that has been established in the literature for solving the instability problems that are caused by saddlepath situations if the dynamics evolve from predetermined initial conditions.

3.5 Dynamics according to the jump variable technique

We now discuss the dynamics of the good news and the bad news cases from the perspective of the jump-variable technique solution procedures applied by Blanchard (see also Blanchard and Fischer (1989) for a closely related approach to the real-financial interaction).

Figure 3a shows for the bad news case the locally unstable saddle path situation that always results in this case from an attracting IS- and a repelling LM-Curve. This instability is overcome (following the methodology proposed by Sargent and Wallace (1973)) by assuming that the fast variable \( q \) (or better \( p_e \)) is considered as infinitely fast in its adjustment and that this adjustment always puts this variable onto one of the stable arms shown in Figure 3a in the case of unanticipated shocks, in particular fiscal and monetary ones. Thus, although there is a well specified law of motion for this variable \( q \), this law is temporarily switched off in the moment where the shock occurs and \( q \) undergoes infinitely fast vertical motion onto the only path that is capable of bringing this variable back to the steady state position that is relevant in the after shock situation.

Blanchard (1981, p.135) himself seems to imply some doubt about how convincing the jump variable assumption really can be.6 Burmeister (1980) also stresses many of the conceptual issues that such saddle point instability raises. Oxley and George (1994) give a cogent critique of the jump-variable methodology. But by and large the literature has uncritically accepted the jump variable technique methodology and indeed no longer bothers to give any rationalization of its meaningfulness and use.

Let us briefly remark that the good news case of Figure 3b adds another difficulty to the application of the jump variable technique since this case may exhibit a meaningful further (lower) equilibrium to which the economy may converge depending on the choice of initial conditions. One of the unstable arms of the upper saddlepath situation may indeed converge to this lower equilibrium (as indicated in Figure 4) and can thus not be dismissed from consideration by referring to the explosive nature of the saddlepath dynamics up to the stable arms. If a dynamic economic model is properly specified then equilibrium selection should be an outcome of the operation of the dynamics engendered by its laws of motion. If the equilibrium selection is the result of an arbitrary assumption on the placement of a certain dynamic variable onto a particular point in the phase space of the dynamics, then in our view dynamics of the model have not been properly or fully specified.7

From our perspective the jump variable technique has shortcomings on a number of fronts. First, it gives to the agents complete knowledge of the model and unlimited computational ability to calculate

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6To quote Blanchard's exact words on p.135 at the end of section II: "Following a standard if not entirely convincing practice, I shall assume that \( q \) always adjusts so as to leave the economy on the stable path to equilibrium".

7We here stress that all of our considerations apply only to descriptive approaches to macrodynamics, such as the present one, and not to model types where optimizing behavior puts the actions of economic agents onto a certain point in the considered phase space. In this latter type of approach the very nature of the optimizing procedure reduces the relevant phase space to a submanifold of the full phase space and thus does not really allow the consideration of neighbouring explosive dynamics as is usually the practice in the macrodynamics of the type considered in this paper.
the precise magnitude and nature of the jump required to arrive precisely on the stable manifold of the saddlepoint.\(^8\) Of course none of this may seem a problem if one accepts uncritically all the tenets of the rational expectations paradigm. Second, there is an assumption of infinite speed of adjustment to asset return differentials coupled with an assumption of perfect substitutability of assets. Whilst we agree that agents adjust very rapidly to return differentials this adjustment may not be infinite and may vary according to market conditions. Also some imperfect substitutability between assets may exist for a variety of motives. Third, is the assumption of perfect foresight of expectations. This assumption became to be uncritically accepted because the jump-variable methodology developed in the era when only the totally rational and omniscient economic agent inhabited the world of macroeconomic theorizing. In an era when some place is also being found for boundedly rational agents, it would seem appropriate to consider the impact of less than perfect foresight of expectations.

4 State-of-Market Dependent Reaction Speed — An Alternative to the JVT

As we pointed out in the previous section the jump variable solution technique relies on perfect asset substitutability and on agents adjusting infinitely rapidly to asset return differentials. Perfect foresight expectations are also assumed. A relaxation of both of these assumptions is the basis of the alternative to the jump variable technique (JVT) in Chiarella (1986) and Flaschel and Sethi (1999) in relation to the monetary dynamics model, in Chiarella (1990) for the Dornbusch (1976) model of exchange rate dynamics and, in Chiarella, Flaschel and Semmler (2000a) in relation to the present Blanchard model. This approach essentially relies on the imperfect substitutability between alternative assets and in particular on budget constraint considerations that must become binding as the economy moves further and further from steady state. The result of this alternative scenario in the context of the present model is a close to relaxation oscillation in the \(q, z\) phase plane with the relaxation oscillation occurring in the perfect substitutability/perfect foresight (i.e.\(\beta_q = \beta_z = \infty\)) limit. It turns out that it is the expectations, \(z\), which adjust with a jump and Tobin's \(q\) adjusts continuously but not smoothly. The \((q, z)\) phase plane for this situation is depicted in Figure 5a and the resulting dynamics for \(q\) and \(z\) in Figures 5b and 5c respectively. We refer the reader to Chiarella, Flaschel and Semmler (2000a) for details of the analysis behind these figures. The solution procedure to the saddle point instability problem just discussed relies very heavily on the eventual operation of budget constraints, but these may take a long time to become binding. Also the close to relaxation cycles to which it gives rise may be seen as too extreme, even when the model is perturbed by noise and other real world features. Furthermore it leaves economic agents no scope to react to changing market conditions. The solution to the saddle point instability problem that we develop in this section seeks to address these three shortcomings.

We shall assume that our economic agents are less perfect than those who inhabit the world of rational expectations. Recall that in the present context perfection means the agents have full knowledge of the model and display speed of reaction coefficients \(\beta_z = \infty\) and \(\beta_q = \infty\). However we do assume that they are not too far removed from this (economists') ideal perfect state, in that their value of \(\beta_z\) is still very high so that they are operating with close to perfect foresight expectations. We remove from the agents full knowledge of the model (i.e. they do not know the laws of motion) but do allow them to have some reasonable knowledge of its steady state. Our most radical departure from the world of rational expectations is our assumption about the reaction coefficient \(\beta_q\), which changes as a function of market conditions. When market conditions are such that \(q\) is close to its steady state \(q_0\),

\(^8\)This computational burden may not seem too onerous in the simple model discussed in this paper, but we suggest it would become massive in a higher order model involving any degree of nonlinearity. These computations are manageable in a completely linear (or linearised) framework and there is a developed methodology; see e.g. Buiter(1984).
the reaction coefficient $\beta_q$ is rather high so that agents are reacting strongly to the return differential. However the high $\beta_q$ (coupled with the high $\beta_z$) causes the steady state to be locally unstable and hence a rise (or fall) in the stock market. Agents initially are happy to go with this general movement in the stock market, however as it proceeds further and further they are conscious that the economy is moving ever further from its steady state (of which they are assumed to have some reasonable idea) and they start to react more cautiously to the return differential. This cautiousness is reflected in a gradual lowering of the value of the coefficient $\beta_q$, which eventually becomes sufficiently low to cause a turn-around in the dynamics that once again become stable towards the steady state. Eventually $\beta_q$ returns to former high levels and the possibility of another upward (or downward) stock market movement is established. The behaviour of $\beta_q$ as a function of the difference in $q$ from its steady state value $q_0$ is illustrated in Figure 6. We have drawn this function somewhat skewed to the right to indicate greater (less) caution when the share market is below (above) its steady state value. This relation may also exhibit both euphoric (the higher graph) and depressed (the lower graph) states depending on particular news events arriving in the market.\footnote{We do not specifically model the arrival of news in this paper, though this could be done for example by using a Poisson jump process.} A similar type of reaction function has been introduced into the model of monetary dynamics by Chiarella and Khomin (2000).

We thus have fast adaptively formed expectations and a fast adjustment of share prices to the return

Figure 5: The relaxation cycle solution to the saddlepoint instability problem.
differential close to the steady state. However, far from the steady state we assume that agents are aware that the economy is approaching some sort of extreme situation and become increasingly cautious and thus only more and more sluggishly continue to adjust into a direction that they believe cannot continue for much longer. We now seek to analyse the qualitative nature of the dynamics that this state-of-market dependent reaction coefficient brings about.

4.1 Local stability analysis

Referring to equations (5), (8) and (9) we now consider the following 3D dynamical system:

\[
\begin{align*}
\dot{Y} &= \beta_y(c-1)Y + iq + A, \quad c - 1 < 0 \\
\dot{q} &= \beta_q(1-u)Y/K - \delta + z - r, \quad r = (kY + h_o - m)/h_1 \\
\dot{z} &= \beta_z(q - z), \quad z = \hat{q}^e 
\end{align*}
\] (15) (16) (17)

We thus now allow, via equation (16), for a rate of return differential between short-term bonds and shares, resulting from the current level of Tobin's $q$ and expected capital gains $z = \hat{q}^e$, which can now differ from actually realized capital gains. This differential drives asset demands and with them share prices (all other things kept fixed) into the direction of a reduction of this return differential (as long as expectations remain fixed); for example up and the dividend rate of return down if the short-term rate of interest falls short of the expected rate of return on equities. Subsequently, however, expected capital gains are revised on the basis of the actual increase in share prices via equation (17), which gives further momentum to the process of increasing capital gains and so forth. We note that the traditional JVT analysis of the dynamics as discussed in section 3.5 comes about by setting both $\beta_q$ and $\beta_z$ equal to infinity, resulting in algebraic conditions on rates of return and expectation formation in the place of the above two laws of motion in equation (16) and (17).

The steady states of this system are the same as those of the limit case $\beta_q = \beta_z = \infty$. The Jacobian of the dynamical system (15) – (17), calculated at a steady state turns out to be

\[\text{Jacobian} = \begin{pmatrix}
\end{pmatrix}\]
\[
J^{3D} = \begin{bmatrix}
J_{11} & J_{12} & 0 \\
-\beta_q J_{21} & -\beta_q J_{22} & \beta_q q_0 \\
-\frac{\beta_q \beta_s}{q_0} J_{21} & -\frac{\beta_q \beta_s}{q_0} J_{22} & \beta_z (\beta_q - 1)
\end{bmatrix}
\] (18)

where the elements \( J_{11}, J_{12}, J_{21} \) and \( J_{22} \) were defined at equation (14). Routine calculations yield

\[
det J^{3D} = \beta_q \beta_s det J^{2D},
\]

and that the principal minors of \( J^{3D} \) satisfy

\[
J^3_1 = \beta_z \beta_s J_{22} > 0, \quad J^3_2 = \beta_z (\beta_q - 1) J_{11} \quad \text{and} \quad J^3_3 = -\beta_q det J^{2D}.
\]

(20)

We recall the Routh-Hurwitz necessary and sufficient conditions for the local asymptotic stability of a steady state of the 3D dynamical system (15) - (17) (see Brock and Malliaris (1989, p.75) or Gantmacher (1959)), namely

\[
trace(J^{3D}) < 0, \quad \Theta \equiv J^3_1 + J^3_2 + J^3_3 > 0, \quad \text{det} J^{3D} < 0,
\]

and

\[
\Delta \equiv -trace(J^{3D})(J^3_1 + J^3_2 + J^3_3) + \text{det} J^{3D} > 0.
\]

(21)

At the steady state \( E_1 \) considered by Blanchard (where the IS-curve is steeper than the LM-curve) we know from proposition 3 that \( \text{det} J^{2D} < 0 \). Hence by equations (19) and (20) we must have at such a steady state that \( \text{det} J^{3D} < 0 \) and \( J^3_3 > 0 \).

The local stability properties of the steady state are going to be very much determined by the signs of the quantities \( \Theta \) and \( \Delta \). We readily calculate that

\[
\Theta = \beta_q \left| \text{det} J^{2D} \right| + \beta_z \left| J_{11} \right| - \beta_q \left| J_{11} \right| - J_{22} \right|.
\]

(22)

Unfortunately it is not possible to make any definite assertion about the sign of the quantity \( J_{11} \left| - J_{22} \right| = (1 - c) \beta_y - r_o \); either + or - being plausible depending on the relation between the speed of adjustment in the output market, the propensity to consume and the steady state interest rate. Experimenting with empirically plausible values\(^\text{11}\) of \( c, \beta_y \) and \( r_o \) would suggest that \( J_{11} \left| - J_{22} \right| \) is in the range -0.04 to +0.04. In Figures 7a, b we indicate the curves in the \((\beta_q, \beta_z)\) plane along which \( \Theta = 0 \) as well as the sign of \( \Theta \) off these curves. In Figure 7b clearly \( \Theta > 0 \) for all positive \( (\beta_q, \beta_z) \). Consider Figure 7a, and in particular the asymptote at \( \beta_q = \beta_z \equiv | J_{11} / | J_{11} \left| - J_{22} \right| \), whose minimum value would be expected to be around 2.5 for the plausible empirical values just discussed. Values of \( \beta_q \) beyond this value would be considered rather high in the dynamic story we are going to describe below. Indeed empirical estimations\(^\text{12}\) of a model similar to the present one on US data

\(^\text{11}\)For example with \( c = 0.6, r_o = 0.06, \beta_y = 0.25 \) we obtain \( | J_{11} \left| - J_{22} \right| = 0.02 \). Reasonable variations around these values would give the range indicated.

for the period 1960-1993 suggest an average value of $\beta_q$ over that period of around 2. Thus whether the relation between $(1 - c)\beta_y$ and $r_0$ is such that Figure 7a or 7b prevails, we argue that $\Theta > 0$ for low to moderate values of $\beta_q$.

\begin{equation}
\beta^*_q = \frac{|J_{11}|}{|J_{11}| - J_{22}}
\end{equation}

(7a): $|J_{11}| - J_{22} > 0$.

(7b): $|J_{11}| - J_{22} < 0$.

Figure 7: The Sign of the Quantity $\Theta$
Consider next the quantity $\Delta$, which may be expressed in the form

$$\Delta = \alpha_0 + \alpha_1 \beta_z + \alpha_2 \beta_z^2$$

where

$$\alpha_0 \equiv -\beta_y(1-c) + \beta_q r_0)\beta_q \det J^{2D}, \quad \alpha_1 \equiv \beta_y^2 \det J^{2D} + (\beta_y(1-c) + \beta_q r_0)(|J_{11}| + \beta_q (J_{22} - |J_{11}|)), \quad \alpha_2 \equiv (1 - \beta_q)(|J_{11}| + \beta_q (J_{22} - |J_{11}|)).$$

We note that $\alpha_0 > 0$ but it is not possible to give a definite sign to $\alpha_1$ and $\alpha_2$, both of whose signs depend on the sign of $(J_{22} - |J_{11}|)$ and in the case of $\alpha_2$ also on the sign of $(1 - \beta_q)$.

Proposition 4:

"We consider steady states of Blanchard type solely. For the dynamics (17) – (19) there holds:

1. The steady state of the dynamics (17) – (19) is locally asymptotically stable for sufficiently large $\beta_z$ if $\beta_q < 1$.

2. In the case $1 < \beta_q < \beta_q^*$ local asymptotic stability is lost for sufficiently large $\beta_z$.

Proof: The crucial quantity is $\alpha_2$ which can also be written

$$\alpha_2 = (1 - \beta_q)(\beta_q^* - \beta_q)(|J_{11}| - J_{22})$$

where

$$\beta_q^* \equiv |J_{11}|/(|J_{11}| - J_{22}).$$

1. Consider $\beta_q < 1$:

(a) Suppose $J_{22} > |J_{11}|(\Rightarrow \beta_q^* < 0)$, then $\alpha_2 > 0$ and the relationship between $\Delta$ and $\beta_z$ must be as shown in Figure 8a. For $\beta_z$ sufficiently large $\Delta > 0$ and all Routh-Hurwitz conditions are satisfied. Hence the steady state is locally asymptotically stable.

(b) Suppose $J_{22} < |J_{11}|(\Rightarrow \beta_q^* > 0)$. Given that for empirically plausible parameters (as discussed above) $\beta_q^* > 2.5$, then we still have $\alpha_2 > 0$ in this case. Again local asymptotic stability follows.

2. Consider $\beta_q > 1$:

(a) Suppose $J_{22} > |J_{11}|(\Rightarrow \beta_q^* < 0)$, then $\alpha_2 < 0$. Now the relationship between $\Delta$ and $\beta_z$ must be as shown in Figure 8b. For $\beta_z$ sufficiently large $\Delta < 0$ and so local asymptotic stability is lost.

(b) Suppose $J_{22} < |J_{11}|(\Rightarrow \beta_q^* > 0)$. Now $\alpha_2 < 0$ if $\beta_q < \beta_q^*$; and again local asymptotic stability is lost for $\beta_z$ sufficiently large.
Thus with expectations adjusting very rapidly (though not necessarily infinitely fast) we have established that Blanchard type steady states are attracting (repelling) in states of the market in which speed of adjustment on the market for equities is relatively low (high).

Furthermore the steady state position $E_2$ in Figure 3b is now always unstable since $detJ^{3D} > 0$ there. Its counterintuitive comparative static properties are thus no longer of any relevance.

4.2 The qualitative behaviour of the financial dynamics – phase diagram switching

In order to focus on the financial market dynamics we make the observation that typically they will be much faster than the real market dynamics, particularly during periods of bull and bear markets\textsuperscript{13}. Thus we can obtain an understanding of the qualitative features of the dynamics of (15) - (17) by holding output $Y$ at its steady state value $Y_0$. We are then able to use the perspective of a two-dimensional phase space to obtain a more transparent description of the dynamics.

We therefore now consider the dynamics of the financial sector from the perspective of the two laws of motion\textsuperscript{14}

\[
\hat{q} = \beta_q \left( \frac{r_0 q_0}{q} + z - r_0 \right) \quad (23)
\]
\[
\hat{z} = \beta_z (\hat{q} - z) \quad (24)
\]

where we have used the fact that $\rho_0 = r_0 q_0$. In order to simplify the notation we choose units of output measurement so that $q_0 = 1$.

\textsuperscript{13}In the subsequent discussion we shall use the term bull (bear) market to describe a situation in which $q$ is rising (falling).

\textsuperscript{14}Since the real part of the model is deactivated, we have a uniquely determined steady state solution $q_0, z_0 = 0$ for this two-dimensional subsystem.
The Jacobian of the dynamics (23), (24) at the steady state \( q_0 = 1, z_0 = 0 \) reads:

\[
J = \begin{pmatrix}
-\beta q r_0 & \beta q \\
-\beta z \beta q r_0 & \beta z (\beta q - 1)
\end{pmatrix}
\]

and we readily calculate \( \det J = \beta q \beta z r_0 > 0 \).

Thus for all adjustment speeds \( \beta z \) that are sufficiently large, the dynamics have two regimes:

- **Regime 1**: \( \det J > 0 \), trace \( J < 0 \) for \( \beta q < 1 \): Stable nodes or foci.
- **Regime 2**: \( \det J > 0 \), trace \( J > 0 \) for \( \beta q > 1 \): Unstable nodes or foci.

These two regimes correspond to the two regimes discussed in proposition 4.

In regime 1 we assume that we are always close to \( z = \tilde{q} \) (due to our assumption of a high adjustment speed \( \beta q \)), so that the dynamics will be largely dominated by the perfect foresight law of motion:

\[
\tilde{q} = \beta q \left( \frac{r_0}{q} + \tilde{q} - r_0 \right) \quad \text{or} \quad \tilde{q} = \frac{1}{1 - \beta q} \left( \frac{r_0}{q} - r_0 \right)
\]

(25)

in such a situation Figure 9 shows the movements (with double arrowheads) according to (25) along the \( \tilde{z} = 0 \) isocline towards the steady state \( E \), either with \( q > 1, \tilde{q} < 0 \) (a tranquil bear market) or \( q < 1, \tilde{q} > 0 \) (a tranquil bull market). The dynamics of (23), (24) will be quite close to this perfect foresight path, say the path \( D_0D_1 \) in Figure 9. This motion will be relatively slow (compared to the speed of adjustment of \( z \) away from neighbourhoods of this curve) as it is largely determined by \( \beta q \).

In regime 2 we have however an explosive situation and thus a situation where \( \tilde{q} \) and \( z \) depart from each other in a more or less extreme way.\(^{15}\) The motion of \( q \) will speed up compared to regime 1, either rising rapidly or falling rapidly.

Let us consider the tranquil case \( \beta q < 1 \) first. In this case we have as phase portrait of the above 2D dynamics the situation shown in Figure 9. The long(short) arrowheads in the \( z(q) \) direction indicate fast(slow) motion of that variable.

Consider the motion from \( D_0 \) to \( D_1 \) in Figure 9. This is the tranquil bull market phase with \( q \) rising fairly slowly. However as this motion proceeds \( q \) moves closer to its steady state value and so (see Figure 6) the coefficient \( \beta q \) increases in value, so that the tranquil bull market turns into an active bull market, i.e., \( \beta q > 1 \) is established\(^{16}\) as adjustment speed of share prices and Tobin's \( q \). The above law of motion (25) now describes an explosive dynamics and the adaptive expectations then lose contact with the formerly stable close to perfect foresight dynamics. In such a case the phase diagram of the two-dimensional dynamics switches to the one shown in Figure 10 that describes the activated bull market phase.

\(^{15}\)It is easy to show that returns are rising (falling) with respect to \( q \) in this unstable situation if \( \beta q > 2 \) \((\beta q \in (1, 2))\) holds respectively.

\(^{16}\)Such a change in \( \beta q \) could be further accentuated by some news event that causes market participants to react more strongly to the return differential. This is represented by the move from the middle to the upper curve in Figure 6.
\[ \dot{z} = 0 : q = \frac{r_0}{(1-\beta_0)^z/\beta_0^z + r_0} \]

\[ \dot{q} = 0 : q = \frac{r_0}{r_0 - z} \]

Figure 9: A tranquil bull market (with rising q less than one).

Figure 10: An activated bull market (starting from the position \(D_1\) in Figure 9).
\[ \dot{z} = 0 : q = \frac{r_0}{(1-\beta_q)z/\beta_q + r_0} \]

\[ \dot{q} = 0 : q = \frac{r_0}{r_0 - z} \]

Figure 11: A tranquil bear market (with near myopic perfect foresight established).

\[ \dot{z} = 0 : q = \frac{r_0}{(1-\beta_q)z/\beta_q + r_0} \]

\[ \dot{q} = 0 : q = \frac{r_0}{r_0 - z} \]

Figure 12: An activated bear market and the rapid decline of \( q \).

\[ \dot{z} = 0 : q = \frac{r_0}{(1-\beta_q)z/\beta_q + r_0} \]

\[ \dot{q} = 0 : q = \frac{r_0}{r_0 - z} \]

Figure 13: Return to a tranquil bull market.
Starting from $D_1$ we are now on an explosive trajectory $D_1D_4$ (for $\beta_2$ sufficiently large). At $D_1$ both $z$ and $q$ are changing rapidly (as indicated by the arrowheads of roughly equal length). This situation continues at $D_2$ which may be regarded as a very activated phase of the bull market. By $D_3$ the growth in $q$ is starting to slow a little and market participants at the same time start to reduce a little the value of $\beta_q$ (hence the shorter arrowhead in the $q$ direction). By $D_4$ the share market is declining and market participants reduce $\beta_q$ much further and eventually $\beta_q < 1$ is again reestablished. At the same time the motion in the $z$ direction dominates the dynamics. This induces a switch in the phase diagram back to the tranquil phase diagram of Figure 9 with the resulting dynamics shown in Figure 11.

Figure 11 shows that expectations adjust fairly rapidly (indicated by the double arrowheads) from $D_4$ to $D_5$ (close to the now again attracting $\dot{z} = 0$ isocline), a (nearly) horizontal adjustment to the region of close to perfect foresight dynamics. The tranquil bear market phase now begins, where $q(>1)$ is declining and $\dot{q} = z < 0$ is increasing. As this process proceeds it will bring the economy back to the region of steady state where $q \approx 1$. But here $\beta_q$ starts to increase again and the activated bear market phase is eventually set off, say at $D_6$ in Figure 11. The slope of the $\dot{z} = 0$-isocline becomes positive again so that the dynamics are governed by the arrowheads in Figure 10 giving motion from $D_6$ to $D_7$ as depicted in Figure 12.

During this phase the downward motion in $q$ becomes quite rapid. The agents are more risk averse in bear markets than in bull markets, so there will fairly soon be a return to cautiousness. The slope of $\dot{z} = 0$ again becomes less than one and thus induces a switch in the phase diagram back to the asymptotic stability case. The position meanwhile reached is $D_7$, from which the close to myopic perfect foresight is reestablished quickly. We are then back in the position of a tranquil bull market at $D_0$ in Figure 13 from which we started as shown in Figure 9. We conclude from Figure 13 that activated bear markets will recover automatically and may thus lead us back to the situation of tranquil bull markets (or other scenarios, if tranquillity is replaced by other types of activity).17

The entire cycle from tranquil bull market, to activated bull market, to tranquil market back to tranquil bull market is shown in Figure 14. Figure 15 shows the corresponding time series pattern for $q$.

Thus the changing adjustment speed $\beta_q$ of stock market valuations is at the heart of the rhythm that takes us from tranquillity to activated markets and back and from bull markets to bear markets and back, because activated markets take us into positions of the phase space from where close to myopic perfect foresight and tranquillity is reestablished again. There may be other intermediate situations in the change of the slope of equity price adjustment speeds which however only make the considered dynamics more refined and detailed without changing the general picture here outlined. Note finally that other switching sequences between tranquil or activated bull or bear markets are conceivable that may exhibit other regime switches and different accelerating rhythms between them.

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17 Note that this approach does not imply that markets are always in extreme positions as would be the case if a standard relaxation oscillations approach was applied, see Chiarella, Flaschel and Semmler (2000a) for example.
5 Conclusions

We have developed an extension to the Blanchard (1981) model of stock market-real market interaction. The key features of this extension are imperfect substitution between alternative assets, less than perfect foresight expectations and (most importantly) a state-of-market dependent coefficient of reaction to the expected return differential between different assets. The original Blanchard model can be recovered as the limiting case when there is perfect substitution and perfect foresight.

Because the reaction coefficient is relatively high close to the steady-state the extended Blanchard model also exhibits local instability. However as there is movement away from the steady state the state-of-market dependent reaction coefficient operates in such a way that motion eventually is back towards steady state where the reaction coefficient rises again. In this way the alternance between bull market and bear market phases is established. Also an economically plausible method to the poorly motivated jump-variable technique to "close" the dynamics of an otherwise unstable economic
process has been established.

A number of generalizations and extensions of the model developed here are possible.

First, the framework developed here could be further extended to allow a study of the dynamic impact of anticipated and unanticipated monetary and fiscal shocks, as Blanchard (1981) did in his original analysis based on the jump-variable technique.

Second, the proper inclusion of the bond price dynamics needs to be considered. As we pointed out in section 2, there is no feedback of the bond price dynamics into the rest of the system. This could be done, for example by allowing interest rate effects in the aggregate demand function.

Third, the shape of the reaction coefficient $\beta_q$ could be modelled in a more sophisticated way than displayed in Figure 6. Empirically we observe stock markets to remain close to steady state more frequently than Figure 15 suggests. It is thus more likely that $\beta_q$ would be lower close to $q_0$, then rise slightly away from $q_0$ before falling off again further away from $q_0$. One could imagine a valley in the peak in Figure 6. With this shape for $\beta_q$ the bull and bear markets would be set-off by sufficiently large random shocks to the market. Also in this way the alternance between bull and bear regimes would not be so regular, depending rather on the timing of shocks and their sign.

Fourth, heterogeneous agents (fundamentalists and chartists) could be introduced into the model and so give more rationale to the story of variable reaction coefficient. Fundamentalists would presumably be dominant during tranquil phase, whilst chartists would became more dominant during activated phases. The ideas introduced by Brock and Hommes (1997) could be used to model the changing role of fundamentalists and chartists during the various phases of the cycle.

Finally here we have relied on a variable reaction coefficient ($\beta_q$) to expected return differentials as a means to "close" the dynamics of an otherwise unstable system. An alternative approach would involve leaving $\beta_q$ constant and instead allowing the speed of adjustment of expectations ($\beta_e$) to vary according to market conditions.

6 References


Money, Credit and Banking, 12, 217-228.


