Factors Affecting the Probability of Bankruptcy

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Abstract
The majority of classification models developed have used a pool of financial ratios combined with statistical variable selection techniques to maximise the accuracy of the classifier being employed. Rather than follow an "ad hoc" variable selection process, this paper seeks to provide an economic basis for the selection of variables for inclusion in bankruptcy models, which are based on accounting information. Variables which occur in bankruptcy probability expressions derived from the solution of an stochastic optimising model of a firm are 'proxied' by variables constructed from financial statement data. The random nature of the life time of a single firm provides the rationale for the use of duration or hazard-based statistical methods in the validation of the derived bankruptcy probability expressions. The Cox (1972) proportional hazards model is used to estimate the coefficients and standard errors that are required for the validation of the derived bankruptcy probability expressions. Results of the validation exercise confirm that the variables included in the empirical hazard formulation behave in a way that is consistent with the model of the firm.

M. Peat
1 Introduction

A continuing focus of academic interest has been bankruptcy classification models, which are capable of differentiating between a company which will become bankrupt and one which continues to operate. The MDA model of Altman et. al. (1977) and the Logit model of Ohlson (1980) continue to be the focal point for empirical bankruptcy modeling. The early literature is surveyed in papers by Jones (1987) and Zavgren (1983). Recent empirical studies using statistical approaches to bankruptcy classification seek to, generalise the Altman model, Grice and Ingram (2001), or compare the Altman and Ohlson models, Lennox (1999) and Mossman, Bell, Swartz and Turtle (1998). A problem which is mentioned in a number of studies is the lack of a generally accepted theory which explains bankruptcy. In the absence of such a theory, researchers have used statistical variable selection methods on large sets of variables, derived from financial statements, to maximise the number of firms correctly classified. Another approach has been the use of factor analysis to determine a set of explanatory variables for bankruptcy models. Factor analysis of the variables used in prior bankruptcy prediction models by Pinches, Mingo & Caruthers (1973) and Chen and Shimedra (1981), identified seven factors in the variables that have been used in bankruptcy prediction models. Zavgren (1985) used these factors as the inputs to a logit based study.

There have been a number of attempts to provide a theoretical model for bankruptcy behaviour. Wilcox (1971, 1976) proposed a ”gamblers ruin” framework, which was not supported in empirical testing. Scott (1981) derived an expression for the probability of failure from a 2 period model of a firm with access to capital markets. The variables in this model broadly match those used in the Altman (1977) Zeta model. In Scapens, Ryan and Fletcher (1981), Catastrophe Theory is used to connect a firm’s financial position, as reflected by financial ratios, to the behaviour of creditors in deriving a bankruptcy probability expression. Recent work has used the ideas of contingent claims and option pricing formula to provide a bankruptcy probability expression. Charitou and Trigeorgis (2000) derive variables from a compound option model which are included in a logistic regression framework. They found that the market based variables from the option pricing formula
had explanatory power. Hillegeist et. al. (2002) tested a modified option based bankruptcy probability formula that included the expected market return on the firms assets rather than the risk free rate. They used a discrete hazard function approach proposed by Shumway (2001) for empirical testing, and find that all the variables have explanatory power. One of the aims of tests of the option-based approach to bankruptcy prediction is to demonstrate that market-based variables are superior to financial statement information in bankruptcy prediction models. To this end Hillegeist et. al. (2002) added a Z-sore value as an explanatory variable to their model. This variable is found to be significant, as is the expected return on assets, which is constructed from financial statement information. These findings support the conclusion that appropriate financial statement information should be included in empirical bankruptcy models.

The main objective of this study is to outline the process of linking an economic model which includes the possibility of bankruptcy to variables derived from financial statement information, which capture the features of the model and are suitable for inclusion in predictive models. The study takes as its starting point the bankruptcy probability expressions derived from a model of the firm in which management make both investment and financing decisions. The effect of the variables in these expressions on the probability of bankruptcy is then explored. Following this exploration approximating variables are constructed from standard financial statements. These approximating variables are used in a probabilistic regression framework to verify the effects of the variables defined on the probability of bankruptcy. The results of these probabilistic regressions indicate that the variables defined are significant in determining a firms probability of bankruptcy and behave in the manner predicted.

2 The Determinates of Bankruptcy

A model of a firm facing an uncertain environment with the possibility of bankruptcy is developed and analysed in Peat (2002 Ch. 3). In the model, a firm is created with given initial equity, \( K_0 \). These funds can be invested in productive resources, \( c_0 \), or held as cash balances, \( m_0 \).
The productive resources are used to earn random earnings, \((\theta f[c_t]) dt + \epsilon dz_t\)\(^1\), in any period. If earnings are positive, they can be used to pay dividends to shareholders, \(w_t\), invest in new productive resources, \(v_t\), repay outstanding debt, \(x_t\), or increase the firm’s cash balance. The firm is able to borrow and repay funds up to a credit limit; \(0 \leq \frac{x_t}{c_t + m_t} \leq h\) where \(0 \leq h \leq 1\). When the cash position of the firm falls to zero the firm is bankrupt. The firm attempts to maximise the stream of dividends paid to shareholders during its random lifetime, \(\tau\). There are seven possible solutions to the model, each with a bankruptcy probability expression, derived by application of the dynamic programming algorithm. A summary of these solutions follows.

**Regime 1. The firm borrows, improves the cash position and distributes dividends**

The firm pays the maximum allowable dividend, \(w = \theta f(c) - rx\). The firm borrows up to its debt limit, then \(u = a - x\). Further the firm does not invest, so \(v = 0\) and, consequently, the stock of productive resources remains constant. The cash position of the firm changes by the random component of earnings plus the amount borrowed, resulting in \(dm = (a - x)dt + \epsilon dz\).

Combining these results and applying the Euler-Mautyama approximation method, a probability of bankruptcy for this regime can be found. The probability of bankruptcy, \(\Pr(M_{t+1} < 0)\), will be the left tail probability

\[
\Pr (M_{t+1} < 0) = \Pr \left( \Delta z < \frac{-[M_t + (a - x)]}{\epsilon} \right).
\]

This condition is most likely to occur when the firm has low cash reserves and a low borrowing limit. A low borrowing limit can come from a low \(h\) value imposed by lenders, or a small asset

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\(^1\) The value \(\theta f[c_t]\) is equivalent to the expected component of the earnings of the firm. This variable represents the earnings that the firm will generate using the current stock of productive equipment, \(c_t\). The function \(f(\bullet)\) is concave with \(f(0) = 0\). The function is scaled by a constant parameter \(\theta\). The product \(\epsilon dz_t\) is the extent of the earnings surprise observed at time \(t\).

\(^2\) This limit is set by constraining the debt to assets ratio of the firm. This constraint, in conjunction with the interest rate on debt, \(r\), provides a mechanism that financial institutions can use to manage their exposure to the risky earnings of the firm.

This constraint also provides a bound on the amount that can be borrowed or repaid in any period. If the firm has not borrowed \((x_t = 0)\), then the maximum it can borrow is \(h(c_t + m_t)\). If the firm has borrowed up to its limit, the maximum repayment that the firm will make will be \(h(c_t + m_t)\). So the change in the firm’s level of debt is bounded by

\[
|u_t| \leq h[c_t + m_t],
\]

which is derived from the non negativity constraint on debt and the upper bound on borrowing. For convenience the constraint

\[
|u_t| \leq a_t \text{ where } a_t = h(c_t + m_t) \text{ and } 0 \leq a \leq \infty
\]

will be used.
base, $(c + m)$. A large negative random component of earnings could lead to bankruptcy under these conditions.

**Regime 2. The firm repays debt and distributes dividends** The firm pays the maximum allowable dividend, $w = \theta f(c) - rx$. Additionally, the firm repays as much debt as possible, namely $u = -x$. The firm does not invest so, $v = 0$, and, consequently, the stock of productive resources remains constant. The cash position of the firm changes by the random component of earnings minus the amount of debt repaid $dm = -xdt + \epsilon dz$. The probability of bankruptcy will be the left tail probability

$$
\Pr (M_{t+1} < 0) = \Pr \left( \Delta z < \frac{-[M_t - x]}{\epsilon} \right).
$$

This condition is most likely to occur when the firm has low cash reserves and a large debt level. In this case the debt repayment depletes the cash reserve, a large negative random earnings element would then lead to bankruptcy.

**Regime 3. The firm borrows and builds up the cash reserve** The firm pays no dividend, so $w = 0$. The firm borrows as much debt as possible, $u = a - x$. The firm does not invest so, $v = 0$, and, consequently, the stock of productive resources remains constant. The cash position of the firm is increased by the level of earnings plus new borrowings less interest payable. That is,

$$
dm = [\theta f(c) + (a - x) - rx] dt + \epsilon dz.
$$

The probability of bankruptcy will be the left tail probability

$$
\Pr (M_{t+1} < 0) = \Pr \left( \Delta z < \frac{-[M_t + (\theta f(c) + (a - x) - rx)]}{\epsilon} \right).
$$

This condition is most likely to occur when the firm has low cash reserves and a low level of productive resources. In this case the ability of the firm to borrow is limited by the small asset base, $(c + m)$. Earnings, $\theta f(c)$, are also limited by the level of productive resources. The interest payment will also deplete the cash reserve.

**Regime 4. The firm repays debt and builds up the cash reserve** In terms of the model, the firm pays no dividend, so $w = 0$. The firm repays at the maximum allowable rate so, $u = -x$. The firm does not invest so, $v = 0$, and, consequently, the stock of productive resources remains
constant. The cash position of the firm is increased by the level of earnings less interest payable and the amount of debt repaid. This is given by, \(dm = [\theta f(c) - x - rx] dt + \epsilon dz\). The probability of bankruptcy will be the left tail probability

\[
Pr(M_{t+1} < 0) = Pr\left(\Delta z < \frac{-[M_t + (\theta f(c) - x - rx)]}{\epsilon}\right).
\]

This condition is most likely to occur when the debt repayment and interest payment are large relative to the cash reserve and level of expected earnings. A large negative random component of earnings can then wipe out the remaining cash and cause bankruptcy.

**Regime 5. The firm borrows and invests** That is, the firm pays no dividend so \(w = 0\). The firm borrows at the maximum allowable rate with \(u = a - x\). The firm invests at the maximum allowable rate, given by \(v = m + \theta f(c) + (a - x) - rx\), and, consequently, the stock of productive resources changes by \(v\). The cash position of the firm changes by the random component of earnings minus the portion of the cash reserve that has been allocated to investment in productive equipment, \(dm = -mdt + \epsilon dz\). The probability of bankruptcy will be the left tail probability

\[
Pr(M_{t+1} < 0) = Pr(\Delta z < 0).
\]

As the firm is devoting all possible resources to increasing its stock of productive resources, any unexpected negative earnings result will cause bankruptcy. This type of bankruptcy can be thought of as a working capital shortage in an environment of rapid growth.

**Regime 6. The firm repays debt and invests** The firm pays no dividend, so \(w = 0\). The firm repays as much debt as possible with \(u = -x\). The firm invests at the maximum allowable rate, \(v = m + \theta f(c) - x - rx\). Consequently, the stock of productive resources changes by \(v\). The cash position of the firm changes by the random component of earnings minus the portion of the cash reserve that has been allocated to investment in productive equipment, which is given by \(dm = -mdt + \epsilon dz\). The probability of bankruptcy will be the left tail probability

\[
Pr(M_{t+1} < 0) = Pr(\Delta z < 0).
\]
As the firm is devoting all possible resources to increasing its stock of productive resources and repaying debt, any unexpected negative earnings result will cause a bankruptcy. This was also the case in regime 5, where the firm was attempting to grow at the maximum rate. This type of bankruptcy can be thought of as a working capital shortage in the case of rapid growth.

**Regime 7. The firm borrows, invests and pays dividends** In terms of the model, the firm pays the maximum allowable dividend, \( w = \theta f(c) - rx \). The firm borrows at the maximum allowable rate, which is \( u = a - x \). The firm invests as much as possible \( v = m + (a - x) \), and, consequently, the stock of productive resources changes by \( v \). The cash position of the firm changes by the random component of earnings minus the portion of the cash reserve that has been allocated to investment in productive equipment, \( dm = -mdt + cdz \). To find the probability of bankruptcy for this regime, we combine these results with the specification of earnings uncertainty in the model. The probability of bankruptcy will be the left tail probability

\[
Pr (M_{t+1} < 0) = Pr (\Delta z < 0).
\]

This condition is most likely to occur because the cash reserve has been allocated to investing for future growth. Under these circumstances a negative random component of earnings will then cause bankruptcy. This regime describes firms that have embarked on an aggressive growth program. This type of failure is related to working capital management problems.

Five unique bankruptcy probability expressions can be derived from the seven possible solutions of this model. They are collected in Table 1. Each of these expressions is conditional on the firm being a going concern at the beginning of the period of interest.

<table>
<thead>
<tr>
<th>Failure Probability Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Pr \left( z &lt; -\frac{M_t + (a-x)}{\epsilon} \right) )</td>
</tr>
<tr>
<td>( Pr \left( z &lt; -\frac{M_t - \epsilon}{\epsilon} \right) )</td>
</tr>
<tr>
<td>( Pr \left( z &lt; -\frac{[M_t + \theta f(c) + (a-x) - rx]}{\epsilon} \right) )</td>
</tr>
<tr>
<td>( Pr \left( z &lt; -\frac{[M_t + \theta f(c) - x - rx]}{\epsilon} \right) )</td>
</tr>
<tr>
<td>( Pr (z &lt; 0) )</td>
</tr>
</tbody>
</table>

Table 1: Five unique Failure Probability Expressions

The variables that make up these expressions are summarised in Table 2.
Variable | Description
--- | ---
a | maximum change in debt allowed under a debt to equity constraint (see note 2)
x | the debt level at time \(t\)
\(M_t\) | cash balance at \(t\)
\(rx\) | interest paid in period \(t\), where \(x\) is the debt level and \(r\) the interest rate
\(\theta f(c)\) | expected earnings in period \(t\)
\(\epsilon\) | the variability of earnings.

Table 2: Variables in Failure Probability Formula

Analysis of the model solutions suggests two groups of variables need to be considered in empirical bankruptcy prediction models. The first group are the control variables of the model; dividend paid, investment level and change in the debt level, which are the decision variables of the firms management. The second group are those variables which appear in the expressions for the probability of bankruptcy, listed in table 2.

2.1 The Firms Decision Variables

Values of control variables, the first group of variables to be considered in the validation of the model, determine which policy regime the firm is operating in. This information is important as the probability of bankruptcy expressions are regime specific. Combinations of the control variables; change in the level of debt, \(u\), investment in productive equipment, \(v\), and the dividend paid, \(w\), determine the current regime.

A dividend will only paid if earnings are larger than expenses. Thus, firms paying dividends are generating earnings that are larger than the expenses incurred. Management of these firms has judged the cash balance of the firm to be large enough to offset negative earnings surprise, otherwise the funds available would have been allocated to increasing the cash balance. That is, dividend paying firms will have adequate cash reserves and be less likely to fail. The ability to pay a dividend will reduce the probability of a firm going bankrupt.

Sound financial management practices dictate that firms should not undertake investments that have nonpositive value, when the project cash flows have been properly defined to include the value of management flexibility. That is, the discounted stream of revenue from the project must be greater than the cost of the investment. Investments in productive equipment increase the certain part of the random earnings stream, providing a larger buffer against negative earnings
shocks. This effect helps to reduce the probability of failure. Thus, the coefficient of the investment variable should be negative.

The effect of a change in the level of debt on the probability of failure will depend on whether the change is an increase or decrease in debt. A firm increasing its debt level within its debt constraint, a positive change, will be judged by lenders as being able to repay these funds. This positive assessment by lenders can be taken as a sign of financial health. The implication is that an increase in the level of debt should reduce the probability of bankruptcy in the short term. Over the longer term the increased interest commitments may increase the probability of bankruptcy. When a firm pays back debt, it does so out of its free cash. When free cash is used to repay debt it is not available for other purposes. Forgoing investments reduces the pool of productive equipment available in later periods. This equipment generates the certain part of the random earnings stream. If the certain part of the random earnings is falling, the chance of a large negative earnings shock sending the firm bankrupt increases. So a decrease in debt should increase the probability of bankruptcy. Combining these two effects we can conclude that the sign of the coefficient of the change in debt will be negative. That is, an increase in debt multiplied by the negative coefficient indicates that the probability of failure is reduced. Negative changes in debt, resulting from repayments, are multiplied by the negative coefficient to produce a positive effect increasing the probability of failure.

2.2 Variables from the Bankruptcy Probability Expressions

A list of the variables suggested by analysing the failure probability expressions in Table 1 can be found in Table 2.

The effect of these variables on the probability of bankruptcy can be determined through an analysis of the impact on the value of the bankruptcy probability expression for a given regime when the variable is changed, holding the value of all other variables constant. All of these expressions describe the area in the left tail of a normal probability distribution, the shaded area in Figure 1. The larger the value of the expression ($Z$) the smaller the left tail area, which is the probability of failure.
Figure 1: Bankruptcy Probability - Left Tail

The cash position of the firm, $M_t$, is negatively related to the probability of failure. Cash holdings provides a buffer against negative values of the random component of earnings. The larger the cash holding, the greater the buffer. This buffer effect reduces the probability of failure. This effect can be further explored by looking at the first failure probability expression. The larger the value of $\frac{M_t + (a-x)}{\epsilon}$, the smaller the probability of failure. Holding $\epsilon$ and $a - x$ constant, an increase in the cash position of the firm makes the expression $\frac{M_t + (a-x)}{\epsilon}$ larger, decreasing the area in the left tail of the distribution. The cash position of the firm has a negative relationship with the probability of bankruptcy.

The constant, $\epsilon$, defines the variability of the earnings process of the firm. This variable can not be directly observed, however earnings surprise, $\epsilon dz_t$, can be computed. The constant, $\epsilon$, is a function of the firm specific and economy-wide influences on the earnings of the firm. A firm with a high level of earnings uncertainty will be characterised by high values of $\epsilon$. That is, a large $\epsilon$ is consistent with a large amount of variability in the observed earnings of the firm. In the definition of earnings a Wiener process is multiplied by the constant, $\epsilon$, the product is the unexpected or "suprise"component of earnings. If the constant is large the possibility of a negative earnings surprise increases. A large enough negative earnings suprise will wipe out the cash holdings of the firm, leading to bankruptcy. The probability of bankruptcy increases with increases in the constant, $\epsilon$. If the cash position, $M_t$, and change in debt, $a - x$, are held constant constant in
the first bankruptcy probability expression, \( \frac{M_t + (a - x)}{\epsilon} \), then increasing \( \epsilon \) causes the expression to become smaller, increasing the left tail area of the probability distribution. This is consistent with earnings surprise, \( \epsilon dz_t \), being negatively associated with the probability of bankruptcy.

Interest payable on outstanding debt, \( rx \), has to be paid from the expected part of earnings, before the firm can distribute a dividend or invest in productive equipment. Interest payable is the product of the debt level and the lending interest rate. When the interest payable is larger than the expected part of earnings, it has to be funded from the cash reserve. The greater the value of interest payable, the larger the potential drain on the cash reserve. Reductions in the cash reserve decrease its ability to act as a buffer against negative values of the random component of earnings, leading to an increase in the probability of bankruptcy. This effect can be confirmed using the third bankruptcy probability expression. By keeping cash, \( M_t \), the expected part of earnings, \( \theta f(c) \), and the increase in debt, \( a - x \), constant in the expression, \( \frac{M_t + (\theta f(c) + (a - x) - rx)}{\epsilon} \), the effect of an increase in interest payable, \( rx \), can be determined. An increase in interest payable, all else constant, makes the value of the expression smaller. This increases the area in the left tail of the distribution, increasing the probability of bankruptcy. Interest payable is positively related to the probability of bankruptcy.

The expected part of earnings after interest payments, \( \theta f(c) - rx \), provides funds that the firm can use to distribute as a dividend, invest in productive equipment, increase the cash holding or repay debt. The larger the expected part of earnings, \( \theta f(c) \), relative to interest payable, \( rx \), the greater the funds available. The availability of funds to offset negative values of the random component of earnings will decrease the probability of bankruptcy. In the expression, \( \frac{M_t + (\theta f(c) + (a - x) - rx)}{\epsilon} \), let the expected part of earnings, \( \theta f(c) \), increase while holding all other variables in the expression constant. Increases in \( \theta f(c) \) will cause the value of the expression to increase, decreasing the area in the left tail of the distribution. The expected part of earnings, \( \theta f(c) \), is negatively related to the probability of bankruptcy.

The effect of the level of debt, \( x \), can be direct or indirect. The direct effect is that the higher the level of debt, the smaller the amount that can be borrowed in regimes requiring borrowing. In regimes where debt is repaid the larger the level of debt the greater the drain on the firms
cash reserve. The impact of the level of debt on interest payments provides the indirect effect. Both effects work in the same direction. The level of debt will have a positive association with the probability of bankruptcy.

3 Variables for Statistical Testing

To test the usefulness of the theory in empirical bankruptcy prediction, the variables defined have to be approximated using the financial statements of actual companies. All variables will be constructed book values. The accuracy of the approximations will determine the extent to which the theory is useful in practice. A factor which may affect the quality of the approximations is the construction of earnings in the theoretical model as a random variable with expected and surprise\(^3\) components. Earnings are not broken up in this way in standard financial statements. As well, the model has no corporate tax, unlike the actual corporate environment.

The proxy variables for the control variables flow directly from standard financial statements. The dividend paid, \(w\), is measured by the actual dividend paid by the firm in the period of the financial statements. Investments in productive equipment, \(v\), are made to increase the earnings of the firm. Investing in the stock of productive equipment of the company is required to allow for the generation of larger expected earnings, \(\theta f(c)\). The productive equipment of firms is recorded in the Plant, Land and Equipment account of the balance sheet. Changes in this account from one year to the next, after adjustments for depreciation charges, provide a measure of the amount that a company is investing. Debt is measured by the total debt of the firm, which is the sum of short term and long term borrowings. The change in the debt control variable, \(u\), is therefore measured by the change in total debt from one balance sheet to the next.

A firm is bankrupt when its cash position, \(m\), falls below zero. By this definition, bankruptcy is caused by the firm not having enough cash to meet current commitments. The definition of liquidity is limited in the model by an assumption of irreversible investment and a prohibition on equity raising. The cash variable used in empirical testing is operating cash flow, calculated by the indirect method. Negative values of operating cash flow indicate the firm is facing a short term

\(^3\) These components can also be thought of as corresponding to permanent and temporary components of earnings.
liquidity problem. The inability to meet debt servicing requirements that arise from a liquidity
problem is often an important factor in the instigation of bankruptcy proceedings.

The interest paid variable, \( rx \), has a direct counterpart in a firm’s financial statements. The
amount of interest that a firm has paid is recorded in the Profit and Loss statement. This value
will be used in the testing exercises.

Earnings has two components, a expected element based on a production function which maps
productive resources to dollars and a random ”surprise” component which may increase or decrease
realised earnings. As Earnings Before Interest Depreciation and Taxes (EBDIT) is the item which
records the results of use of productive resources in monetary terms, it will be used to in the
construction of the expected portion of earnings, \( \theta f(c) \). Averages are associated with expected
values in statistics. In this work the expected part of earnings is computed as the average of the
this and the last periods EBDIT.

The earnings surprise, \( dz_t \), will have to be approximated from some combination of financial
statement elements. A simple dispersion measure, the deviation from a two period av-
erage \( \left( EBDIT_t - \frac{EBDIT_t + EBDIT_{t-1}}{2} \right) \), is used to measure earnings surprise. This measure
is a simplification derived from the Mean Absolute Deviation of earnings, \( MAD = \frac{1}{n} \sum_{t=1}^{n} |EBDIT_t - EBDIT| \). Firms with EBDIT that is above the two period moving average of EBDIT,
have experienced a positive earnings surprise. Such values will decrease the probability of failure.
Where the observed EBDIT is below the two period moving average, the firm has experienced a
negative earnings surprise and is more likely to fail. This measure is used for two reasons. Firstly,
the calculation of the measure only requires two sets of financial statements, so it is economical
on data and, secondly, the measure is based on the average used in the constructing expected
earnings.

Table 3 summarises how these variables are operationalised using the standard financial state-
ment variables.

All these variables are divided by Total Assets, as this is the simplest consistent method
available to standardise financial statement information. Using Total Assets, that is a measure
of the size of a company, as a standardising variable rules out its inclusion as an explanatory
The dividend paid in period $t$. 

The difference in Plant, Land & Buildings (PLB) over the last 2 Balance Sheets, $(\text{PLB}_t - \text{PLB}_{t-1})$, adjusted for depreciation.

The difference in total debt (Short Term Debt + Long Term Debt) over the last 2 Balance Sheets, $(\text{Total Debt}_t - \text{Total Debt}_{t-1})$.

Operating Cash Flow, calculated by the indirect method.

Interest paid from the P&L.

The average of this and last periods EBDIT.

A simple earnings dispersion measure, the deviation from a 2 period average, is used.

Total debt (Short Term Debt + Long Term Debt).

### Table 3: Proxy Variables from Financial Statements

<table>
<thead>
<tr>
<th>Variable</th>
<th>Proxy Variable Constructed from Financial Statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>The dividend paid in period $t$.</td>
</tr>
<tr>
<td>$v$</td>
<td>The difference in Plant, Land &amp; Buildings (PLB) over the last 2 Balance Sheets, $(\text{PLB}<em>t - \text{PLB}</em>{t-1})$, adjusted for depreciation.</td>
</tr>
<tr>
<td>$u$</td>
<td>The difference in total debt (Short Term Debt + Long Term Debt) over the last 2 Balance Sheets, $(\text{Total Debt}<em>t - \text{Total Debt}</em>{t-1})$.</td>
</tr>
<tr>
<td>$M_t$</td>
<td>Operating Cash Flow, calculated by the indirect method.</td>
</tr>
<tr>
<td>$rx$</td>
<td>Interest paid from the P&amp;L.</td>
</tr>
<tr>
<td>$\theta f(c)$</td>
<td>The average of this and last periods EBDIT.</td>
</tr>
<tr>
<td>$\epsilon dz_t$</td>
<td>A simple earnings dispersion measure, the deviation from a 2 period average, is used. $\left(\frac{\text{EBDIT}_t - \frac{\text{EBDIT}<em>t + \text{EBDIT}</em>{t-1}}{2}}{\text{EBDIT}_t} \right)$.</td>
</tr>
<tr>
<td>$x$</td>
<td>Total debt (Short Term Debt + Long Term Debt).</td>
</tr>
</tbody>
</table>

As Total Assets do not appear in the expressions for bankruptcy probability in Table 1, this would not appear to present a problem in the analysis.

The expected signs of the variables are summarised in Table 4.

### Table 4: Summary of Hypotheses to be Tested

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standardised Proxy Variable</th>
<th>Effect on Bankruptcy Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>DIV / TA</td>
<td>-</td>
</tr>
<tr>
<td>$v$</td>
<td>INV / TA</td>
<td>-</td>
</tr>
<tr>
<td>$u$</td>
<td>DDEBT / TA</td>
<td>-</td>
</tr>
<tr>
<td>$M_t$</td>
<td>OCF / TA</td>
<td>-</td>
</tr>
<tr>
<td>$rx$</td>
<td>INT / TA</td>
<td>+</td>
</tr>
<tr>
<td>$\theta f(c)$</td>
<td>MA / TA</td>
<td>-</td>
</tr>
<tr>
<td>$\epsilon dz_t$</td>
<td>DISP / TA</td>
<td>-</td>
</tr>
<tr>
<td>$x$</td>
<td>TDEBT / TA</td>
<td>+</td>
</tr>
</tbody>
</table>

### 4 Development of a Test Statistic

To verify the effect of the variables defined above on bankruptcy probabilities, a statistical technique is required that can estimate failure probabilities, allow for explanatory variables and deal with interval-censored data. The Cox (1972) proportional hazard method, as extended by Fleming and Harrington (1991), is a probabilistic regression model that allows for censored data. This approach to the modeling and estimation of failure probabilities will be used in this study.

A hazard rate is the probability that a subject (firm) will fail in the next increment of time. It is derived from the distribution of firm durations (lifetimes). The hazard is defined as a function of explanatory variables and estimated parameters.

The model specification problem that presents itself here has two distinct parts. The proba-
bility of bankruptcy is defined as

$$Pr(Bankruptcy = true) = f(\beta X),$$ (1)

where $f(\bullet)$ is a function that returns a value between 0 and 1, with $\beta$ a vector of parameters to be estimated and $X$ a vector of firm-specific variables. Then the first part of the model specification process is to identify the functional form of $f(\bullet)$. Once the function $f(\bullet)$ has been defined, the second task is to find a statistical technique to estimate the unknown parameters, $\beta$.

### 4.1 Hazard functions

As a starting point, we define the probability of bankruptcy in more detail. At any time, $t$, the firm can be in one of two states: a going concern (the firm has a positive cash balance, $m_t \geq 0$) or the firm is bankrupt (the firm has a negative cash balance, $m_t < 0$). The quantity of interest is the probability that the firm will switch from being in the going concern state, to one of bankruptcy. This probability is conditional, the state transition can only occur if the firm is a going concern at the beginning of the period of interest. The definition of bankruptcy becomes,

$$Pr(Bankruptcy) = Pr(Bankrupt \ in\ period \ t \ | \ going \ concern \ at \ beginning \ of \ period \ t).$$

$$= \lambda(t)\Delta$$

The function, $\lambda(t)$, is known as a hazard function for the random variable firm lifetime, $\tau$, and $\Delta$ is the length of the time period. Roughly, $\lambda(t)$ is the rate at which firms will fail at time $t$, given that they last until $t$. This function, like the probability of bankruptcy, is conditional, as a firm can only be at risk of failing at time $t$ if it was a going concern at the end of period $t - 1$.

Further development of a statistical argument requires the availability of a sample containing both going concerns and bankrupt firms. In such a sample of firms, there is a distribution of firm lifetimes. The lifetime of any firm is a realisation of the random variable, $\tau$. The probability distribution of these lifetimes can be specified by the distribution function.

$$F(t) = Pr(\tau < t).$$ (2)

---

4 $m_t$ is the firm’s cash balance at time $t$.

5 The variable $\tau$ denotes the first time that the cash balance falls below zero, $m_t < 0$. That is, the length or duration of the firm’s lifetime.
It specifies that the probability of the random lifetime, $\tau$, is less than some given value, $t$. This is the probability that the firm goes bankrupt before time $t$. The corresponding probability density function for this distribution is
\[ f(t) = \frac{dF(t)}{dt}. \]

The probability of surviving longer than $t$ periods is given by the survival function
\[ S(t) = 1 - F(t) = \Pr(\tau \geq t), \]
which defines the upper tail area of the distribution of firm lifetimes.

The definition of the probability of bankruptcy can now be made more precise in terms of the probability distributions that have been introduced
\[ \lambda(t) = \lim_{\delta \to 0} \frac{P(t \leq \tau \leq t + \delta | \tau \leq t)}{\delta}. \]
This is the likelihood that the firm will become bankrupt in the next instant, given that it was a going concern at time $t$. The statistical analysis of this hazard function will form the basis for the verification of the bankruptcy conditions presented in section 2.

### 4.2 Explanatory variables and the form of the hazard function

The Proportional Hazard specification of Cox (1972) will be used to estimate bankruptcy probabilities. This model includes explanatory variables and partitions the hazard function in a way that lends itself to the verification of the variables derived from the explanatory model.

In the Cox (1972) proportional hazards model, the hazard function depends on a vector of explanatory variables, $X$, with unknown coefficients $\beta$, and $\lambda_0$. It is defined as
\[ \lambda(t, X) = \phi(X, \beta)\lambda_0(t), \]
where $\lambda_0(t)$ is a "baseline", or underlying, hazard faced by all firms at time $t$. It is conventional when working with this model, to scale the $X$’s so that $\phi(\bullet)$ equals one at their mean values.

---

6 The hazard function can also be written as $\lambda(t) = \frac{f(t)}{S(t)}$.
7 The hazard rate is not strictly a probability as it is not bounded above by one.
8 The statistical theory underlying the analysis of lifetime data can be found in Kalbflesch and Prentice (1980).
9 The transformation used to scale the explanatory variables is $x = X - \bar{X}$. 
Under these conditions, \( \lambda_0(t) \) can be interpreted as the hazard function for the average firm. The baseline hazard is an unknown which has to be estimated before the model can be used to generate bankruptcy probabilities. In this form, the effect of the firm specific variables is to multiply the baseline hazard, \( \lambda_0(t) \), by a factor \( \phi(\bullet) \) which does not depend on time. The form of \( \phi(\bullet) \) which was specified by Cox (1972) is

\[
\phi(X, \beta) = \exp(\beta'X).
\]

This allows \( \phi(\bullet) \) to be non negative without requiring restrictions on the \( \beta ' s \) in the estimation process. To interpret the parameters, \( \beta \), take the derivative of the log of the hazard function with respect to \( X \). Then,

\[
\frac{\partial \ln \lambda(t)}{\partial X} = \frac{\partial}{\partial X} \ln \left( \exp(\beta'X) \right) + \frac{\partial}{\partial X} \lambda_0(t) = \beta.
\]

The underlying hazard drops out as it does not depend on \( X \). The coefficients, \( \beta \), describe the constant proportional effect of the explanatory variables on the conditional probability of bankruptcy.

5 Data and Sample Properties

5.1 Data

The published financial statements of listed firms has been the source of data for the classification and data reduction studies reported in the literature. Financial data for Australian firms derived from Annual Reports filed with the Australian Stock Exchange(ASX) between 1966 and 1994 are used in this study. Firms are able to enter the study from the time they are listed. Each firm has one set of financial statements for each year, a P&L statement and a Balance Sheet. All sets of financial statements that are available for a firm are used in the study.

The end of a firm’s lifetime can be caused by a number of different events. These include delisting caused by financial distress or the appointment of a liquidator, voluntary liquidation, takeover, or a voluntary delisting. Only firms that are surviving at the end of 1994, or have been delisted due to financial distress or appointment of a liquidator, are used in this analysis.
Companies which have been delisted are flagged through the inclusion of a delisting date in their ASX company information. Those companies with a delisting date were then cross checked against the Nothman (1999) Record of Delistings. Only those firms identified as being distressed or having a liquidator appointed are included as bankrupt. This definition of bankruptcy is a legal definition, based on the listing rules of the Australian Stock Exchange. The bankruptcy definition in the model is that the firm is bankrupt when its cash holding is negative. This is an economic definition based on liquidity. Severe liquidity problems in a firm are often the trigger to have liquidators appointed. This is a first step in the formal delisting process.

The analysis is run as time-on-study, not in calendar time. The age of a firm at the start of any observation period is the difference between the current balance sheet date and the firm’s listing date. This means that firms of equal age are compared no matter what date their statements were released. There are 146 delisted companies and 1505 companies that are going concerns in the sample.

A number of variables in the model are expressed in first-difference terms. To calculate these variables, two consecutive financial statements are required. Firms not having two consecutive statements to work with have been excluded from the study. This requirement may introduce a bias. Companies which fail in the first two years of their lifetimes, that is companies who only report once then fail, will not appear in the study.

5.2 Censoring and Time Variation

In the sample of financial statements for the companies included in the study there are 146 records with known failure times. Each of the companies that has failed contributes one of these records. For the companies that are still going concerns at the study termination date, December 31, 1993, we do not know whether they fail. The information on these companies is said to be right-censored. The listing date, and the date of their first financial statement included in the sample, is known for all companies. That is, there are no companies which are left-censored in the sample.

Each set of statements contain the public information about a company, until the issue of

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10 For example, in 1995 a company which was listed in 1990 will have a lifetime of 5 years, a company listed in 1985 will have a lifetime of 10 years. The 1995 statements of these companies will not be analysed together, but with other 5 year old statements and 10 year old statements, respectively.
the next statement or exit of the firm from the study due to failure. The information in these statements is unknown before the statement date and not applicable following the release of the next financial statements, or delisting of the company. Observations which apply to a specific interval of time are known as interval-censored data. Intervals commence on the release of a set of financial statements. They end when a new set of financial statements is released or the company is delisted from the exchange. All the observations used in model validation are interval-censored. This interval-censoring allows for the introduction of time varying effects. The financial position of companies in the sample is repeatedly measured over the period they remain on study. Each of these measures captures the firm in a different stage of evolution. The use of all the observed intervals allows the full history of the financial status of the firm to be incorporated into the statistical analysis of failure probabilities\(^{11}\). There are 8045 intervals not ending in delisting, and 146 intervals ending in delisting available for use in the model validation exercise. The formation of the risk sets used in the estimation of the Cox (1972) proportional hazards model allows for the inclusion of interval-censored observations. At each failure time the risk set constructed will contain all the records in the sample whose time on study matches the age of the company that has failed. Construction of the risk sets in this way allows for time variation in the estimation of the Cox (1972) model.

### 5.3 Descriptive Statistics

Descriptive statistics of seven variables identified in the probability expressions derived from the model are described in Figure 2. The statistics reported for each variable are the minimum value, the mean, median, the maximum value and its standard deviation.

The statistics presented in the top panel of the display, labeled Continuing, relate to those intervals which do not end in a delisting. The results in the bottom panel of the display, labeled delisted, present the statistics of the variables for firms which were delisted at the end of an interval.

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\(^{11}\) The use of repeated measures in the Proportional Hazards model is analogous to the analysis of Panel Data by regression analysis. Chapter 12 of Johnston and DiNardo (1997) describes regression methods for panel data. It is also consistent with the discrete hazards approach adopted by Shumway (2000).
Figure 2: Figure 2 - Descriptive Statistics

6 Cox Regression Results

The results of the Cox regression is presented in this section. All results reported in this section were generated using the S+ software package and the data set described above.

The first model presented is based on the control variables, these variables determine the solution regime. Figure 3 presents the results from the estimation of the $\beta$ coefficients of the Cox (1972) proportional hazards model, $\lambda(t, X) = \exp(\beta'X)\lambda_0(t)$. The results reported do not arise from a stepwise variable selection approach to the Cox model. All variables that are included are specified in advance. The statistics reported are, the estimated coefficient, $\beta_i$, for each variable included, an estimated relative risk for a one unit change in the variable, $\exp(\beta_i)$, the standard error of the estimated coefficient, a $z$-test which tests the null hypothesis, $H_0: \beta_i = 0$, $i = 1, 2, \ldots, 6$, and its $p$-value. The likelihood ratio test presented is analogous to the F-test for significant coefficients in the regression framework. It tests the null hypothesis $H_0: \beta_1 = \beta_2 = \cdots = \beta_7 = 0$, using the test statistic, $\chi^2_{LR} = 2[\ln(L(\beta)) - \ln(L(0))]^{12}$, which is distributed chi-squared with $m$ degrees of freedom, where $m$ is the number of estimated parameters. The $p$-value for this test is also presented.

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Continuing

<table>
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<th>INV.TA</th>
<th>DDEBT.TA</th>
<th>OCF.TA</th>
<th>INT.TA</th>
<th>MA.TA</th>
<th>DISP.TA</th>
<th>EBIT.TA</th>
<th>TDEBT.TA</th>
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<td>Median:</td>
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Delisted

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<th>INT.TA</th>
<th>MA.TA</th>
<th>DISP.TA</th>
<th>EBIT.TA</th>
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<td>Median:</td>
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<td>0.003</td>
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<td>Max:</td>
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---

12 The function $L(\bullet)$ used in the calculation of this test statistic is the partial likelihood function of the Cox proportional hazards model.
### Figure 3 - Control Variables

The results of the likelihood ratio test, with a p-value of 0.000026, indicate that at least one of the coefficients is significantly different to zero. The sign of the coefficient for the dividend variable is as expected, and the effect of the variable is significant. In this regression the variables representing investment and change in debt are not significant. The result is consistent with the argument that dividend paying firms have adequate cash reserves and are less likely to fail. However changes in the firms’ level of debt, that is capital structure, and investment decisions are not significant in the determination of bankruptcy probability.

The model presented in Figure 4 tests the effects of the variables included in the derived bankruptcy probability expressions. The results of the likelihood ratio test indicate that at least one of the coefficients is significantly different to zero.

### Figure 4 - Probability Expression Variables

In this model the signs of the estimated coefficients correspond with their hypothesised directions. This indicates that the variables are affecting the probability of bankruptcy in a manner consistent with the theoretical model. The cash, interest, expected earnings and earnings surprise variables are all significant at \( \alpha = 0.05 \). The debt variable is not significant. This indicates
that the direct effect of debt, like the change in debt, is not significant in the determination of bankruptcy probabilities. The indirect effect of debt, in determining the level of interest payable is however significant. This is consistent with the theoretical models focus on the cash position of the firm. The result further supports the finding, in Figure 3, that capital structure is not as important as liquidity issues in the determination of bankruptcy probability.

The final model presented in Figure 5 combines the significant variables from the previous models.

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n= 8191
Likelihood ratio test= 42.4 on 5 df,  p=4.95e-008

Figure 5 - Combined Model

The results of the likelihood ratio test indicate that at least one of the coefficients is significantly different to zero. All the coefficients of the variables included in this model are significant. The estimated coefficients (Column 2) all have the expected sign. This combination of variables indicates that the inability to meet loan payments caused by liquidity problems is the primary mechanism causing firms to become bankrupt. Liquidity problems arise when the sum of a negative earnings surprise and the interest due exceeds, the sum of cash plus expected earnings, the firms cash position.

7 Summary

The Cox regression results are summarised in Table 5. The effect on the probability of bankruptcy of the dividend paid is significant and as predicted. The effects for the change in debt and investment are not significant. Tests for the direction and significance of the relationship between variables included in the bankruptcy probability expressions show that the effect on the probability of bankruptcy of the firm’s cash position is significant. The effect of interest payable is as predicted.
as is the effect of the expected part of earnings. The measure of earnings surprise also affects the probability of bankruptcy as the model indicates.

<table>
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Table 5: Summary of Cox Regression Results

These results are consistent with the theoretical model developed in indicating that variables affecting cashflow, operating cashflow, expected earnings and earnings surprise, combined with interest payments are the primary determinates of bankruptcy. The effect firms capital structure on bankruptcy probability is indirect, through the impact of debt on interest payable. Changes in capital structure, measured by the change in debt have no significant effect on bankruptcy probability.

In conclusion the results of the Cox regression analysis on the sample of listed Australian firms demonstrates that the variables suggested by a cash based model with stochastic earnings have a significant effect on the probability of bankruptcy. They therefore provide a set of explanatory variables derived from an economic model which can be used in empirical bankruptcy modeling.
References


