Endogenous Money, Non-neutrality and Interest-sensitivity in the Theory of Long Period Unemployment

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Abstract

This paper investigates the role played by endogenous money in models with interest-sensitive expenditures. In particular, it examines the impact of endogenous money on a baseline neoclassical model arguing against the frequently asserted claim that traditional neoclassical macroeconomics is compatible with endogenous money. It demonstrates firstly that endogenous money is a sufficient condition to render unstable a neoclassical model characterised by interest-sensitive expenditures, full employment and money neutrality. Secondly, it shows that the introduction of either money illusion on the part of workers or a Taylor rule governing monetary policy are alternative methods of stabilising models with interest-sensitive expenditures and endogenous money, though with different implications for the full employment and neutrality characteristics of the standard model. Thirdly, it raises questions about whether models which incorporate Taylor rules can be properly characterised as containing endogenous money and it provides an alternative interpretation of such models. The paper concludes by arguing that money supply endogeneity of the extreme or accommodationist type is of fundamental significance for the construction of a theory of long period unemployment but it identifies a set of remaining questions which need to be addressed in the advancement of this project.

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Keywords: Endogenous money, money neutrality, unemployment, interest-sensitivity.

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Introduction
The interest-sensitivity of expenditures has been a central feature of neoclassical macroeconomics since at least the time of Wicksell (1898). It lies at the heart of the neoclassical mechanism which establishes full employment as the centre of gravitation for an economic system, and it is integral to the doctrine of money neutrality. But these features of neoclassical theory were seriously challenged in the second half of the twentieth century. In the 1960s, the capital debates crippled the idea that investment, in particular, could be a monotonically inverse function of the rate of interest (see Harcourt 1972; Garegnani 1978a). An economic system in a state of persistent unemployment might well generate nominal interest rates below the natural rate at which full employment obtains but since these lower rates would fail to elicit any expenditure response, the system would fail to move closer to full employment and the problem would persist. From the 1970s, the endogenous money school challenged the idea of money neutrality. An important part of the neoclassical money neutrality mechanism is the exogenous nature of the money supply. Variations in money demand against an exogenously fixed money supply cause movements in the nominal interest rate that shift the real interest rate back to the natural rate. If the money supply is endogenous, leverage for nominal interest rate movements is unavailable and rates remain stable. According to Kaldor (1970, 1986), Moore (1979, 1988), Lavoie (1996), Rochon (1999) and Docherty (2005) the money supply is not only endogenous but it is the rate of interest that is exogenous, fixed by the central bank to avoid destabilizing fluctuations that may cause severe economic downturns. The result of this approach is that interest rates are not free to perform the function required of them by money neutrality and that rates of interest chosen by the central bank above the natural rate are capable of generating persistent unemployment.

Interestingly, both the results of the capital debates and the principle of endogenous money appear compatible with certain interpretations of Keynes’ The General Theory. According to these interpretations, money is non-neutral and unemployment constitutes an essential characteristic of the equilibrium to which an economy gravitates rather than a short run disequilibrium phenomenon.¹ But the separate emergence of the two challenges to neoclassical

¹ Milgate (1977, 307) commented some time ago on the relationship between the negative enterprise of criticising neoclassical theory and the positive enterprise of constructing an alternative theory. If the negative enterprise is successful, the question is begged as to how a capitalist system operates if not along the lines suggested by neoclassical theory. Here theorists have tended to be thrown back on some particular interpretation of Keynes (e.g. Pasinetti 1974; Garegnani 1978a,b; 1984; and Eatwell 1983). There are, of course, alternative interpretations of The General Theory. Those offered by the neoclassical synthesis and the New Keynesian school find in the logic of The General Theory various impediments to the efficient operation of the mechanism which pushes an economic system from a position of unemployment towards full employment. But unemployment in these interpretations is a distinctly disequilibrium phenomenon that can be alleviated by freeing up a market economy from strictures preventing the efficient operation of the full employment mechanism. Other interpretations find in The General Theory an explanation of persistent or long period unemployment.
economics described above raises a number of questions about their relationship with each other that have yet to be squarely addressed. Firstly, does endogenous money constitute a sufficient condition for long period unemployment? While the endogenous money school identified above suggests that it does, other theorists, including some non-marginalist writers (e.g. Pivetti 1985; 1991; and 2001) have explicitly rejected this proposition arguing that a range of neoclassical models, including that of Wicksell (1898) himself, contain endogenous money and demonstrate a capacity to deliver their traditional results despite its inclusion.

A second question relates to the precise nature of endogenous money itself. Docherty (2005, 292 ff) explores the impact of a simple form of endogeneity associated with a naïve central bank rule that fixes the nominal interest rate. But central banks adjust interest rates in response to the interaction of particular economic goals and circumstances, and their attention ultimately focuses on real interest rates rather than nominal ones. For example, the famous Taylor (1993, 202) rule, requires adjustment of the real rate in relation to deviations of inflation and output from target values. The question is thus whether the operation of a Taylor rule has any material impact on the results of the endogenous money argument and how this affects the answer to the first question posed above.

A third question arises from the response of neoclassical theorists themselves to the concept of money non-neutrality. Friedman (1968), for example, was highly critical of traditional Phillips curve analysis, which was characterised by non-neutrality, because it assumed money illusion on the part of workers. The question then is: what is the precise role of money illusion in generating non-neutrality and what relationship does it have with endogenous money?

This paper offers answers to these questions using a set of simplified macroeconomic models which variously embody the theoretical features identified above. A baseline neoclassical model is first established using a version of Friedman’s (1968) analysis. The characteristics of equilibrium and the conditions for its stability are examined. The model is then modified to incorporate a simple form of money supply endogeneity and the impact of this change on equilibrium and stability is considered. This allows the first question posed above on the sufficiency of endogenous money for an unemployment equilibrium to be answered. The model is then examined when a Taylor rule is used in place of a naïve interest rate rule. This allows the second question above to be answered. Finally money illusion is added to the model in two different forms and the results compared with results from the preceding versions of the model, allowing the third question above to be answered. Results from all of the models are summarised and some conclusions drawn in the final section of the paper.
The Baseline Model: Friedman’s (1968) Approach

Friedman (1968) argues that in a world where workers do not suffer from money illusion in the formulation of their wage claims and where demand for money has a transactions component, the economy will gravitate to full employment and inflation will depend only on the rate of growth of the money supply.

Friedman’s model may be represented in terms of equations (1) to (6) below. Equation (1) is a standard IS curve with real output, $Y_t$, depending on a multiple, $a$, of autonomous expenditures, $\overline{A}$, and expected real interest rates, where $i_t$ is the nominal interest rate, $\pi_t^e$, is the expected rate of inflation and $b$ is the sensitivity of expenditures (principally investment) to the real interest rate. Unemployment ($UE_t$) is determined in equation (2) as a result of some minimum level of unemployment $UE^*$ associated with market frictions and Okun’s law, where $Y^*$ is potential output and $\gamma$ is a scaling parameter assumed to be strictly positive. The inflationary process is set out in equations (3) and (4). Equation (3) determines inflation as the rate of change in the money wage, $W_t$, per period less growth in labour productivity, $p$. Equation (4) outlines the wage determination process in terms of a basic claim for increased wages $w_t$, which is assumed to be constant, compensation for expected inflation and an adjustment for the level of unemployment which undermines the bargaining position of labour and reduces the size of any successful wage claim. $\beta$ is a parameter assumed to be strictly positive.

\begin{align*}
Y_t &= \alpha \overline{A} + ab \cdot \pi_t^e - ab \cdot i_t \tag{1} \\
UE_t &= UE^* + \gamma \cdot (Y^* - Y_t) \tag{2} \\
\pi_t &= \frac{W_t}{\bar{W}_t} - p \tag{3} \\
\frac{W_t}{\bar{W}_t} &= w + \pi_t^e - \beta \cdot UE_t \tag{4} \\
\frac{d\pi_t^e}{dt} &= j \cdot [\pi_t - \pi_t^e] \tag{5} \\
\frac{di_t}{dt} &= k_1 \cdot \frac{dY_t}{dt} + k_2 \cdot \pi_t - k_2 \cdot \frac{\bar{m}}{m} \tag{6}
\end{align*}
The rate of change of expected inflation, $\pi^e_t$, is determined in (5) by a standard adaptive process which compares the actual rate of inflation, $\pi_t$, with expected inflation (where $j$ is a parameter assumed again to be strictly positive). The nominal interest rate is determined in (6) as the result of a market process which increases the interest rate when demand for money exceeds supply and vice versa. Nominal demand depends on both nominal income, $P_tY_t$, and the interest rate, so that changes in demand result from real growth in income, inflation and monetary growth. The supply of money is assumed to grow at an exogenous rate, $\dot{m}/m$.\(^2\) The adjustment parameters $k_1$ and $k_2$ are assumed to be strictly positive.

This system may be reduced to the following set of two differential equations in expected inflation and unemployment (this is shown in the Appendix):

$$-ab \cdot \frac{d\pi^e_t}{dt} = \frac{1}{\gamma} \cdot (1 + abk_1) \cdot \frac{dUE_t}{dt} + abk_2 \cdot \pi^e_t - ab \beta k_2 \cdot UE_t = abk_2 \cdot \left[ \frac{\dot{m}}{m} - (w - p) \right]$$  \(7\)

$$\frac{d\pi^e_t}{dt} + j\beta \cdot UE_t = j \cdot (w - p)$$  \(8\)

to which the solution is:

$$\begin{bmatrix} \pi^e_t \\ UE_t \end{bmatrix} = \begin{bmatrix} \frac{\dot{m}}{m} \\ \frac{(w - p)}{\beta} \end{bmatrix}$$  \(9\)

And this solution is stable provided:

$$k_2 > j$$  \(10\)

This, of course, confirms the standard result that expected inflation will, in equilibrium, be equal to actual inflation which will in turn be equal to the rate of monetary growth. Unemployment is given by the excess of the basic money wage claim over labour productivity growth adjusted for the extent to which unemployment affects money wage growth. This is the natural rate of unemployment or the non-accelerating inflation rate of unemployment (NAIRU) in the standard monetarist-neoclassical model. Long run money neutrality is evident in this solution.

\(^2\) To simplify the mathematics, the time derivative of interest rates is a function in equation (6) of the time derivative of real income and the growth rates of prices and the nominal money supply. To make the scaling of variables consistent, the time derivative on the RHS is given a different parameter to the growth rates. The effect of this approach is that inferences about system stability must be interpreted as being local rather than global.
both from the fact that money has no impact on the NAIRU and that money is the sole determinant of long run inflation.

The stability condition requires interest rates to respond more quickly to changes in inflation than revisions to inflation expectations do to discrepancies between actual and expected inflation rates. Consider, for example, a positive shock to monetary growth. This would generate additional inflation by initially reducing the nominal interest rate and raising aggregate demand above potential output. As prices begin to rise after this shock, upward pressure is placed on the nominal interest rate in equation (6) which increases the real interest rate in equation (1) and pulls aggregate demand back towards potential output, thus stabilising the economy. At the same time however, the increased inflation increases inflation expectations in equation (5) and reduces the real interest rate in equation (1), increasing aggregate demand and pushing it further away from potential output, thus destabilising the economy. When (10) is satisfied, the first stabilising effect outweighs the second destabilising effect and the economy settles down to a new equilibrium with all nominal variables increased.

This example highlights the importance of expenditure interest-sensitivity for this system’s version of money neutrality. A second example highlights the importance of interest-sensitivity for the model’s full employment result. Let the economy initially be in a situation where there is insufficient aggregate demand to warrant the employment of all available labour. In this case, unemployment will be above the NAIRU, nominal wage growth will be below its equilibrium growth rate and inflation will be lower than monetary growth. In equation (6), nominal interest rates will fall which will stimulate aggregate demand in equation (1) and reduce the level of unemployment. This will continue until unemployment reaches the NAIRU.

These two examples together indicate the significance of expenditure interest-sensitivity for the key features of the baseline model. We turn now to examine the impact of introducing endogenous money into this framework.

**Endogenous Money: The Pure Credit Case**

The nature of endogenously determined money as described by theorists such as Kaldor (1970, 1986) is relatively complex. The essential principle, however, is that central banks cannot exert strict monetary control except over the most narrow money aggregates. This obtains because failure to accommodate variations in money demand leads to variations in the quantity of wider aggregates via extensions or reductions in bank credit, and corresponding variations in the velocity of narrower aggregates. Since variations of this kind may generate significant variations in interest rates, failure to accommodate has the potential to cause financial instability.
Accommodationists thus argue that representing endogenous money in terms of an elastic money supply curve at a fixed interest rate is a reasonable representation of central bank behaviour designed to avoid financial instability (see Docherty 2005, 248-249). The theoretical consequences of this approach have been argued to be essentially negative for the full employment and neutrality characteristics of models such as the baseline model considered in the previous section. Some have challenged this argument, however, claiming that neoclassical economics has the capacity to cope perfectly well with endogenous money. Wicksell’s (1898) famous model of pure credit money is frequently cited as an example of this capacity (see Pivetti 2001). The objective of this section is to explore these conflicting propositions by introducing endogenous money into the baseline model and examining its impact.

The first variation of Friedman’s baseline model to be considered, therefore, incorporates endogenous money in the simplest way possible. The interest rate is set according to the rule shown in equation (11) which implies that demand for money is perfectly accommodated at this interest rate by appropriate central bank market operations.

\[ i_t = \bar{i} \]  

The full model is then made up of equations (1) to (5) and (11). Once again, this model may be reduced to a system of two differential equations in expected inflation and unemployment. Equation (12) below is the first of these equations and differs from equation (7) in the baseline model because of the replacement of equation (6) with the simple interest rate rule. The derivation of this equation is shown in the Appendix. The second equation is simply equation (8) from the baseline model reproduced below for the sake of exposition and is obtained in the same manner as before.

\[
\frac{1}{\gamma} \cdot UE_t + ab \pi^e_t = Y^* + \frac{UE^*}{\gamma} - (\alpha A - ab \bar{i}) \\
\frac{d\pi^e_t}{dt} + j\beta \cdot UE_t = j \cdot (w - p)
\]  

Equation (12) below is the first of these equations and differs from equation (7) in the baseline model because of the replacement of equation (6) with the simple interest rate rule. The derivation of this equation is shown in the Appendix. The second equation is simply equation (8) from the baseline model reproduced below for the sake of exposition and is obtained in the same manner as before.

\[
\begin{bmatrix}
\pi^e_t \\
UE_t
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{ab} Y^* - \left(\frac{A}{b} - \bar{i}\right) \\
\frac{(w - p)}{\beta}
\end{bmatrix}
\]  

The solution to this system is:
This is an important and at first glance surprising result. It indicates that a nominal interest target has no impact on equilibrium unemployment which retains the same NAIRU value as for Friedman’s baseline model in (9) above while the rate of expected inflation depends on aggregate demand (which is affected by the interest rate) relative to potential output. This result is surprising because one might have expected unemployment rather than inflation to have depended on the nominal interest target. Some reflection on this outcome is thus warranted.

The equilibrium value for unemployment reflects equations (3) and (4) which together generate the expectations augmented Phillips curve (EAPC):

$$\pi_t = (w - p) + \pi^e_t - \beta \cdot UE_t$$

Since equation (5) indicates that actual and expected inflation will be identical in equilibrium, the terms for actual and expected inflation cancel out in the above expression leaving only the rate of unemployment as an unknown which has the following value:

$$UE^* = \frac{w - p}{\beta}$$

This argument also applies to the baseline model and explains why the results for unemployment are identical for these two models.

The inflation result in the baseline model is driven from equations (1), (2), (5) and (6). Having determined the equilibrium unemployment rate from the EAPC, equation (2) determines potential output. In equilibrium, the time derivative of this output level will be zero as will the time derivative of the nominal interest rate. In addition, equation (5) implies that actual and expected inflation must be the same in equilibrium. All of this, together with equation (6), implies that expected inflation must be equal to the rate of monetary growth. From equation (1), the equilibrium value of the nominal interest rate is that value which reconciles potential output with aggregate demand driven by autonomous expenditures and the expected inflation rate. Thus the system is determinate.

If a demand shock were to increase autonomous expenditures from a position of equilibrium, the nominal interest rate required to reconcile aggregate demand with a given level of potential output would be higher. But the dynamic process required to deliver this higher interest rate would have the higher aggregate demand initially increasing output at the old level of the nominal interest rate. This would reduce unemployment and temporarily increase inflation. Together the higher output and inflation would raise demand for money and interest rates would rise via
equation (6). The higher nominal interest rate would dampen aggregate demand via equation (1) which would pull the system back towards equilibrium where output would return to its potential level, unemployment would return to the level specified in equations (10) and (13), and inflation would be equal to the rate of monetary growth. But the nominal interest rate would be permanently higher.

Equation (6) in the baseline model is central to this dynamic process. In the present model, where equation (11) replaces equation (6), this mechanism is inoperable. Any shock to aggregate demand implies that one of the components of the real interest rate in equation (1) must adjust to return the real interest rate to its natural or full employment level. But the nominal interest rate cannot perform this function as it does in the baseline model. Any increase in money demand associated with an initial increase in output responding to the demand shock will now be implicitly accommodated by the central bank according to equation (11) to leave the nominal interest rate unaffected. As a result, equation (1) no longer determines the nominal interest rate but the expected rate of inflation, its function being to reconcile potential output with aggregate demand given the components of aggregate demand already determined by the fixed nominal interest rate and autonomous spending. This is the meaning of the expected inflation solution in equation (13). It tells us that a higher level of autonomous aggregate demand requires a higher expected inflation rate to make the interest-sensitive components of aggregate demand smaller, thus reconciling them with a fixed level of potential output.

But this value of expected inflation is, of course, simply an equilibrium requirement. Whether this requirement is met and how it is met is a dynamic issue. The analysis outlined in the Appendix indicates that this equilibrium is in fact unstable, depending on the solutions to the characteristic equation of the system made up of equations (8) and (12). The Appendix shows that there is only one characteristic root in this case given as follows:

\[ r = \gamma_{\text{adj}} \beta \]  

(14)

This root is unambiguously positive indicating that the system is unstable without cycles. It signals the fact that if the nominal interest target is set above the nominal rate determined by equation (1) in the baseline model (other things equal), the real rate will also be higher than the natural (real) rate (given initial inflation expectations). In this case, aggregate demand will be below potential output, output will fall and unemployment will rise above the NAIRU. In the Phillips curve equation shown above, higher unemployment will reduce actual inflation which will fall below expected inflation causing a downwards revision to expected inflation. Thus whereas a higher nominal interest rate requires a higher rate of expected inflation to establish
equilibrium, the mechanics of the model generate a lower rate of expected inflation. This lower rate of expected inflation further increases the real rate of interest, depressing aggregate demand and output further, raising unemployment and reducing actual inflation. Further reductions in actual inflation cause further downward revisions to expected inflation and the system moves away from the equilibrium specified in equation (13) rather than towards it. Precisely the opposite effect occurs if the nominal interest target is below the natural rate.

Figure 1 shows the result diagrammatically. The equilibrium given in (13) is shown at point A. The demarcation curve (Chiang 1984, 629) for expected inflation is given by equation (8) which indicates the time derivative of expected inflation in terms of unemployment. This is zero when unemployment is at the NAIRU so that the demarcation curve for expected inflation is vertical at this level. For levels of unemployment below the NAIRU, expected inflation is rising and for levels of unemployment above the NAIRU, it is falling. These trajectories are shown by the phase paths to the right and the left of the vertical demarcation curve for expected inflation. Strictly, the demarcation curve for unemployment is undefined in this case. Equation (A2) in the Appendix shows the time path of unemployment in terms of expected inflation. Taking the time derivative
of unemployment generates an expression in the time derivative of expected inflation alone. However, equation (A2) constitutes the expectations augmented Phillips curve and indicates that the time path of unemployment depends on the time path of expected inflation. This is drawn into Figure 1. At any point on the EAPC to the left of A, unemployment is below the NAIRU and the phase paths for expected inflation indicate that it should increase, moving us further to the left along the EAPC and away from point A. In fact what happens when inflation increases is that expected inflation is revised upwards by equation (5) so that the intercept of the EAPC is increased and the curve itself shifts upwards. At any point on the EAPC to the right of A, exactly the opposite adjustment occurs. Point A is thus inherently unstable and while an intelligible equilibrium exists for Friedman’s model with endogenous money, the economy will move systematically away from this equilibrium.

These results, obtained when endogenous money is injected into the baseline model, represent Wicksell’s famous pure credit model. Some argue that they reflect the ability of neoclassical economics to cope with endogenous money and to explain inflationary and deflationary phenomena. But this is a fundamentally mistaken interpretation. The cumulative process in Wicksell’s pure credit model represents an indeterminate price level due to the absence of an effective nominal anchor for the system (cf. McCallum 1986). That is, the system fails to explain the basic economic phenomena at hand: the level of output, unemployment and price level. And Docherty (1995) demonstrates that Wicksell himself does not regard this model as an operational endpoint but as an expositional tool designed to show the correct operation of the quantity theory in a ‘modern’ financial system with fractional reserve banking. In the end, Wicksell reintroduces an exogenous money base, and thus supply, to render the price level determinate thus effectively returning us to a model similar in spirit to the baseline model considered above. Thus if the simple interest rate rule in equation (11) is a reasonable representation of the essential principles of endogenous money, the results of this section suggest that endogenous money, as a structural feature of modern financial systems, is antithetical to basic neoclassical principles. Capitalist economic systems do not have a tendency to automatically gravitate towards full employment when the money supply is endogenously determined. An alternative explanation of the operation of this system must therefore be sought.

**Resolving Indeterminacy: The Role of Money Illusion**

We consider now the first of two possible instruments by which the unstable neoclassical model examined in the previous section may be rendered stable. This is the instrument of money
illusion. A second instrument, the Taylor rule for monetary policy, will be considered in the next section.

Instability in the neoclassical model with pure endogenous money arises because changes to the nominal interest rate which push the system towards equilibrium cannot be generated when the system is away from equilibrium. Money illusion provides a complex alternative source of stability firstly by rendering the Phillips curve stable in the face of inflation or deflation, and secondly by allowing unemployment to be variable with respect to inflation through the affect of real interest rates on expenditures rather than being fixed at the natural rate. The result is an equilibrium with unemployment different from the NAIRU and inflation dependent on the rate of base money wage growth relative to productivity growth and the level of unemployment.

When considering money illusion we may make one of two assumptions. We may either assume that workers suffer from money illusion by allowing the unemployment rate to affect their money wage demands rather than their real wage demands, or that firms suffer from money illusion by basing investment decisions on the money rate of interest rather than the real rate of interest. We take each of these possibilities in turn beginning with worker illusion. This involves a reformulation of equation (4) in the baseline model to remove expected inflation:

\[ \frac{\tilde{W}}{W_t} = w - \beta \cdot UE_t \]  

(15)

The third model we consider then is made up of equations (1)-(3), (15), (5) and (11). This system may be reduced to two equations by substituting (15) into (3) to generate the traditional Phillips curve shown in (16) below, and by recognizing from (5) that expected and actual inflation will be the same in equilibrium. We then substitute (11) into (1) and the resulting version of (1) into (2) to produce (17) after some rearranging.

\[ \pi_t = (w - p) - \beta \cdot UE_t \]  

(16)

\[ UE_t = UE^* + \gamma \cdot [Y^* - (\alpha \bar{A} - \alpha \bar{b})] - \gamma \alpha b \cdot \pi_t \]  

(17)

These two equations express the interdependence of unemployment and inflation in this model. Inflation depends on unemployment because the former is driven by wage growth, and unemployment inversely determines the size of wage claims, while unemployment depends on the level of output via Okun’s law which in turn is driven by the level of aggregate demand (itself
affected by the inflation rate through the real rate of interest). Equations (16) and (17) may be solved simultaneously to obtain the following solution vector:

\[
\begin{bmatrix}
\pi_i \\
UE_i
\end{bmatrix} = \begin{bmatrix}
\frac{\beta \gamma}{1 - \beta ab \gamma} \cdot [(\alpha \bar{A} - \alpha b \bar{t}) - Y^*] \\
UE^* + \frac{\gamma}{1 - \beta ab \gamma} \cdot [Y^* - (\alpha \bar{A} - \alpha b \bar{t})]
\end{bmatrix}
\]

(18)

According to this solution, equilibrium unemployment is made up of two components. The first is some base level unemployment which we have so far interpreted in terms of the NAIRU. The second depends on the excess of autonomous expenditures (including that associated with the fixed nominal interest rate) over potential output (a kind of adjusted output gap) corrected for the labour requirement in production and the fact that spending depends on the real rather than nominal interest rate. A clear and important aspect of this result is that the equilibrium value for unemployment is positively related to the nominal interest rate, \( \bar{i} \). Equilibrium inflation is driven by the size of this adjusted output gap corrected for the labour requirement in production and the impact of unemployment on the basic wage claim. This follows from the basic conception of the simple Phillips Curve. Within this conception, inflation depends on the basic money wage claim over growth in labour productivity and the degree of unemployment which undermines labour’s bargaining power. Since unemployment is made up of the base and output related components described above, there are effectively three components driving inflation. But the first two exactly offset each other when we assume that base unemployment, \( UE^* \), is the NAIRU, leaving only output-related unemployment having a direct effect on the level of equilibrium inflation.

The solution to this model reflects the argument advanced by some theorists that endogenous money represents a sufficient condition for the generation of equilibrium unemployment in an otherwise neoclassical model (see Lavoie 1996; Docherty 2005, 343ff). It may be seen however, that the nature of the condition must be understood clearly. Endogenous money renders the price level indeterminate in the neoclassical model requiring an alternative theory of prices and inflation. One such possibility is the traditional Phillips Curve in which prices implicitly depend on money wages reflecting in turn the costs of production. In this sense the “nominal anchor” is the social convention regarding money wages which depend on the relative bargaining strength of labour with appropriate adjustment for the technical characteristics of the production process in terms of productivity. Unemployment is characteristically Keynesian in this result depending on aggregate demand and varying positively with the nominal interest rate.
The solution in equation (18) is represented in Figure 2 below. Both the Phillips curve, representing equation (16), and the unemployment curve, representing equation (17), are inverse relations between inflation and unemployment. They intersect at point $E_1$ in Figure 2 which represents the equilibrium of the system. An increase in the nominal interest rate would shift the unemployment relation upwards to the dashed relation, moving its point of intersection with the Philips curve from $E_1$ to $E_2$. The new equilibrium would thus be characterised by a higher level of equilibrium unemployment but a lower level of equilibrium inflation holding everything else constant.

![Figure 2: Equilibrium and Stability Features of the Model with Endogenous Money and Worker Money Illusion](image)

The equilibrium, $E_1$, depicted in Figure 2 is stable provided that the slope of the Phillips curve is smaller in absolute terms than the slope of the unemployment relation. Consider a demand shock which temporarily increases the nominal rate of interest and moves the unemployment relation to the dashed line. Unemployment is now higher at $UE_1$ than its equilibrium value $UE^{**}$. The higher value of unemployment will, in turn, reduce the inflation rate via the Phillips curve to $\pi_1$. Stability of the original equilibrium, $E_1$, hinges on the value of unemployment that corresponds to this new value of inflation. If this value of unemployment is less than $UE_1$, (for example, $UE_2$) a stabilizing dynamic adjustment process will ensue. Since unemployment now
falls, inflation increases via the Phillips curve, higher inflation reduces the real rate of interest, increasing aggregate demand and reducing unemployment. The system thus moves from $E_2$ back towards $E_1$ as indicated by the arrowed adjustment paths. However if the level of unemployment corresponding to $\pi_1$ is above $U_E$, a dynamically unstable adjustment path ensues, taking the economy away from $E_1$. In this case unemployment would rise, inflation would be reduced via the Phillips curve, output would fall due to the increased real rate of interest that the fall in inflation represents, and unemployment would rise further. Additional increases in unemployment would lead to further increases in inflation in an unstable spiral.

The condition for stability then is that the unemployment rate corresponding to $\pi_1$ must be to the left of point $E_2$. This in turn requires that the slope of the Phillips curve be smaller in absolute terms than the slope of the unemployment relation. Expression (19) outlines this condition:\(^3\)

$$\beta < \frac{1}{ab\gamma}$$

Money illusion, therefore, plays an important role in the context of an otherwise neoclassical model with endogenous money. It delivers stability to an unstable model by providing the nominal anchor function usually preserved for the exogenous money supply. It also ensures that the equilibrium to which the model gravitates is characterised by unemployment different to the NAIRU and sensitive to the level of aggregate demand.

When firms suffer from money illusion, the result is somewhat different. If firms make spending decisions by taking into account the nominal interest rate rather than the real interest rate we may revise equation (1) to exclude the expected rate of inflation. This is done in equation (20) below:

$$Y_t = \alpha \bar{A} - ab \cdot i_t$$

The fourth model for consideration is thus made up of equations (20), (2) to (5) from the original model and (11) reflecting endogenous money. This model may be reduced to three equations by substituting (4) into (3) to obtain equation (8), the EAPC as previously, (11) into (20) and the resulting version of (20) into (2) to obtain (21), and equation (5):

$$\pi_t = (w - p) + \pi^*_t - \beta \cdot U_E$$

$$UE_t = UE^* + \gamma \cdot [Y^* - (\alpha \bar{A} - ab \bar{d})]$$

\(^3\) The Appendix confirms this result with stability analysis identical to that used for the models previously considered.
\[
\frac{d\pi^*_t}{dt} = j \cdot [\pi_t - \pi^*_t]
\]

(5)

Equation (5) indicates that in equilibrium expected and actual inflation are equal. This implies from equation (8) that unemployment will be at the NAIRU. However equation (21) also specifies the level of unemployment. Given that:

\[UE^* = \frac{w - p}{\beta}\]

the two equations are consistent only if:

\[Y^* = \alpha \bar{A} - ab\bar{I}\]

(22)

That is, if aggregate demand is equal to potential output, the output gap will be zero and unemployment will be at the natural rate. If the nominal interest rate is set too low so that aggregate demand exceeds potential output, unemployment falls below the natural rate, inflation rises above expected inflation and, by equations (5) and (8), inflation rises in a permanent upward spiral. Since inflation has no way of feeding back onto the expenditure equation, which depends only on the nominal rate of interest due to money illusion on the part of firms, this spiral makes the system dynamically unstable. Exactly the opposite result occurs if the nominal interest rate is set above the level that equates aggregate demand with output. This outcome is reflected in the formal solution for inflation derived in the Appendix and shown in equation (23):

\[
\pi_t = \{j(w - p) - j\beta \cdot UE^* - j\beta \gamma \cdot [Y^* - (\alpha \bar{A} - ab\bar{I})]\} \cdot t + C
\]

(23)

According to this equation, the equilibrium value of inflation is time varying where \(C\) is a constant determined from some boundary condition. If, for example, inflation is observed to be \(\pi_0\) at time \(t = 0\), \(C = \pi_0\). This equilibrium value for inflation is stable only if the coefficient of \(t\) in braces is zero, and the condition for this is precisely (22) above. In this case the equilibrium value for inflation is \(\pi_0\) and unemployment is at the NAIRU. If this condition is not met, unemployment will differ from the NAIRU and the system will continuously inflate or deflate without limit. The system is thus dynamically unstable when firms suffer from money illusion.

**A Model with Endogenous Money and a Taylor Rule**

A second approach to making the Friedman system with endogenous money stable is the imposition of a Taylor rule for monetary policy. Taylor rules function to replace the automatic
interest rate adjustment mechanism associated with equation (6) in the baseline model with a central bank-operated mechanism with essentially the same characteristics. A variety of forms could be adopted for this rule but we examine a form closely related to Taylor’s (1993, 202) original suggestion represented in equation (24):

\[ i_t = \bar{r} + \pi_t + \theta_1(\pi_t - \bar{\pi}) - \theta_2(U_E - \bar{U_E}) \]  \hspace{1cm} (24)

According to (24), the central bank sets the nominal interest rate in terms of some base real rate, \( \bar{r} \), plus the current inflation rate plus an upwards adjustment of \( \theta_1 \) for each percentage point that the inflation rate exceeds the target inflation rate, \( \bar{\pi} \), less an adjustment of \( \theta_2 \) for each percentage point that the unemployment rate exceeds the target unemployment rate \( \bar{U_E} \). Ideally the base real interest rate used in this calculation is the natural or full employment rate. This kind of system is argued to represent a form of endogenous money since the central bank is targeting the interest rate and must accommodate money demand to conduct monetary policy in this fashion. The question is whether this more sophisticated approach to policy has the same theoretical impact on the neoclassical system as the naïve policy rule associated with endogenous money throughout this paper.

The fifth model for consideration thus examines this question and is made up of equations (1)-(5) of the original model and (24). The Appendix shows how this new system can be reduced to two complex differential equations, again in expected inflation and unemployment:

\[
\begin{align*}
[1 + \gamma ab \theta_2 + \gamma ab(1 + \theta_1)\beta] \cdot U_E + [\gamma ab - \gamma ab(1 + \theta_1)] \cdot \pi^e \\
= U^* + \gamma ab(1 + \theta_1)(w - p) + \gamma [Y^* - (\alpha A - ab\bar{r}) - \gamma ab(\theta_1\bar{\pi} - \theta_2\bar{U_E})]
\end{align*}
\]  \hspace{1cm} (25)

\[
\frac{d\pi^e}{dt} + j\beta \cdot U_E = j \cdot (w - p) \]  \hspace{1cm} (26)

The solution to this system is:

\[
\begin{bmatrix}
\pi^e_t \\
U^*_{E_t}
\end{bmatrix} =
\begin{bmatrix}
\bar{\pi} \\
\frac{w - p}{\beta}
\end{bmatrix}
\]  \hspace{1cm} (27)

\[\text{Taylor (1993) uses GDP rather than unemployment and lagged inflation and GDP rather than current values for these variables in his original exposition.}\]
The Taylor rule thus gives us the same unemployment result as the baseline model with inflation at the central bank’s target level provided we assume that target unemployment is the NAIRU and equal weight is given to the two targets in the policy rule (the Appendix shows how the above result depends on these assumptions). Stability of this equilibrium requires:

\[ -\frac{j\beta y\sigma b \theta_1}{1 + \gamma ab \left[ \theta_1 + (1 + \theta_1) \cdot \beta \right]} < 0 \]  

(28)

which is shown in the Appendix to be reasonable.

Thus the neoclassical system with endogenous money and a Taylor rule possesses an equilibrium similar in nature to the equilibrium of the baseline model and is stable under reasonable conditions. Some also interpret this result as implying that neoclassical economics has the capacity to handle endogenous money. But one must be very careful of this interpretation in the light of the analysis in the rest of the paper. Friedman’s baseline model reflects the traditional core claims of neoclassical economics that capitalist systems tend to full employment automatically, that is, without the assistance of government, and that money is neutral in its effects on the system. The results above indicate that endogenous money renders the first of these propositions invalid. A neoclassical system with endogenous money does not automatically gravitate to full employment and lacks the capacity to determine the absolute price level.

The system is, however, rendered stable and determinate if the central bank performs the interest rate adjustment function that markets perform in the baseline model. But the operation of this function implies no fundamental change to the monetary nature of the economy compared with the first model considered above. Alterations to the interest rate simply become the mechanism by which the quantity of money is adjusted in a manner consistent with the central bank’s inflation target: when interest rates are increased, the quantity of money available in the money market is reduced and vice versa. It is true that the economy reconciles its demand for money to the new supply because expenditures are interest-sensitive, and in this sense money is endogenous. But a more accurate depiction of this mechanism is that of an alternative operating procedure for delivering a specific quantity of money to the system rather than a change to the principle that money supply growth determines inflation.

If the central bank decides to relax its attitude to inflation and raises the inflation target, \( \pi \), this will lead to a reduction in the policy-determined interest rate via equation (24). If money demand is stable, this interest rate change will correspond to a higher level of the money supply. In fact, the lower interest rate will be achieved by means of a larger volume of money made available to the system. The quantity of money thus continues to determine the price level in this kind of
model or put in dynamic terms, inflation is determined by the rate of monetary growth as with the baseline model. The monetary characteristics of this mechanism can only be described as “endogenous” in a semantic sense and not in the sense that aggregate demand determines the volume of money without the necessary consequence of feedback to demand which re-reverses money-income causality, causing it to flow in the traditional direction. In this sense, a Taylor rule does not represent endogenous money at all but a central bank mechanism for identifying and delivering the stock of money necessary for and consistent with a particular inflation target, leaving the quantity theory of money firmly in place.

The idea that Taylor rules incorporate endogenous money and at the same time ensure the stability and successful operation of the neoclassical model must, therefore be rejected. Taylor rules do ensure the stability of the neoclassical system but they achieve this outcome by reasserting the quantity theory of money in an operationally effective way, not by manipulating a genuinely endogenous money supply. In addition, Taylor rules cannot be used to assert the self-correcting nature of a market economy since their very nature requires the co-ordinating hand of government.

**Conclusion**

Consideration of the five models in this paper demonstrates a number of things. Firstly, endogenous money is not a theoretical structure that the basic neoclassical model has the capacity to handle. The proposition advanced in the literature by such theorists as Pivetti (1991, 2001) that it does, must be dismissed. Endogenous money in the sense of Kaldor (1970, 1986), Moore (1979, 1989), Lavoie (1996), Rochon (1999) and Docherty (2005) completely undermines the ability of the basic neoclassical model to automatically adjust to full employment and deliver a determinate absolute price level. If endogenous money constitutes a phenomena that should to be incorporated into macro models, an alternative model must be sought to explain the natural or automatic tendencies of a macro system. The role of Taylor rules in rendering the neoclassical system determinate do not reverse this conclusion. A fundamental principle of neoclassical economics is that economic systems automatically gravitate to full employment. Deliberate policy action may deliver full employment in a range of models some of which may also explain why an economy will settle down to an equilibrium characterised by unemployment in the absence of such policy action. Such models would have to be regarded as superior to one in which a specific form of policy action is required not simply to deliver the specific objective of full employment but to make the model itself intelligible.
One possibility for such an alternative is the third model considered in this paper which assumes money illusion on the part of workers in framing their wage claims. This model was shown to be stable but the equilibrium to which it argues the economy gravitates is characterised by unemployment different to the NAIRU and sensitive to aggregate demand and the level of interest rates. It is thus characterised by money non-neutrality. Inflation in this model is determined not by money supply growth but by money wage growth in excess of growth in labour productivity corrected for the level of unemployment.

This model with endogenous money and money illusion is not surprisingly, therefore, Keynesian in nature. It suffers, however, from two problems. Firstly, while money illusion is useful in rendering a Friedman-type, baseline model stable with Keynesian results, it surely does not represent a sensible form of economic behaviour on the part of workers. Friedman’s objection to money illusion in this sense is correct. Secondly, it incorporates interest-sensitive expenditures about which the capital debates raise important theoretical objections. Additional evidence from studies such as Fazzari, Petersen & Hubbard (1988) that cost of capital effects on investment spending are empirically weak, support these objections.

Resolving these two problems requires careful consideration given the important role played by both money illusion and interest-sensitive expenditures in the unemployment model considered in this paper. A model containing endogenous money but without interest-sensitive investment spending or money illusion on the part of workers raises a further set of questions. The first is whether endogenous money in a model with interest-insensitive expenditures is necessary to deliver an equilibrium unemployment result. It has been argued in this paper that endogenous money is sufficient to deliver this result when expenditures are interest-sensitive, but since interest-insensitive expenditures are usually argued to constitute an alternative sufficient condition for unemployment (Cottrell 1994, 591), the status of endogenous money in the presence of interest-insensitive expenditures must be carefully considered. A second question is whether equilibrium in a model without money illusion will be stable given that money illusion ensures stability in the Keynesian model considered in this paper. These questions represent challenges for the development of a coherent alternative to neoclassical economics but the negative work of criticising an existing body of ideas is always easier than the positive work of creating new and better theoretical structures. Both, of course, are important for improving our ability to manage the economy and improve human welfare.
References


Appendix

This appendix outlines the solution and stability procedures for each of the models considered in the text.

Model 1: Friedman’s (1968) Baseline Approach

Equations (1) to (6) in the text may be simplified considerably. Differentiating (1) and (2) with respect to time and substituting the resulting version of (2) into the resulting version of (1) and rearranging gives:

\[ -\frac{1}{\gamma} \cdot (1 + \alpha bk_1) \cdot \frac{dUE_t}{dt} = \alpha b \cdot \frac{d\pi^e_t}{dt} + \alpha bk_2 \pi_t = \alpha bk_2 \frac{\dot{m}}{m} \]  

(A1)

Substitution of (4) into (3) yields Friedman’s expectations-augmented (EAPC):

\[ \pi_t = (w - p) + \pi^e_t - \beta \cdot UE_t \]  

(A2)

Equation (A2) may be substituted into (A1) and (5) to remove inflation. This generates the following system of two first order differential equations in expected inflation and unemployment:

\[ -\alpha b \cdot \frac{d\pi^e_t}{dt} - \frac{1}{\gamma} \cdot (1 + \alpha bk_1) \cdot \frac{dUE_t}{dt} + \alpha bk \pi^e_t - \alpha b \beta k_2 \cdot UE_t = \alpha bk_2 \left[ \frac{\dot{m}}{m} \cdot (w - p) \right] \]  

(A3)

\[ \frac{d\pi^e_t}{dt} + j\beta \cdot UE_t = j \cdot (w - p) \]  

(A4)

This system may be expressed in matrix form as:

\[
\begin{bmatrix}
-\alpha b & -\frac{1}{\gamma} \cdot (1 + \alpha bk_1) \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
\pi^e_t \\
UE_t
\end{bmatrix}
+ \begin{bmatrix}
\alpha bk_2 \\
0
\end{bmatrix}
\begin{bmatrix}
\pi^e_t \\
UE_t
\end{bmatrix}
= \begin{bmatrix}
\alpha bk_2 \left[ \frac{\dot{m}}{m} \cdot (w - p) \right] \\
0
\end{bmatrix}
\]

or in more compact notation as:

\[ J \cdot \mu + M \cdot \nu = g \]  

(A5)

Constant solutions to (A5) are given by:

\[
\begin{bmatrix}
\pi^e_t \\
UE_t
\end{bmatrix} = \begin{bmatrix}
\frac{\dot{m}}{m} \\
\frac{(w - p)}{\beta}
\end{bmatrix}
\]  

(A6)
Note from the definition of potential output $Y^*$ that when unemployment is at its equilibrium value $Y = Y^*$. Thus from equation (2):

$$UE^* = \frac{w - P}{\beta}$$

The stability of this long run equilibrium is ascertained by inspection of the roots of the characteristic equation of (A5). We obtain this characteristic equation from the reduced equation corresponding to (A5):

$$J \cdot \mu + M \cdot \upsilon = 0$$

A non-trivial solution to this equation of the form $\pi^e_t = me^{\gamma t}$ and $UE_t = ne^{\gamma t}$ requires the following vanishing determinant:

$$|r \cdot J + M| = 0$$

which after substitution for $J$ and $M$, rearranging and normalisation generates the characteristic equation:

$$r^2 + \frac{ab\gamma \beta \cdot (k_2 - j)}{1 + abk_1} \cdot r - \frac{j ab\gamma \beta k_2}{1 + abk_1} = 0$$

The roots of this characteristic equation are:

$$r_1 = -\frac{1}{2} \cdot \frac{ab\gamma \beta \cdot (k_2 - j)}{1 + abk_1} + \frac{1}{2} \cdot \sqrt{\left(\frac{ab\gamma \beta \cdot (k_2 - j)}{1 + abk_1}\right)^2 - 4 \cdot \frac{j ab\gamma \beta k_2}{1 + abk_1}}$$

$$r_2 = -\frac{1}{2} \cdot \frac{ab\gamma \beta \cdot (k_2 - j)}{1 + abk_1} - \frac{1}{2} \cdot \sqrt{\left(\frac{ab\gamma \beta \cdot (k_2 - j)}{1 + abk_1}\right)^2 - 4 \cdot \frac{j ab\gamma \beta k_2}{1 + abk_1}}$$

Stability hinges on the term under the radical signs on the LHS of (A9) and three cases are relevant depending on the following condition:

$$\left(\frac{ab\gamma \beta \cdot (k_2 - j)}{1 + abk_1}\right)^2 \geq 4 \cdot \frac{j ab\gamma \beta k_2}{1 + abk_1}$$

In Case 1, (A10) is satisfied non-strictly. In this case the term under the radical signs in (A9) is positive and we have distinct real roots. Since all of the parameters are strictly positive, the following is true:
\[
\frac{ab\gamma\beta \cdot (k_2 - j)}{1 + abk_1} > 0 \quad (A11)
\]

if
\[
k_2 > j \quad (A12)
\]

This condition requires interest rate adjustmen in response to inflation and monetary growth to be greater than the adjustment of incorrect inflation expectations. However if this condition is met, the radical term in each root will be smaller than the first expression on the RHS of (A9), both roots will be negative and the system converges to its equilibrium values.

In Case 2, (A10) is met strictly and the radical terms vanish in both roots. Given (A12) both roots are identically negative and we have stability. In Case 3, (A10) is not met and we have complex roots with cyclical fluctuations. Stability then hinges on whether the real part of the roots is negative. This real component is simply the first term on the RHS of (A9) and this is negative given (A11). This in turn depends on (A12) as with Case 2. We may thus conclude that a sufficient general condition for stability of the Friedman system is given by (A12).

**Model 2: The Baseline Model with Endogenous Money**

The second model in the text made up of equations (1) to (5) and (11) may be simplified by substituting (2) into (1), (11) into the resulting equation, and rearranging. This yields equation (A13) below.

\[
\frac{1}{\gamma} \cdot UE_i + ab\pi_i^{*} = Y^{*} + \frac{UE^{*}}{\gamma} - (\alpha\bar{A} - ab\bar{t}) \quad (A13)
\]

Substitution of (4) into (3) and the resulting equation into (5) gives (A4) as before. Equations (A13) and (A4) now constitute the model’s system which in matrix form is given by (A14):

\[
\begin{bmatrix}
0 & 0 \\
1 & 0 \\
\end{bmatrix} \begin{bmatrix}
\pi_i^{*}
\end{bmatrix} + \begin{bmatrix}
ab \\
\gamma
\end{bmatrix} \begin{bmatrix}
\pi_i^{*}
\end{bmatrix} = \begin{bmatrix}
Y^{*} + \frac{UE^{*}}{\gamma} - (\alpha\bar{A} - ab\bar{t}) \\
\beta \end{bmatrix}
\]

(A14)

The solution to (A14) is:

\[
\begin{bmatrix}
\pi_i^{e} \\
UE_i
\end{bmatrix} = \begin{bmatrix}
\frac{1}{ab} \cdot Y^{*} + \frac{UE^{*}}{ab\gamma} \cdot \left(\frac{\bar{A}}{b} - \bar{t}\right) - \frac{w - p}{\beta \cdot ab\gamma} \\
\frac{(w - p)}{\beta}
\end{bmatrix}
\]

(A15)
But recognising that:

\[ UE^* = \frac{w - p}{\beta} \]

this becomes:

\[
\begin{bmatrix}
\pi_t^* \\
UE_t
\end{bmatrix} = \begin{bmatrix}
\frac{1}{ab}y^* - (\frac{A}{b} - \bar{i}) \\
\frac{(w - p)}{\beta}
\end{bmatrix} \tag{A16}
\]

The stability of this equilibrium depends on the solution to the following characteristic equation implied by the relevant version of (A7) for the present system:

\[ abj\beta - \frac{r}{\gamma} = 0 \tag{A17} \]

The solution to (A17) is straightforward:

\[ r = \gamma abj\beta \tag{A18} \]

The term on the RHS of (A18) is unambiguously positive indicating that (A16) is unstable without cycles.

**Model 3: Endogenous Money and Worker Money Illusion**

The model with money illusion on the part of workers is made up of equations (1)-(3), (15), (5) and (11). Substituting (15) into (3) generates the traditional version of the Phillips curve which may be substituted into (5) to obtain (A19) below. Rearranging (2) and substituting the resulting expression along with (11) into (1) and normalising gives (A20). Equations (A19) and (A20) thus constitute our two equation summary of the system.

\[
\frac{d\pi_t^*}{dt} + j \cdot \pi_t^* + j\beta \cdot UE_t = j \cdot (w - p) \tag{A19}
\]

\[
UE_t + ab\gamma \cdot \pi_t^* = UE^* + \gamma \cdot [Y^* - (\alpha\bar{A} - ab\bar{i})] \tag{A20}
\]

In matrix form this may be written as:

\[
\begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix} \cdot \begin{bmatrix}
\pi_t^* \\
UE_t
\end{bmatrix} + \begin{bmatrix}
j & j\beta \\
ab\gamma & 1
\end{bmatrix} \cdot \begin{bmatrix}
\pi_t^* \\
UE_t
\end{bmatrix} = \begin{bmatrix}
j \cdot (w - p) \\
UE^* + \gamma \cdot [Y^* - (\alpha\bar{A} - ab\bar{i})]
\end{bmatrix} \tag{A21}
\]
The solution to this system is:

\[
\begin{bmatrix}
\pi_e \\
UE_e
\end{bmatrix} = \begin{bmatrix}
\frac{\beta \gamma}{(1 - ab\gamma \beta)} \cdot [(\alpha \bar{A} - \alpha b\bar{i}) - Y^*] \\
\frac{\gamma}{(1 - ab\gamma \beta)} \cdot [Y^* - (\alpha \bar{A} - \alpha b\bar{i})]
\end{bmatrix}
\] (A22)

The stability of this intertemporal equilibrium depends on the root of the characteristic equation associated with the following determinantal condition:

\[
\begin{vmatrix}
 r + j & j\beta \\
\gamma ab & 1
\end{vmatrix} = 0 
\] (A23)

There is only a single root for this equation:

\[ r = j \cdot (ab\beta \gamma - 1) \] (A24)

which must be negative for stability. Negativity is ensured under the following condition:

\[ \beta < \frac{1}{ab\gamma} \] (A25)

**Model 4: Endogenous Money and Money Illusion on the Part of Firms**

The model with money illusion on the part of firms is made up of equations (20), (2) to (5) and (11). Following the same procedure as previously, we may substitute (11) into (20) and the resulting expression for output into (2). This gives the following expression for unemployment after rearranging:

\[ UE_e = UE^* + \gamma Y^* - \gamma \alpha \bar{A} + \gamma ab\bar{i} \] (A26)

The EAPC is also obtained the same way as before by substituting (4) into (3) to obtain (A2). When (A2) is substituted into (5) we obtain (A4) once again. But now we may substitute (A26) for unemployment to obtain the following first order differential equation in expected inflation:

\[ \frac{d\pi_e}{dt} = j(w - p) - j\beta \cdot UE^* - j\beta \gamma \cdot [Y^* - (\alpha \bar{A} - \alpha b\bar{i})] \] (A27)
This delivers a much simpler expression than in the models previously considered and the solution for unemployment is immediately apparent. The solution is obtained by straight integration (Chiang 1984, 473) and is given by:

\[ \pi_i^* = \{j(w - p) - j\beta \cdot UE^* - j\beta \gamma \cdot \{Y^* - (\alpha A - ab\xi)\}\} \cdot t + C \]  

(A28)

where \( C \) is a constant that depends on some boundary condition. Given that \( UE^* = \frac{w - p}{\beta} \), the term in braces on the RHS of (A28) is zero when aggregate demand is equal to potential output, that is when:

\[ Y^* = \alpha A - ab\xi \]  

(A29)

In this case expected inflation is constant at the boundary condition. Otherwise the value of expected inflation rises without limit if \( Y^* < \alpha A - ab\xi \) or falls without limit if \( Y^* > \alpha A - ab\xi \).

**Model 5: Friedman’s Model with a Taylor Rule**

Friedman’s model with a Taylor rule is made up of equations (1) to (5) and (24). We follow the now familiar procedure of substituting (4) into (3) to obtain the EAPC. Substituting the EAPC into (5) gives the familiar differential equation in (A30) below. Then substituting (24) into (1) and the resulting equation along with the EAPC into (2) gives (A31) below:

\[ \frac{d\pi_i^e}{dt} + j\beta \cdot UE_i = j \cdot (w - p) \]  

(A30)

\[ [1 + \gamma ab \theta_1 + \gamma ab(1 + \theta_1)\beta] \cdot UE_i + [\gamma ab - \gamma ab(1 + \theta_1)] \cdot \pi_i^e \]

\[ = UE^* + \gamma ab(1 + \theta_1)(w - p) + \gamma \{Y^* - (\alpha A - ab\xi)\} - \gamma ab(\theta_1\pi - \theta_2\overline{UE}) \]  

(A31)

Reversing the order of these equations and putting them into matrix form gives:

\[
\begin{bmatrix}
0 & 0 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
\pi_i^e \\
UE_i
\end{bmatrix}
\begin{bmatrix}
-\alpha b \gamma \theta_1 & 1 + \alpha b \gamma (\theta_2 + \beta \cdot (1 + \theta_1)) \\
0 & j\beta
\end{bmatrix}
\begin{bmatrix}
\pi_i^e \\
UE_i
\end{bmatrix}
\]

\[
\begin{bmatrix}
UE^* + \gamma \{Y^* - (\alpha A - ab\xi)\} \\
\gamma ab(1 + \theta_1)(w - p) - \alpha b \gamma (\theta_1\pi - \theta_2\overline{UE})
\end{bmatrix}
\]

(A32)
The solution to this system of equations is given by:

\[
\begin{bmatrix}
\pi^e_i \\
UE_i
\end{bmatrix} = \left[ \frac{\theta_i \pi^e - \theta_i \bar{UE}}{\theta_1} - \frac{UE^*}{ab \gamma \theta} + \frac{(w - p)}{\beta} \right] \left[ \frac{\theta_2}{\theta_1} + \frac{1}{ab \gamma \theta_1} \right] \cdot \left[ \frac{1}{ab \theta_1} \cdot [Y^* - (\alpha A - ab \bar{r})] \right]
\]

\[
= \left[ \frac{w - p}{\beta} \right]
\]

(A33)

The Taylor rule generates the same unemployment result as does the original Friedman system although the inflation result is complex and difficult to interpret. This may, however, be simplified by making three assumptions or observations: Firstly we follow Taylor (1993) by assuming \( \theta_i = \theta_2 = \theta \); secondly we note from (1) that \( Y_i = \alpha A - ab \cdot (i_i - \pi^e_i) \). By definition when the bracketed term is equal to the natural rate of interest, \( \bar{r} \), output is at its potential level, thus \( Y_i^* - (\alpha A - ab \cdot \bar{r}) = 0 \); Thirdly, we assume that the unemployment target, \( \bar{UE} \), is the NAIRU, which in our model is given by \( UE^* \). Under these conditions, the equilibrium inflation is simply the central bank’s policy target:

\[ \pi^e_i = \bar{\pi} \]  

(A34)

The stability of these results depends on the root of the characteristic equation associated with the following determinantal condition:

\[
\left| \begin{array}{cc}
-\gamma ab \theta_1 & 1 + \gamma ab \cdot [\theta_2 + (1 + \theta_1) \cdot \beta] \\
r & j \beta
\end{array} \right| = 0
\]

(A35)

There is only a single root for this equation:

\[ r = \frac{-j \beta \gamma ab \theta_1}{1 + \gamma ab \cdot [\theta_2 + (1 + \theta_1) \cdot \beta]} \]

(A36)

Since all of the parameters are strictly non-negative, this expression is strictly negative and the intertemporal equilibrium in (A33-34) is stable.