Money and Monetary Policy in a Kaldor-Pasinetti-Sraffa-Keynes Framework

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Abstract
This paper investigates the role of money and the transmission of monetary policy in a model characterised by interest-insensitive expenditures and unemployment equilibria. It first outlines the structure of what is called a Kaldor-Pasinetti-Sraffa-Keynes (KPSK) model where output is determined by the principle of effective demand, expenditures are determined by income distribution, and prices are determined along Sraffian lines. It secondly explores the role of money in this kind of model, assuming first an exogenous money supply and then an endogenous one, and compares the difference between these two sets of monetary arrangements for the model’s key macroeconomic variables: output, employment and prices. This exploration provides evidence that endogenous money is a necessary condition for the attainment of an unemployment equilibrium. The paper thirdly examines the operation of a Taylor rule in the conduct of monetary policy within a KPSK model, comparing its impact with that in a neoclassical macro model where such rules generate full employment. The paper concludes with some observations about the relationship between the concepts of money supply endogeneity, the interest-insensitivity of expenditures and money non-neutrality in models of the KPSK variety.

JEL Classification Numbers: E11, E12, E42, & E52.

Keywords: Endogenous money, money non-neutrality, long period unemployment.

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1. Introduction

The role of money and monetary conditions in explaining the existence of long period unemployment has received increasing attention in recent years. The original objective of Keynes’ *The General Theory* was, of course, to explain why unemployment could characterise an economy’s equilibrium rather than simply representing an intermediate state on an adjustment path towards equilibrium. At the centre of Keynes’ theory of output and employment was the principle of effective demand. But unfortunately the core system built around this principle gravitates to full employment with the passage of sufficient time. According to the *Keynes effect*, wage deflation in the face of unemployment reduces the demand for money, reduced demand for money causes reductions in interest rates, and these reductions stimulate interest-sensitive investment spending until unemployment is eliminated.

Cottrell (1994, 581) identifies three possible mechanisms capable of circumventing this effect: nominal wage rigidity; investment or expenditure interest-insensitivity; and a structurally fixed interest rate. The Cambridge critique provides theoretical justification for the second of these mechanisms (see Garegnani 1978, 348) while Fazzari, Hubbard & Petersen (1988, 143) provide empirical support in that they find little evidence for the role of cost of capital effects on investment spending. Cottrell’s paper very usefully explores the third mechanism which has come to be associated with endogenous money of the accommodationist form.

Docherty (2006) re-examines the role of endogenous money in preventing the gravitation of systems with interest-sensitive expenditures to full employment and concurs with Cottrell (1994) that endogenous money is sufficient to disrupt such gravitation. He also argues that the combination of endogenous money and money illusion on the part of workers is sufficient to generate Keynesian unemployment equilibrium. Such models are not, however, very satisfactory for explaining long period unemployment since the assumption of money illusion is problematic and since such models ignore the importance of the Cambridge critique. Recognition of the Cambridge critique, however, by designing expenditures to be interest-insensitive raises questions about the role of endogenous money. Since interest-insensitivity is sufficient to generate unemployment equilibria, is endogenous money also required in alternative macro models?

This paper casts further light on this question and its implications for monetary policy. It examines the monetary characteristics of a simple Kaldor-Pasinetti-Sraffia-Keynes (KPSK) model built around the principle of effective demand and incorporating interest-insensitive expenditures with endogenous money. Section 2 outlines the structure of the model and its equilibrium and stability features. Section 3 examines the role of endogenous money within the model. Section 4
then considers a series of monetary policy approaches within the model that illuminate its operation. The argument and main findings are summarised in the final section.

2. The Kaldor-Pasinetti-Sraffa-Keynes (KPSK) Model

Three features are central to the model outlined in this section. It has at its core the principle of effective demand, reflecting the major contribution of Keynes’ *The General Theory*. Secondly, unlike *The General Theory*, it reflects the central insight of the Cambridge critique so that investment does not have the rate of interest as an argument. Thirdly, it contains endogenous money, reflecting Cottrell’s (1994) observation discussed above and the contributions of Kaldor (1970, 1986) and Moore (1988) in particular. In order to outline the essential features of the model, a highly simplified form is assumed. The economy is static with zero real growth over time. There is only one productive sector. Workers’ income arises solely from their supply of labour and capitalists’ income arises solely from profits. Endogenous money is simply portrayed in terms of a fixed interest rate in the Kaldor tradition, leaving the quantity of money implicit. Each of these assumptions omits significant insights from the contributing theories but the implications of the resulting model for output and employment are much easier to interpret given these simplifying assumptions. The approach thus represents a good first step towards the construction of a more complicated model along these lines.

The model is outlined in equations (1) to (15) below. Equations (1) to (6) encapsulate the theory of demand drawn from Keynes (1936), Kaldor (1956) and Pasinetti (1974). Equation (1) represents the principle of effective demand by which output is determined as the sum of lagged total consumption spending ($C_t$), lagged investment spending ($I_t$) and autonomous government spending ($G$). Total consumption spending is given in equation (2) as the sum of consumption by workers and capitalists respectively. The former, in the first set of square brackets on the right hand side of (2), is made up of autonomous worker consumption ($wC$) and a proportion ($1 - s_w$) of aggregate real wages ($W_t$) net of taxes (where $h$ is the single marginal and average tax rate). The latter, in the second set of square brackets, is similarly comprised of autonomous capitalist consumption ($cC$) and a proportion ($1 - s_c$) of aggregate real profits ($P_t$) net of taxes. Aggregate real wages are determined in equation (3) as the product of the real wage rate per unit of labour ($w_t$) and the volume of employment, itself the product of the unit labour requirement, or labour-output ratio, ($\ell$) and the level of output ($Y_t$). Aggregate profits are determined residually in equation (4) from an aggregate budget constraint according to which aggregate real wages and profits sum to aggregate output ($Y_t$).
Investment spending is determined in equation (5) and represents one possibility for reflecting the Cambridge critique. In a dynamic context this expression would allow changes in output to affect investment spending via a classic acceleration mechanism but in the present, static context investment simply replaces capital as it wears out, maintaining the economy’s ability to produce the equilibrium level of output. Thus equation (5) determines investment as the product of the rate of economic depreciation of the capital stock ($\partial$), the unit capital requirement, or capital-output ratio, ($k$), and the current level of output. Both $\partial$ and $k$ are assumed to be technologically given. Equation (6) determines the rate of unemployment ($UE_t$) as the number of unemployed, being the exogenously given labour supply ($L^*$) less the demand for labour, as a proportion of the exogenous labour supply. The demand for labour is the product of the unit labour requirement and the volume of real production.

$$Y_t = C_{t-1} + I_{t-1} + G$$  \hspace{1cm} (1)

$$C_t = [\overline{C}_w + (1 - s_w) \cdot (1 - h) \cdot W_t] + [\overline{C}_c + (1 - s_c) \cdot (1 - h) \cdot P_t]$$  \hspace{1cm} (2)

$$W_t = \ell \cdot Y_t$$  \hspace{1cm} (3)

$$Y_t = W_t + P_t$$  \hspace{1cm} (4)

$$I_t = \partial k \cdot Y_t$$  \hspace{1cm} (5)

$$UE_t = (L^* - \ell \cdot Y_t) / L^*$$  \hspace{1cm} (6)

$$w_t = \frac{1}{\ell} \cdot [1 - k \cdot (r_t + \partial)]$$  \hspace{1cm} (7)

$$r_t = r_{n,t} - \pi_t$$  \hspace{1cm} (8)

$$r_{n,t} = i_t + \sigma_t$$  \hspace{1cm} (9)

$$i_t = i^*$$  \hspace{1cm} (10)

$$i^* = l_1 + \pi_{t-1}$$  \hspace{1cm} (11)

$$\pi_t = MWG_t - \lambda_{t-1}$$  \hspace{1cm} (12)

$$MWG_t = \alpha + \beta_1 \pi^e_t - \beta_2 UE_{t-1}$$  \hspace{1cm} (13)

$$\pi^e_t = \pi^{e}_{t-1} + j \cdot (\pi_t - \pi^{e}_{t-1})$$  \hspace{1cm} (14)

$$\lambda_t = \frac{w_t - w_{t-1}}{w_{t-1}}$$  \hspace{1cm} (15)
Equations (7) to (11) encapsulate the Sraffian theory of distribution and are constructed using similar methodology to Aspromourgos (1991). The real wage rate is given by equation (7). This is a version of the standard Sraffa pricing equation derived from the expression for the rate of profit \( r_t \) (shown in the Appendix) and determines the real wage in terms of the technical conditions of production and the real rate of profit. The real rate of profit is determined by equations (8) to (11). Equation (8) defines the real rate of profit as the nominal rate of profit less the inflation rate \( \pi_t \). Equation (9) determines the nominal rate of profit \( n_{nr} \) using the nominal rate of interest \( i_t \), as a benchmark to which an exogenous risk premium \( \sigma_t \) is added. Equation (10) reflects the model’s implicit assumption of endogenous money. The central bank supplies whatever volume of money is required by the system to deliver its target nominal interest rate \( i^* \). This target rate is determined in equation (11) where the central bank targets a constant real rate of interest of \( i^* \). The nominal rate in the current period is thus the target real rate plus the observed inflation rate from the previous period.

The model’s inflation dynamics are given in equations (12) to (15) in terms of a conflicting claims framework. The usual form for such a framework is to specify two equations (cf. Lavoie, 2002, 179) such as those in (16) and (17). Equation (16) determines the money wage \( MW_t \) pursued by workers in terms of the difference between their desired real wage \( w^* \) and the lagged real wage \( w_{t-1} \), and the expected price level \( p^*_t \). \( \Omega_1 \) and \( \Omega_2 \) are positive adjustment parameters. Equation (17) specifies a price equation used by firms to pursue a target rate of return or mark-up over input costs represented by the expected money wage \( MW_t^* \). Thus prices are increased whenever the target real rate of profit \( r^* \) exceeds the lagged rate of profit \( r_{t-1} \). \( \Omega_3 \) and \( \Omega_4 \) are also positive adjustment parameters.

\[
MW_t = \Omega_1(w^* - w_{t-1}) + \Omega_2 p^*_t \quad (16)
\]
\[
p_t = \Omega_3(r^* - r_{t-1}) + \Omega_4 MW_t^* \quad (17)
\]

Interaction between these two equations determines price and inflation dynamics. As Rowthorn’s (1977, 219) original exposition indicates, precise dynamics depend on the coefficient values which reflect the relative market power or bargaining strengths of workers and capitalists. These will vary from place to place and time to time and are very different in the 2000s compared to their values in the 1970s for most western economies. Equations (16) and (17) could also be specified in terms of the rates of change of money wages and prices rather than their levels.
The form taken by wage-price dynamics in the KPSK model is slightly different to this standard representation. The rate of profit is determined by monetary policy and the exogenous risk premium in equations (8) to (10). We assume that firms always realise the rate of profit in equation (8) by the appropriate setting of output prices. These prices are determined in line with the real wage corresponding to the target rate of profit given the money wage. In growth terms, the rate of price inflation ($\pi_t$) will be equal to the rate of money wage growth ($MWG_t$) less the rate of change in real wages ($\lambda_t$). This is shown in equation (12) and is a dynamic version of equation (17) above.

The money wage dynamics of equation (16) may also be expressed in growth terms as follows:

$$MWG_t = \Omega_e (\lambda^* - \lambda_{t-1}) + \Omega_\pi \pi^*_t$$

An alternative formulation, however, would be for workers to specify a real wage or rate of real wage growth as the basis for money wage claims. Money wage growth would then be equal to target real wage growth plus expected inflation. Equation (19) follows this specification where $\beta_i$ captures the extent to which workers have the power to achieve their target. While the value of this parameter would vary as argued above, it could generally be expected to be less than one.

$$MWG_t = \beta_i [\lambda^*_t + \pi^*_t]$$

Rowthorn (1977, 219-224) further explores the determinants of his counterpart of $\lambda^*_t$ and argues that the level of aggregate demand is significant in this respect. Equation (20) sets out the possible impact of aggregate demand on target real wage growth in terms of Okun’s Law and unemployment:

$$\lambda^*_t = \varepsilon - \theta_1 UE_{t-1}$$

According to (20), workers have a basic real wage growth target of $\varepsilon$ but as aggregate demand in the economy falls, output falls and unemployment rises. Workers realise that their bargaining power may be weaker on account of competition between unemployed workers and they moderate aspirations for real wage growth by $\theta_1$ for each percentage point of the workforce that is unemployed. Equation (20) may be substituted into (19) to obtain a single expression for the rate of money wage growth as follows:

$$MWG_t = \varepsilon \beta_i + \beta_i \pi^*_t - \beta_1 \theta_2 UE_{t-1}$$
Equation (13) in the model is identical to equation (21) except for a recalibration of parameters with $\alpha = \varepsilon \beta_1$ and $\beta_2 = \beta_1 \theta_1$ for the sake of simplicity. Equation (14) determines the expected rate of inflation by a simple adaptive expectations mechanism and equation (15) simply defines the rate of real wage growth.

To summarise the model, equations (1) - (5) encapsulate the essential elements of Keynes’ principle of effective demand and the Kaldor-Pasinetti theory of income distribution. This approach to distribution stresses the impact of aggregate distributive shares on consumption and hence output. It is, however, reconcilable with Sraffa’s (1960) approach to value and distribution which determines distributive prices rather than aggregate shares (cf. Garegnani 1984). Equation (3) explicitly links the two approaches by showing the aggregate share of wages in terms of the Sraffian real wage rate and technical production coefficients given the level of output determined by the principle of effective demand. The Sraffian equation also allows us to incorporate Sraffa’s suggestion that the free distributive variable be the wage rate with the rate of profit being determined by the rate of interest (Sraffa 1960, 33). But here the Kaldor-Pasinetti dimension of the theory of distribution provides the transmission mechanism by which this closure of the distributive story also affects output. Essentially monetary policy determines the interest rate via equations (10) and (11) and the rate of profit by equation (8). This determines the real wage by equation (7) and the aggregate real wage by equation (3) which then has an impact on consumption via equations (2) and (4). Unemployment is determined by a version of Okun’s Law expressed in equation (6). The Sraffian component of the model essentially determines income distribution via monetary policy while the Kaldor-Pasinetti component of the model examines the impact that this distribution, scaled to the aggregate level, has on effective demand and output. The inflationary process is affected by distribution and output but does not feedback on these variables in long run equilibrium.

The KPSK system represented in equations (1) to (15) may be reduced to a smaller set of equations as described in the Appendix. The reduced form of the model is given in equations (22) to (24) below:

\[ Y_{t+2} - \theta_2 Y_{t+1} - \partial k Y_t = \bar{A} \]  
\[ U E_{t+2} - \theta_1 U E_{t+1} - \partial k U E_t = (1 - \theta_2 - \partial k) - \frac{\ell}{L^* \bar{A}} \]  
\[ (1 - j \beta_1) \pi_{t+1}^c - (1 - j) \pi_{t+1} + j \beta_1 \beta_2 U E_{t+1} = \alpha j \]

Where $\theta_2 = (1 - h) \cdot (1 - s_c) + (1 - h) \cdot (s_c - s_w) \cdot [1 - k \cdot (\bar{I} + \sigma + \bar{\sigma})]$ and $\bar{A} = \bar{C}_{w} + \bar{C}_{c} + \bar{G}$. 
Equation (22) is, however, separable and can be explored independently. This second order linear difference equation in output possesses the intertemporal equilibrium given in expression (25) (once again see the Appendix for the derivation of this expression):

\[ Y^{**} = \frac{1}{1 - \theta_2 - \delta k} \cdot \bar{A} \]  

(25)

This intertemporal equilibrium has a familiar Keynesian form although the multiplier is more complex than in the usual Keynesian case. The expression for \( \theta_2 \) in (22), indicates that the multiplier process in the KPSK model depends on the proportion of income spent with each augmentation of income but this proportion depends on the relative propensities to consume of workers and capitalists, expressions \((1 - s_w)\) and \((1 - s_c)\), and the distribution of income. The distribution of income is in turn determined by monetary policy and the technical conditions of production. The \( \delta k \) term indicates that investment impinges on the multiplier process since the proportion of capital that depreciates each period depends on the volume of production. However, equilibrium output is equal to a multiple of autonomous expenditure where the multiple depends on distribution and monetary conditions in the model. The model is thus characterised by money non-neutrality.

Stability of this equilibrium depends on whether the absolute value of the roots of the characteristic equation corresponding to (22) are less than unity. The expression for these roots involves a complex combination of the model’s parameter values including the capital-output ratio and the rate of depreciation. This expression is shown in the Appendix where it is evaluated for specified values of the parameters (shown in Table 1) and a range of capital-output ratios and depreciation rates. Table 2 shows the results of this evaluation for various combinations of the capital-labour ratio and the depreciation rate. Where the absolute value of both roots is less than

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one, Table 2 indicates that the equilibrium in (25) is stable, and where this condition is not met the equilibrium is indicated to be unstable. The shaded area indicates all combinations for which the equilibrium is stable. Since a capital-output ratio of around one and a depreciation rate of 15% would be reasonable for a small open economy (such as that of Australia), the bordered area indicates what might be regarded as a range of reasonable combinations for these two variables. Since this bordered area is well within the shaded region, we conclude that the KPSK model is stable for reasonably specified parameters values.

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Determination of equilibrium output from equation (22) then feeds into equation (23) to determine equilibrium unemployment, which feeds into equation (24) to determine equilibrium inflation. These equilibrium values are shown in expressions (26) and (27):

\[
UE^{**} = 1 - \frac{\ell}{L^*} \cdot \frac{1}{1 - \theta_2 - \partial k} \cdot \bar{A}
\]  
(26)

\[
\pi^{**} = \frac{1}{1 - \beta_1} \left[ \alpha - \beta_2 + \frac{\ell \beta_2}{L^*} \cdot \frac{\bar{A}}{1 - \theta_2 - \partial k} \right]
\]  
(27)

The conditions for stability of these results are also shown in the Appendix and unambiguously hold for the parameter values specified in Table 1. The KPSK model of output, distribution and inflation thus provides a coherent alternative to the neoclassical paradigm.
3. The Role of Endogenous Money in the KPSK Model

The KPSK model outlined in Section 2 incorporates both endogenous money and interest–insensitive investment expenditure. Given the argument that each of these structural features is capable of delivering an equilibrium characterised by unemployment, it will be instructive to explore money’s role in the model. Our starting point in this exploration is a recognition that money is non-neutral within the KPSK model outlined in equations (1) to (5). The operation of endogenous money, of course, entails fixing the real interest rate. This determines income distribution and, with the technical conditions of production and investment behaviour, the multiplier. An interest rate set too high implies a smaller multiplier and lower equilibrium output. In this way, monetary conditions have real effects.

It is important to note that this result does not depend on money illusion. Docherty (2006) explores the nature of unemployment equilibrium in a neoclassical model modified to incorporate the principle of effective demand and endogenous money, but with interest-sensitive expenditures. Determinacy of the unemployment result in that model requires money illusion on the part of workers which serves as the model’s nominal anchor. Without money illusion, the price level is indeterminate. The reason for this is that the equality of actual and expected inflation in the model’s Phillips curve equation requires a specific value for unemployment. This is, of course, the NAIRU and it implies a vertical long run Phillips curve. If unemployment is different from this level, prices must rise or fall continuously. For example, if unemployment is above the NAIRU, workers will reduce their wage demands and this will reduce the rate of price inflation. As actual inflation falls, expected inflation falls and this further reduces actual inflation. In more standard neoclassical models, this process is usually truncated by the presence of an exogenous money supply. If deflation reduces the demand for money, an exogenous and fixed money supply implies falling interest rates. Investment will then be stimulated and output will rise thus reducing unemployment and moving it closer to the NAIRU. The movement to full employment also ends the movement of prices when the money supply is exogenous and fixed, rendering the price level determinate.

When the money supply is endogenous this mechanism cannot operate. But the price level can be made determinate if the expected rate of inflation is removed from the Phillips curve equation. In this case no particular level of unemployment is associated with equality of actual and expected inflation and there is no NAIRU. Worker concern with the nominal value of wages may be influenced by the level of unemployment and this both locks down the system’s nominal dimensions and allows effective demand to determine unemployment (see Docherty 2006).
However in the KPSK model presented in this paper, there is no money illusion since workers target and pursue a real wage objective. What allows us to combine endogenous money with a lack of money illusion is the conflicting claims structure of the wage-inflation dynamics. These dynamics deliver the same theoretical result as money illusion but without the unsatisfactory nature of that assumption. Workers pursue claims for real wages but the market power of firms limits the success of these campaigns so that no particular level of unemployment is implied by the realisation of expectations. The long run Phillips curve is thus downward sloping rather than vertical even in the absence of money illusion.

Expressions for the short run and long run Phillips curves are shown in equations (28) and (29) respectively (using the original parameter specification in Table 1):

\[
\pi_t = \left[ \beta_1 - \lambda_{t-1} \right] + \beta_1 \pi_t^e - \beta_1 \theta_1 \cdot UE_{t-1} \tag{28}
\]

\[
\pi_t = \frac{1}{1 - \beta_1} \left[ (\epsilon \beta_1 - \lambda_{t-1}) - \beta_1 \theta_1 \cdot UE_{t-1} \right] \tag{29}
\]

The short run Phillips curve in (28) has the usual structure except that the coefficient on expected inflation takes a value different from unity. Thus changes in expected inflation can move the Phillips curve up and down in the usual way. But in the long run when expectations are realised, the curve takes the form shown in equation (29). If \( \beta_1 \) is equal to unity, the denominator in the first term on the RHS will equal zero and the rate of inflation will be undefined. This is the usual neoclassical case and in this case the KPSK model would have no nominal anchor. The bracketed term in (29) would be equal to zero and unemployment would have to be equal to the NAIRU for inflation expectations to be realised. However, when \( \beta_1 \) takes a value less than one, indicating a limit to the ability of workers to realise their real wage objectives, the long run Phillips curve has a negative slope. Thus unemployment determined by the principle of effective demand feeds into the determination of long run inflation via the extent to which workers pursue real wage growth inconsistent with that determined by monetary policy.

This is shown in Figure 1. The slope of the long run Phillips curve in equation (29) is given by \( \beta_1 \theta_1/(1 - \beta_1) \) which must be greater than the slope of the short run curve given by \( \beta_1 \theta_1 \) when \( \beta_1 < 1 \). If we assume that the long run curve is given by LRPC, that the short run curve is initially given by SRPC, and that unemployment is at an equilibrium value of \( UE \), current inflation must be \( \pi_t \) which is above its long run equilibrium value of \( \pi_2 \). This can only be because expected inflation is currently above actual inflation. This will cause a revision of expected inflation downwards by equation (14) above, which will shift the short run Phillips curve in Figure 1 downwards until it
Figure 1: Short Run and Long Run Phillips Curves in the KPSK Model

reaches SRPC₂. Given this short run curve, actual and expected inflation will be identical (at point A) with unemployment of $UE₁$. However a different level of unemployment, determined by a different level of effective demand, would generate a different long run equilibrium value for inflation along the long run Phillips curve.

Money is thus non-neutral in the KPSK model. The fixed interest rate associated with an endogenously determined money supply generates a distribution of income which in turn generates unemployment at a position of long run equilibrium. It is worth pointing out that while investment spending is interest-insensitive in the sense that cost of capital effects play no role in the formulation of firms’ plans for capital expenditure, changes in interest rates do have clear implications for spending. This is central to the nature of the model’s non-neutrality. The nominal dimensions of the model are rendered determinate by the nature of conflicting claims over income distribution and in particular by workers lack of complete control over nominal wage outcomes despite the absence of money illusion.

A second and related dimension of money’s role in the model may be seen by examining the system’s reaction to an exogenous increase in investment spending. Let us assume a permanent increase in the rate of depreciation from 15% to 16%. In order to consider the impact of this
shock to investment we assume the parameter values from Table 1 and a set of values for the system’s exogenous variables in Table 3. These assumptions may be used to calculate initial equilibrium values for the system’s endogenous variables which are also shown in Table 3. We may then shock the system with a 1% increase in the rate of depreciation thus increasing real investment and track the response of key variables. This investment shock simulation was structured so that the system operated at the original equilibrium from periods 1 to 20 before increased investment hits the system in period 21.

The response of key variables is presented in Figures 2 and 3. Panel A in Figure 2 shows the impact that the shock to the rate of depreciation has on the level of investment spending which rises by about 10% from its initial level. Panel B shows the consequential effect of this increase on the level of output which expands by about 3.33%. This increase in production in turn reduces the rate of unemployment as indicated in Panel C. The increased rate of depreciation also reduces the real wage via equation (7). Since more capital is used in the production process each period, less output is available to pay real wages and the real wage rate falls. This translates into reduced aggregate real wages via equation (3) and thus into a reduced wage share. The change in depreciation thus has two opposing influences on equilibrium income. According to the first,

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{C}_w$</td>
<td>1.5</td>
<td>$L^*$</td>
<td>16.00</td>
</tr>
<tr>
<td>$\bar{C}_c$</td>
<td>0.1</td>
<td>$\bar{i}$</td>
<td>0.02</td>
</tr>
<tr>
<td>G</td>
<td>20.00</td>
<td>$Y_t$</td>
<td>100.00</td>
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<tr>
<td>$C_t$</td>
<td>65.75</td>
<td>$\pi_t$</td>
<td>0.057</td>
</tr>
<tr>
<td>$I_t$</td>
<td>14.25</td>
<td>$\pi^*_t$</td>
<td>0.057</td>
</tr>
<tr>
<td>$W_t$</td>
<td>74.35</td>
<td>$MWG_t$</td>
<td>0.057</td>
</tr>
<tr>
<td>$P_t$</td>
<td>25.65</td>
<td>$r_t$</td>
<td>0.120</td>
</tr>
<tr>
<td>$UE_t$</td>
<td>0.05</td>
<td>$i_t$</td>
<td>0.077</td>
</tr>
<tr>
<td>$w_t$</td>
<td>4.891</td>
<td></td>
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Figure 2: The Response of Investment, Saving, Output, and Unemployment to a Permanent 1% Increase in the Rate of Depreciation
Figure 3: The Response of the Rate of Profit, Inflation and Interest Rates to a Permanent 1% Increase in the Rate of Depreciation
investment rises, stimulating aggregate demand and output. According to the second, income is redistributed from workers (with high propensities to consume) to capitalists (with lower propensities to consume), dampening aggregate demand and output. However, the expression for $\theta_2$ in equation (22) above indicates that the first of these effects will always dominate the second, causing the ultimate reduction in unemployment shown in Figure 2.

The second dimension of money’s role in the KPSK model may be seen in the context of the system’s response to this investment shock. The higher level of investment spending which ultimately increases equilibrium income should have monetary effects on the system. At the most basic level, greater income should cause increased real demand for money which should place pressure on real interest rates. In the KPSK model, this would alter income distribution back in favour of capitalists, reducing the impact of the investment expansion. But the model precludes this effect by equations (10) and (11). The fixed real interest rate implicitly assumes that any increase in real demand for money is accommodated by the central bank. Cottrell (1986) argues that there may also be potential financing problems associated with the investment expansion that could force real interest rates to increase. In Panel A of Figure 2, the dotted line shows the response of the sum of private and public saving to the investment shock. At any point in time after the shock but before the economy reaches the new equilibrium, investment runs ahead of saving and this deficit requires financing. In addition, if savers do not wish to hold the securities issued by firms attempting to finance the additional investment, further upward pressure on interest rates may result. Docherty (2005, 222-228) argues, however, that in either of these cases there will be portfolio reallocations in response to such higher interest rates which will eventually result in a greater volume on the market of what ever security the central bank uses to conduct monetary policy. This higher volume will reduce the price of this security and raise its interest rate unless the central bank purchases the unwanted volume, providing liquidity sufficient to finance the savings-investment gap until equilibrium is reached or portfolio preferences are satisfied. All of this is implied by equations (10) and (11) and constitutes the second role of money in the model.

The expansion does, however, have effects on nominal variables. The reduced level of unemployment increases the bargaining power of labour which pushes for higher real wages via equation (20). This feeds into a push for higher nominal wages and inflation via equations (13) and (12) respectively, and becomes self-reinforcing via the effect on expectations in equation (14). The higher rates of money wage growth and the resulting higher equilibrium level of inflation are shown in Figure 3. Higher inflation leads to higher nominal interest rates due to the monetary policy of stabilising the real interest rate. This is also shown in Figure 3.
A third perspective on the role of money in the model is obtained by considering the impact that a switch to a fixed exogenous money supply would have on the model’s key variables. Once again we assume that the model operates at the equilibrium specified in Table 3 for the first 20 periods. In period 21 the central bank switches from supplying money endogenously and attempts instead to enforce a strict money growth rule of 5.7% per period. Nominal interest rates are thus no longer given by equations (10) and (11) but by equation (30) below:

\[ i_t = i_{t-1} + \beta_3 \left( \frac{Y_{t-1} - Y_{t-2}}{Y_{t-2}} + \pi_{t-1} - \pi_t \right) \]  

(30)

According to this equation, interest rates adjust upwards from the previous period as the sum of real GDP growth and the rate of price inflation exceeds the exogenous rate of monetary growth (assumed here to be 5.7% per period). The inclusion of equation (30) transforms the reduced system in equations (A16) and (A17) in the Appendix into a highly complex, nonlinear system in unemployment and expected inflation. Simulations for this system indicate that it is highly unstable and very quickly generates large values for all variables. Variable time paths for these simulations have, therefore, not been reported except for the initial periods following period 21 for interest rates, unemployment and inflation. These are shown in Figure 4. Unemployment even slightly above the original equilibrium causes reductions in workers’ target real wage. This initially reduces money wages and inflation. This can be seen from Figure 4. Lower inflation causes two further effects: a fall in inflation expectations and a fall in nominal interest rates. The effect on real interest rates depends on which of these effects dominates. Panel A in Figure 4 indicates that real interest rates initially fall. Unemployment thus falls and generates a reverse set of effects. But the changes in real interest rates cause fluctuations in output which feedback on both the interest rate and unemployment so that soon large variations in nominal interest rates, real interest rates, inflation, unemployment and output interact to feed each other and cause explosive oscillations in the system. This occurs for a wide range of parameter values.

The KPSK system is thus highly unstable when endogenous money is replaced with exogenous money. Panel A in Figure 4 is curiously reminiscent of the increased volatility of interest rates during the Fed’s 1979-82 monetarist experiment (cf. Mishkin 2006, 381) and the significant increase in both the level and volatility of unemployment in Panel B of Figure 4 is not dissimilar to the pattern of unemployment in the years that followed this experiment. When the money supply is endogenous, the real rate of interest is stabilised and does not quickly react to variations in inflation or activity which such reactions feed in the exogenous money case.
Figure 4: Response of the Interest Rates, Inflation and Unemployment to a Switch from Endogenous to Exogenous Money
Endogenous money is thus an essential part of the KPSK system which is characterised by interest-insensitive investment expenditure and which generates unemployment equilibria despite an absence of money illusion. Endogenous money and the framework surrounding it performs the threefold function of providing interest rate stability, a mechanism by which monetary conditions have real effects and the financial means by which expenditures have their independence from saving in disequilibrium.

The following section explores the implications of how money functions in the KPSK model for the operation of monetary policy. In particular two related questions are considered: why would central banks maintain monetary conditions which generate unemployment? And: what impact would some form of Taylor rule have on an economic system of the KPSK variety since central banks use such rules rather than rules which permanently fix real interest rates?

4. Monetary Policy in the KPSK Model

Central banks do not, of course, run naïve rules which permanently fix the real rate of interest. Instead, they use monetary policy counter-cyclically in response to economic conditions as they stand in relation to specified target values. This section explores the operation of counter-cyclical monetary policy within the KPSK model using a form of Taylor rule. It also examines a second potential use of monetary policy within the KPSK model which is to change income distribution. We consider these issues in turn.

Many central banks follow an inflation targeting regime and set the nominal interest rate according to some variant of Taylor’s (1993) now famous rule. According to this rule, the nominal interest rate should be set equal to a base real rate of interest ($\tilde{i}$) plus lagged inflation plus an upwards adjustment for the extent to which actual lagged inflation exceeds target inflation plus an additional upwards adjustment for the degree to which lagged output exceeds potential output. The central bank in our model will follow a similar rule to this but with the output gap replaced by the deviation of the lagged unemployment rate from some target unemployment rate ($UE^*$) since unemployment is the variable of central interest in this paper rather than output. Equation (31) expresses this form of the Taylor rule and the approach will be to allow the economy to operate at the original equilibrium considered above for twenty periods with the central bank fixing the real rate of interest and then to have it switch to a Taylor rule in period 21 with a target inflation and unemployment both set at their equilibrium levels (5.7% for inflation and 5.0% for unemployment). Thus at period 21 we replace equation (11) in the original model above with equation (31). We will also consider a number of policy changes undertaken in period
Figure 5: Switch to Taylor Rule in Period 21 with Reduced Inflation Target in Period 101.
Figure 6: Switch to Taylor Rule in Period 21 with Reduced Unemployment Target in Period 101.
101. Both the inflation and unemployment targets will be increased and decreased by between 1% and 2% from their initial target values in four separate cases to be considered.

\[ i_t = \bar{i} + \pi_{t-1} + \beta_3 (\pi_{t-1} - \pi^*) - \beta_4 (UE_{t-1} - UE^*) \]  

(31)

Results for the initial switch to a Taylor Rule in period 21 and then a reduction in the inflation target from 5.7% to 4.0% in period 101 are shown in Figure 5. As one might expect, there is little change to the behaviour of any of the model’s variables at period 21 when the central bank switches to the Taylor rule. The equilibrium values of interest rates (both nominal and real) inflation, money wage growth, and the rate of unemployment all remain unchanged between periods 21 and 100. Since targets are met for both inflation and unemployment under the equilibrium of the baseline model, interest rates simply maintain their original values. But the behaviour of the system when target inflation is reduced is more interesting. Panel A shows that since inflation is now above target, the nominal interest rate is raised. This also causes an initial increase in the real interest rate, an increase in the real rate of profit and a reduction in the real wage with the impact being to reduce consumption spending and output and to increase unemployment. The increased unemployment rate implies that unemployment is now above target and this acts to dampen interest rates. The model is, however, very stable and converges to equilibrium relatively quickly. But notice that the equilibrium rates of inflation and unemployment, shown in Panels B and C, are not equal to their target values. Both rates are above target. The reason for this is straightforward. Since inflation above target mandates an interest rate decrease, while unemployment above target mandates an interest rate increase, there comes a point (depending on the relative values of the beta coefficients) where these mandates offset each other leaving the nominal interest rate unchanged. In the case where the inflation target is raised from its initial equilibrium value in the baseline case, just the opposite situation occurs with both variables below target. This case is not shown due to space considerations.

A third case, identical to the first, but where the unemployment target is reduced in period 101 from 5.0% to 4.0% is shown in Figure 6. Panel A indicates that the change in target generates an initial reduction in the interest rate. The lower nominal rate reduces both the real rates of interest and profit and increases the real wage. This redistributes income toward workers and increases aggregate consumption and output. Unemployment thus falls. But worker bargaining power is increased by this reduction in unemployment and both the rate of growth of money wages and the rate of inflation rise. In equilibrium, the same result obtains as for the reduction in the inflation target with both inflation and unemployment settling at values above target because of the
offsetting effects that these positions have on the policy rate. In the case where the unemployment target is raised from its initial equilibrium value in the baseline case, the opposite result to this third case obtains with both variables below target. This case is also not shown.

When monetary policy, therefore, follows some form of Taylor rule in the KPSK model, a stable equilibrium with persistent unemployment may be generated. If the economy begins from a position where inflation or unemployment are both above their target values, offsetting interest rate requirements mandated by the Taylor rule may leave interest rates at what is essentially their equilibrium values. The corresponding equilibrium values for inflation and unemployment would thus be different to their target values, unemployment being higher than desired.

A second possibility for monetary policy to be considered in this section is its use to change income distribution. This could be modeled in a number of ways but the approach taken here is to have the central bank permanently increase the fixed, target, real rate of interest from 2% to 3% in period 21. The reaction of the system to such an innovation depends crucially on the values of parameters $\beta_1$, the ability of workers to translate their inflation expectations into money wage growth, and $\beta_2$, the extent to which unemployment dampens the formulation of target real and, therefore, nominal wages. Two cases are, therefore, considered. In the first, $\beta_1$ and $\beta_2$ are set at relatively low levels (0.65 and 0.50 respectively) that reflect a relatively poor bargaining position of labour but at the same time a small effect of the impact of unemployment on the formulation of target real wages. The impact of the policy change under these circumstances is illustrated in Figures 7 and 8. Panel A in Figure 7 shows that the policy of raising the real rate is successful although the real rate initially overshoots its target before settling down to a new higher equilibrium level. In Panel B of Figure 7, this has the expected effect of increasing the real rate of profit by half a percentage point and reducing the real wage, shown in Panel C. The consequent change in income distribution at the aggregate level reduces consumption spending and thus output, and increases unemployment. This is shown in Panel A of Figure 8. Since unemployment is higher, the bargaining power of labour is reduced, money wage growth falls and inflation subsequently declines. This is shown in Panel B of Figure 8. Lower inflation implies lower nominal interest rates as shown in Panel C and hence the lower nominal rate of profit in Panel B of Figure 7 despite the higher real rate of profit.

In the second case $\beta_1$ and $\beta_2$ are set at much higher levels (0.90 and 0.80 respectively). These values reflect a more intensive conflict over income distribution where any change in inflation expectations, for example, feeds into higher demand for money wages more successfully, raising
Figure 7:  Response of Real Rate of Interest, Rates of Profit, and the Real Wage to a 1% increase in the Target Real Interest Rate. Low Values for $\beta_1$ and $\beta_2$. 
Figure 8: Response of the Nominal Interest Rate, Inflation and Unemployment, to a 1% increase in the Target Real Interest Rate. Low Values for $\beta_1$ and $\beta_2$. 
Figure 9: Response of the Nominal Interest Rate, Inflation and Unemployment to a 1% increase in the Target Real Interest Rate. High Values for $\beta_1$ and $\beta_2$. 
Figure 10: Response of the Nominal Interest Rate, Inflation and Unemployment to a 1% increase in the Target Real Interest Rate. High Values for $\beta_1$ and $\beta_2$ and Taylor Rule.
actual inflation and affecting the real rate of profit. This has real effects through its impact on income distribution and the consequent effect on unemployment feeds back on the formulation of money wages in such a way as to create significant cyclical variation. The impact of the policy change under these circumstances is illustrated in Figures 9 and 10. The increased real rate of interest increases the real profit rate, redistributes income from workers to entrepreneurs and reduces output. Unemployment rises and money wage growth and inflation fall. This fall happens slowly and as it does, nominal interest rates also fall. But the model is constrained by a zero nominal interest bound. Eventually nominal interest rates hit this bound and when this happens a crisis is precipitated. Further reductions in the inflation rate, now increase the real rate of interest, exacerbating the policy change. Deflation feeds itself via the adaptive expectations mechanism (as indicated in Panel C of Figure 9) causing an amplification of the real interest rate effect so that a massive redistribution of income reduces output and increases unemployment dramatically. These effects are shown in Panels A and B of Figure 9. The sign of key variables such as the real wage eventually change as the system explodes, which causes a reversal in the deflationary pattern and the system re-converges toward equilibrium. But the pattern may recur multiple times before stabilising (unless the equilibrium nominal interest rate is negative in which case the pattern will simply recur indefinitely). These results are consistent with the findings of Rowthorn (1977, 227) who finds dynamic instability when distributional conflict is intense depending on the initial position of the system.

A Taylor rule, however, prevents the emergence of the periodic financial crises represented in Figure 9 even with high values for $\beta_1$ and $\beta_2$. The operation of the rule adds an additional stabilising influence on interest rates when inflation or unemployment begin to respond to the strong dynamic forces of the system. Figure 10 depicts the system at its original equilibrium for periods 1 to 20 and then the switch in period 21 to a Taylor rule as before. In period 41, however, the base real rate, $\tilde{r}$, is increased from 2% to 3% with the high values for $\beta_1$ and $\beta_2$. In Panel A of Figure 10 the nominal rate is increased in line with an increase in the base real rate. This increased the real rate of profit, reduces the real wage and redistributes income in a way that reduces consumption and increases unemployment. This is shown in Panel B. This in turn causes the deflation that eventually led the nominal interest bound being hit. However, the model stabilises in this case.

Thus the effect of monetary policy on income distribution depends on the parameters of the conflicting claims part of the model. Where these parameters reflect a low level of conflict, monetary policy is able to engender permanent changes to income distribution with flow-on effects to output and employment. But where these parameters reflect a more intense level of
conflict, changes to monetary policy may generate large variations in nominal interest rates and inflation that are capable of causing significant economic fluctuations. These fluctuations are eventually self-correcting but counter cyclical use of a Taylor rule stabilises the interest rate variations which cause them so that the system itself is stabilised.

**Conclusion**

This paper has explored the behaviour of a model characterised by the Keynesian principle of effective demand adjusted to accommodate the influence of income distribution in the Kaldor-Pasinetti tradition, a Sraffian theory of value and distribution, interest-insensitive expenditures and endogenous money. It has shown that such a model is stable under reasonable assumptions about key parameter values and generates equilibria characterised by persistent unemployment. The model thus displays long run money non-neutrality. The paper has also shown that endogenous money is a necessary condition for stability in a Kaldor-Pasinetti-Sraffa-Keynes (KPSK) system and that the model cannot function with an exogenous money supply. It has also explored the impact of a Taylor rule within the KPSK framework. In contrast to the neoclassical model where a Taylor rule operates to deliver the appropriate money supply for a given inflation target, a Taylor rule in the KPSK model helps to explain why central banks might set interest rates at levels too high for full employment. Monetary policy is also capable of permanently changing income distribution when the intensity of conflicting claims within the model is low but generates significant economic fluctuations when this intensity is higher. These fluctuations are stabilised if counter cyclical use of a Taylor rule is superimposed on the original longer term objective of changing income distribution.
References


Appendix

This appendix derives the real wage equation used in the baseline Kaldor-Pasinetti-Sraffa-Keynes (KPSK) model and outlines the model’s solution and stability features.

Derivation of the Real Wage Equation

The real wage equation in the KPSK model is derived from a standard expression for the nominal rate of profit. This expression is given as follows:

\[ r_{n,t} = \frac{p_t Y_t - W_t L_t - \partial p_t K_{t-1}}{p_{t-1} K_{t-1}} \]  \hspace{1cm} (A1)

where all variables are defined in the text except \( p_t \) which represents the commodity price level and which may be related to the rate of inflation as follows: \( p_t = p_{t-1}(1 + \pi_t) \); and the capital stock which may be explained in terms of its lagged value, depreciation and investment as follows: \( K_t = K_{t-1} - \partial K_{t-1} + I_t = K_{t-1} \) after substitution of equation (3) in the model for investment. Rearranging the first of these expressions for the price level to obtain its lagged value and substituting both expressions into (A1) gives:

\[ r_{n,t} = \frac{(1 + \pi_t) \cdot Y_t}{K_t} - \frac{W_t (1 + \pi_t)}{P_t} \cdot \frac{L_t}{K_t} - \partial \cdot (1 + \pi_t) \]  \hspace{1cm} (A2)

Using definitions of the capital-output ratio (\( k_t \)), the labour-output ratio (\( \ell_t \)) and the real wage (\( w_t \)), (A2) becomes:

\[ r_{n,t} = \frac{(1 + \pi_t)}{k_t} - w_t (1 + \pi_t) \cdot \frac{\ell_t}{k_t} - \partial \cdot (1 + \pi_t) \]  \hspace{1cm} (A3)

If we define the real rate of profit, \( r_t \), as follows:

\[ r_t = \frac{r_{n,t}}{(1 + \pi_t)} \]  \hspace{1cm} (A4)

then (A3) becomes:

\[ r_t = \frac{1}{k_t} - w_t \cdot \frac{\ell_t}{k_t} - \partial \cdot \]  \hspace{1cm} (A5)
Rearranging (A5) to express the real wage in terms of the real rate of profit and the technical conditions of production gives equation (7) in the text:

\[ w_t = \frac{1}{\ell} \cdot [1 - k \cdot (r_t + \theta)] \] 

Equilibrium and Stability Features of the Baseline KPSK Model

The full KPSK system is represented by equations (1) to (14) in the text. This system may be reduced as follows. Lagging (2) and (5) by one period and substituting these into (1) gives:

\[ Y_t = [C_w + C_c + G] + (1 - s_w) \cdot W_{t-1} + (1 - s_c) \cdot P_{t-1} + \theta \cdot k \cdot Y_{t-2} \] 

(A6)

Grouping and designating the autonomous expenditures as \( \bar{A} \) and substituting equation (4) for aggregate profits gives:

\[ Y_t = \bar{A} + \theta \cdot k \cdot Y_{t-2} + (1 - h) \cdot (s_c - s_w) \cdot W_{t-1} + (1 - h) \cdot (1 - s_c) \cdot Y_{t-1} + \theta \cdot k \cdot Y_{t-2} \] 

(A7)

Now straightforward substitutions of (11) into (10), (10) into (9) and (9) into (8) gives the following simple expression for the real rate of profit:

\[ r_t = \tilde{\ell} + \sigma \] 

(A8)

Substituting this expression into the real wage equation (7), the resulting real wage equation into (3) and this into (A7) gives the following second order difference equation in output after further rearranging:

\[ Y_{t+2} - \theta_2 Y_{t+1} - \theta k Y_t = \bar{A} \] 

(A9)

where: \( \theta_2 = (1 - h) \cdot (1 - s_c) + (1 - h) \cdot (s_c - s_w) \cdot [1 - k \cdot (\tilde{\ell} + \sigma + \theta)] \)

This equation doesn’t contain the unemployment variable or any of the inflation terms in the model and may, therefore, be solved independently. Its intertemporal equilibrium is obtained in a straightforward manner assuming a constant value as follows: \( Y^{**} = Y_{t+2} = Y_{t+1} = Y_t \). Thus (A9) becomes:

\[ Y^{**} - \theta_2 Y^{**} - \theta k Y^{**} = \bar{A} \] 

(A10)
And

\[ Y^{**} = \frac{1}{1 - \theta_2 - \partial k} \cdot \vec{A} \]  \hspace{1cm} (A11)

The stability of this equilibrium depends on the behaviour of the reduced equation given by:

\[ Y_{t+2} - \theta_2 Y_{t+1} - \partial k Y_t = 0 \]  \hspace{1cm} (A12)

A solution of the form \( Y_t = Ab^t \) transforms (A12) into:

\[ Ab^{t+2} - \theta_2 Ab^{t+1} - \partial k Ab^t = 0 \]  \hspace{1cm} (A13)

which may be factorised as follows:

\[ Ab^t (b^2 - \theta_2 b - \partial k) = 0 \]  \hspace{1cm} (A14)

A non-trivial solution to (A13) requires that the bracketed term be zero and this condition constitutes the characteristic equation of (A9). There will be two values of \( b \) that satisfy this condition as follows:

\[ b_{1,2} = \frac{\frac{\theta_2 \pm \sqrt{\theta_2^2 + 4\partial k}}{2}} \hspace{1cm} (A15)\]

Tables A1 and A2 show the values of the first and second roots for specified parameter values and a range of values for the capital-output ratio and the depreciation rate. Table 2 in the text shows the combinations of the capital-output ratio and the depreciation rate for which the absolute value of both roots is less than one. The equilibrium value for output in Equation (A11) is stable for these combinations.

The KPSK system may be further reduced by rearranging equation (6) from the text to express output in terms of the rate of unemployment and substituting the resulting expression into (A9). This would give equation (A16) below. In addition to this, equation (13) may be substituted into equation (12) and the resulting expression into (14) to give (A17) below. These two expressions represent the reduced form KPSK model.
Ratios and Depreciation Rates with $s_c = 0.39; s_w = 0.06; i_{bar} = 0.02$

Table A2: Values of Second Characteristic Equation Root for a Range of Capital-Output Ratios and Depreciation Rates with $s_c = 0.39; s_w = 0.06; i_{bar} = 0.02; \sigma = 0.10$

<table>
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<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.75</th>
<th>0.90</th>
<th>1.00</th>
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</table>

Table A1: Values of First Characteristic Equation Root for a Range of Capital-Output Ratios and Depreciation Rates with $s_c = 0.39; s_w = 0.06; i_{bar} = 0.02; \sigma = 0.10$. 

$R_{k/d} = 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.40, 0.50, 0.75, 0.90, 1.00$
\[ U_{E_{t+2}} - \theta_2 U_{E_{t+1}} - \partial k U_{E_t} = (1 - \theta_2 - \partial k) - \frac{\ell}{L^*} \bar{A} \]  
\[ (1 - j \beta_1) \pi_{t+1}^e - (1 - j) \pi_t^e + j \beta_2 U_{E_{t+1}} = \alpha \]  
\[ (A16) \]

Once again equation (A16) does not depend on either inflation or expected inflation and so may be solved separately. The solution to (A17) may then be obtained by substituting the equilibrium value for unemployment and treating this as part of the constant on the RHS of the expression. Following a procedure similar to that followed above for equation (A9), delivers the following intertemporal equilibrium value for unemployment:

\[ UE^{**} = 1 - \frac{\ell}{L^*} \cdot \frac{1}{1 - \theta_2 - \partial k} \cdot \bar{A} \]  
\[ (A18) \]

Substituting this expression for unemployment into (A17) and following a similar procedure delivers the following expression for equilibrium expected inflation:

\[ \pi^{e**} = \frac{1}{1 - \beta_1} \left[ \alpha - \beta_2 + \frac{\ell \beta_2}{L^*} \cdot \frac{\bar{A}}{1 - \theta_2 - \partial k} \right] \]  
\[ (A19) \]

The stability condition for (A18) is identical to that for (A9) since the reduced equation for unemployment has the same form as that for output. The stability condition for (A19) depends on the following reduced equation for (A19):

\[ (1 - j \beta_1) \pi_{t+1}^e - (1 - j) \pi_t^e = 0 \]  
\[ (A20) \]

As with output above, a solution of the form \( \pi_t^e = Bb^i \) transforms (A20) and generates the following characteristic equation:

\[ (1 - j \beta_1) b - (1 - j) = 0 \]  
\[ (A21) \]

For which there is a single solution as follows:

\[ b = \frac{(1 - j)}{(1 - j \beta_1)} \]  
\[ (A22) \]

This root must be less than one in absolute value to ensure stability of the solution in (A19). For the values in Table 1 in the text the value for this root is 0.9589 which is less than one. The solution for inflation is, therefore, stable.