A Revised Exposition of the Methodology for Testing Payments System Risk

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Abstract
Financial economists and central bankers have been concerned for some time about the possibility of financial contagion spreading from bank to bank via interbank exposures within the payments system. The initial study of payments system risk was undertaken by Humphrey (1986) who found significant risk in the U.S. Fedwire system in the mid 1980s. Subsequent studies by Angelini, Maresca & Russo (1996), Kaussaari (1996), Northcott (2002), Furfine (2003) and Wang & Docherty (2006) have found, however, little evidence of systemic risk in the payments systems of Italy, Finland, Canada, Australia and in the U.S. inter-bank market. All of these studies employ a methodology in which the effects of a simulated failure at one institution on other institutions are examined and quantified but no formal statement of the simulation process is usually provided. One exception to this is the study by Wang & Docherty (2006) but it is possible to further refine and sharpen the exposition offered in that study. The objective of this short paper is simply to provide such an updated and refined exposition of the default simulation methodology used in payments system risk research.

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1. Introduction

The possibility of financial crises and the public policy objective of crisis prevention have underpinned a number of significant developments in financial systems around the world over the last twenty years. One development has been the introduction of real time gross settlement (RTGS) systems for the clearing of payments obligations between banks. The aim of this development has been to reduce the possibility of problems at one bank spreading to other banks via a series of defaults on payments system obligations (see, for example, Campbell 1998, 54). A number of papers examine the size and extent of this dimension of contagion risk across a number of countries and the present authors have investigated it in the context of the Australian payments system. All of these papers employ a methodology in which the effects of a simulated failure at one institution on other institutions are examined and quantified, however, while most outline the logic of the methodology, no formal statement of the simulation process is usually provided. One exception to this is offered in the study referred to above by the present authors (see Wang & Docherty 2006) but it is possible to further refine and sharpen the statement provided in that study. That is the simple objective of the present paper. Section 2 recaps some relevant aspects of the literature on testing payments system risk using simulated defaults, Section 3 presents the revised formal model of the simulation methodology and Section 4 offers some concluding remarks.

2. The Simulated Default Literature

Humphrey (1986) provides an early demonstration of payments system risk by simulating the failure of a single participant in the CHIPS system for a randomly selected business day in January 1983. All payments to and from this participant were unwound and new net multilateral settlement positions calculated for each participant in the system. These revisions had the potential to change the initial exposures of other institutions from net credit positions to net debit positions, with the possibility that liabilities associated with such position deteriorations might not be covered by the capital bases of affected

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1 Clearing House Inter-bank Payments System (CHIPS) is a privately operated U.S. dollar clearing system owned by banks located in New York City. CHIPS is more than 30 years old and operated as a DNS system until 2001 when it was upgraded to CHIPS Finality, a continuous netting settlement (CNS) system.
institutions. Humphrey (1986, 104) assumes that such additional failures ensue when two conditions are met:

- An institution’s revised multilateral net position to the system is negative; and
- the institution’s increased system obligation exceeds or equals its available capital.

Additional failures, of course, require further revisions to the multilateral positions of surviving institutions and the reapplication of the above tests. Humphrey repeats this process until all remaining participants are able to settle their net multilateral obligations.

He finds that 50 institutions failed in this simulated crisis, representing 38.6% of the total dollar value of daily payment instructions in the system. Six sets of failure iterations were required before no additional participant met Humphrey’s two knock-on failure criteria. If only 10% of bank capital is assumed available to absorb losses from position deteriorations (rather than 100%), the number of failures increases to 73 representing 76.1% of the total dollar value of daily payment instructions. Humphrey tests the robustness of this result by repeating the failure simulation of the same large institution on a different randomly selected day in January 1983, and by simulating the failure of a large associated participant for two randomly selected days in the same sample period.

He concluded that a significant level of systemic risk was present in the then DNS-based U.S. payments system (Humphrey 1986, 110). The precise level of risk as measured by the number of bank failures and the percentage of the value of daily payment instructions lost, varied from day to day, but the overall level was of the order of multiple institution failures and was well in excess of 30% of daily payment instructions.

Angelini, Maresca and Russo (1996) extend Humphrey’s methodology in the context of the Italian payments system. They distinguish between two types of default caused by external shocks and system-related shocks respectively, and simulate the failure of each institution in the Italian system on each of 21 business days during January 1992. They also examine the role of liquidity (rather than simply capital) in generating knock-on effects as in Humphrey’s approach. Angelini, Maresca and Russo’s find that only 4% of the 288 participants in the Italian system were capable of generating a systemic crisis when subject to an exogenously generated default. The extent of the crises simulated was also limited.
Northcott (2002) applies this methodology to Canada’s DNS system. Using bilateral settlement flows between clearing banks for 231 days between August 2000 and June 2001, she systematically tests the exogenous failure of each institution in the system on each of the days in the data set as do Angelini, Maresca and Russo. However, the framework developed by Northcott to model bilateral and multilateral payment flows between institutions allows for a range of assumptions on such variables as the extent of unwinding from the failed institution, the proportion of lost payments eventually recovered from the failed institution, the value of credit reversals by surviving institutions to customer accounts, and the size of capital and liquidity stocks available to absorb losses. She finds limited potential for contagion in the Canadian system.

Kuussaari (1996), Upper and Worms (2002) and Furfine (2003) apply similar methodology to the systems of Finland, and exposures in the German and U.S. interbank markets respectively, finding little evidence of systemic risk. McAndrews and Wasilyew (1995) use this methodology to identify factors which could significantly increase the level of systemic risk. They find that the number of banks, the size of payments and the probability of exchange payments between any two banks are all positively correlated with systemic risk. Wang and Docherty (2006) also find little evidence of risk in the Australian DNS system although their simulations employ synthetic data constructed from observed features of the actual Australian system.

A well defined methodology thus exists for testing the extent of systemic risk within DNS payments systems although none of the papers considered above, with the exception of Wang and Docherty (2006), provide a formal, detailed exposition of that methodology. This paper sharpens the exposition offered in Wang and Docherty (2006) by redefining end of day payments in terms of payments made progressively through the business day and by more clearly specifying the conditions under which simulation iterations cease. The following section sets out the revised exposition.

3. A Formal Exposition of Systemic Risk Methodology

The structure of the simulations described in this section most nearly parallels the approach of Northcott (2002) but is designed to handle the particular characteristics of data made available to the authors from the Reserve Bank of Australia for the Australian payments system where gross payment flows to and from each institution in the system
were available at 15 minute intervals throughout the day. We denote gross payments from institution \( i \) to institution \( j \) during such a 15 minute interval \( t \) as \( P_{ij,t} \) and gather gross flows between all institution pairs into the following payment flows matrix, \( M_t \):

\[
M_t = \begin{pmatrix}
P_{11,t} & P_{12,t} & P_{13,t} & \cdots & P_{1n,t} \\
P_{21,t} & P_{22,t} & P_{23,t} & \cdots & P_{2n,t} \\
P_{31,t} & P_{32,t} & P_{33,t} & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
P_{n1,t} & \cdots & \cdots & \cdots & P_{nn,t}
\end{pmatrix}
\]

(1)

where \( P_{11,t} = P_{22,t} = \cdots = P_{nn,t} = 0 \). Similar matrices for each of \( T \) such time intervals across the day may be summed to obtain an end of day gross payment flows matrix:

\[
MD = \sum_{t=1}^{T} M_t
\]

(2)

which we shall refer to as the daily payments position matrix. The total value of payments represented by this matrix may be obtained by summing the total value of payments at each 15 minute time interval given by:

\[
S_t = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ij,t}
\]

(3)

across the day to obtain:

\[
S = \sum_{t=1}^{T} S_t
\]

(4)

Gross payment flows from (1) may also be used to calculate bilateral and multilateral positions for individual institutions both for each time interval and for day’s end. Following Northcott (2002), we define the bilateral net position between banks \( i \) and \( j \), for time interval \( t \), \( BNP_{ij,t} \), as:

\[
BNP_{ij,t} = P_{ij,t} - P_{ji,t}
\]

(5)
where $BNP_{ij,t} > 0$ implies a net receipt by bank $i$ from bank $j$. The end of day bilateral net position between banks $i$ and $j$, $FBNP_{ij}$, is given by:

$$FBNP_{ij} = \sum_{t=1}^{T} BNP_{ij,t}$$  \hspace{1cm} (6)$$

And the end of day net multilateral exposure of bank $i$ to the entire system, $FMNP_i$, is given by:

$$FMNP_i = \sum_{j=1}^{n} FBNP_{ij}$$  \hspace{1cm} (7)$$

where $FMNP_i > 0$ implies a multilateral net receipt by bank $i$ from the system.

In the Australian DNS system, payment instructions are generated throughout the day and settled on the morning of the next business day on a net multilateral basis. In practice, each bank sums up its payments to and receipts from each of its counter-parties before submitting instructions to the Reserve Bank of Australia (RBA) at the end of the day. The RBA effectively computes a daily payments position matrix $MD$ from which the bilateral and multilateral positions can be deduced using versions of (6) and (7). Net payments are posted to exchange settlement accounts (ESAs) at 9.00 am on the next business day.

Following Humphrey (1986), Angelini, Maresca and Russo (1996), and Northcott (2002), an exogenous shock may be assumed to cause the failure of bank $d$ ($d \in n$) in the DNS system at the end of the business day. This bank, then, fails to settle its payments system obligations. Payments to and from the failed institution are unwound, and the bilateral and multilateral positions of surviving participants are revised. Deterioration in the payments positions of survivors may be sufficient to cause a second, third or subsequent round of failures, requiring further position revisions. An iterative process of failure, payment position revisions and further failure is set up by the initial failure, reflecting a domino effect through the system. The iterative process ceases when no additional institutions fail. This procedure may be described in terms of 9 steps.
Step 1
Let one institution be assumed to fail at the end of the day. This institution, designated $d$, will have the following failure status index at iteration 0 in the domino process described above and indicated by the second subscript in the following expression:

$$F_{d,0} = 1$$ (8)

A value of 1 for this index indicates that a bank fails and a value of 0 indicates that it does not fail.

Step 2
All payments to and from bank $d$ are unwound from the end of day calculations, and the bilateral and multilateral obligations of all surviving institutions are recalculated. The first step in this recalculation process is to generate what we call settlement adjustments for each surviving bank. The settlement adjustment for surviving bank $i$ in the first round of recalculations (iteration 1) is given in equation (A9) as the difference between gross payments to and from bank $d$:

$$SA_{i,1} = \sum_{t=1}^{T} (P_{i,d,t} - P_{d,i,t}) = -FBNP_{d}$$ (9)

where $SA_{i,1} > 0$ implies an improvement in the bilateral net position of the surviving bank $i$; and $SA_{i,1} < 0$ implies a deterioration. The adjusted end of day multilateral net position for bank $i$ in the system after round one of adjustments, $FMNP_{i,1}^*$, will then be:

$$FMNP_{i,1}^* = FMNP_i + SA_{i,1}$$ (10)

where $FMNP_{i,1}^* > FMNP_i$ implies an improvement in the multilateral net position of the surviving bank $i$; and $FMNP_{i,1}^* < FMNP_i$ implies a deterioration.

Step 3
For each survivor in the system, we next compute the credit exposure caused by the unwinding. We define the credit exposure of bank $i$ at iteration 1 as:
\[ CR_{i,1} = \frac{SA_{i,1}}{K_i} \] (11)

where \( K_i \) is the capital of bank \( i \).

**Step 4**

We also compute the liquidity exposure of each institution in the system associated with the unwinding. We define the liquidity exposure of bank \( i \) at iteration 1 as:

\[ LR_{i,1} = \frac{SA_{i,1}}{LA_i} \] (12)

where \( LA_i \) represents the liquid assets of bank \( i \). The liquidity exposure ratio reflects the bank’s ability to use its liquid assets to cover its payments position deterioration.

**Step 5**

The credit and liquidity exposures calculated in Steps 3 and 4 constitute two criteria that may be used to establish whether the unwinding of payments causes further bank failures in the system. Capital failure after the first round of recalculations is indicated by a value of one for a capital failure status index determined as follows:

\[ F_{Ci,1} = \begin{cases} 1 & \text{when } \begin{cases} FMNP_{i,1} < 0 \\ CR_{i,1} < -1 \end{cases} \\ 0 & \text{otherwise} \end{cases} \] (13)

The second failure criterion is satisfied after the first round of recalculations when a bank’s liquidity exposure is too large to be covered by its liquid assets. This is given as follows:

\[ F_{Li,1} = \begin{cases} 1 & \text{when } \begin{cases} FMNP_{i,1} < 0 \\ LR_{i,1} < -1 \end{cases} \end{cases} \] (14)
Credit failure and/or liquidity failure notwithstanding, a surviving bank only fails if its revised multilateral position is negative. Even where an institution’s net multilateral position is positive, it may struggle to avoid failure where significant net income from the payments system is lost. Humphrey (1986, 111), therefore, argues that the double criteria approach underestimates systemic risk because a bank with a final positive multilateral net position may still experience a reduction in its position too big to be absorbed by its capital or liquidity stocks. However, while Humphrey is correct, it seems better not to over-estimate risk so that the finding of significant systemic risk indicates an unambiguous need for a policy response.

Step 5 thus generates two sets of institutions that fail in the first iteration of position recalculations, $D_{C,1}$ and $D_{L,1}$, corresponding to the credit and liquidity criteria respectively, the first with $\lambda_{C,1}$ elements and the second with $\lambda_{L,1}$ elements:

$$F_{C_{k,1}} = 0 \quad \text{otherwise}$$

$$F_{L_{k,1}} = 0 \quad \text{otherwise}$$

These failures necessitate a further iteration of settlement adjustments, net multilateral position revisions and test applications and the process continues until there are no additional failures. For any iteration $z > 1$, settlement adjustments are calculated in a similar way to those for the first iteration but must be carried out for all institutions failing in the previous round. For the capital simulation these adjustments are given by expression (16), for the liquidity simulation they are given by expression (17). Revised multilateral positions revisions are calculated in the same way as for the first iteration as (18) indicates.
Credit and liquidity exposures for institution $i$ at iteration $z$ are calculated as:

$$ CR_{i,z} = \frac{SA_{i,z}}{K_i} $$

(19)

$$ LR_{i,z} = \frac{SA_{i,z}}{L_i} $$

(20)

and institution $i$ suffers a further knock-on credit or liquidity failure at this iteration according to:

$$ F_{C_{i,z}} = 1 $$

(21)

when \[ \begin{cases} FMNP_{i,z}^C < 0 \\ CR_{i,z} < -1 \end{cases} \]

and

$$ F_{L_{i,z}} = 1 $$

(22)

when \[ \begin{cases} FMNP_{i,z}^L < 0 \\ LR_{i,z} < -1 \end{cases} \]

Iteration $z$ thus generates two further sets of institutions that fail in that iteration, $D_{C,z}$ and $D_{L,z}$, corresponding to the credit and liquidity criteria respectively, the first with $\lambda_{C,z}$ elements and the second with $\lambda_{L,z}$ elements:

$$ F_{C_{k,z}} = 1 \quad \forall \ k \in D_{C,z} \{ 1, \ldots, \lambda_{C,z} \} $$

$$ F_{L_{k,z}} = 1 \quad \forall \ k \in D_{L,z} \{ 1, \ldots, \lambda_{L,z} \} $$

(23)
Step 7

Iterations cease for the credit simulation when $\lambda_{c,z} = 0$ and for the liquidity simulation when $\lambda_{l,z} = 0$. Let these conditions be met at iterations $g$ and $h$ respectively. The total number of credit failures will thus be given by the number of elements in the union of the credit failure sets for each iteration up to $g$ which we may call $D_C = D_{c,1} \cup D_{c,2} \cup \ldots \cup D_{c,g}$. The total number of liquidity failures is given by the number of elements in the union of the liquidity failure sets for each iteration up to $h$ which we may call $D_L = D_{l,1} \cup D_{l,2} \cup \ldots \cup D_{l,h}$. Computationally these sets may be determined by identifying institutions which have a value of one for the relevant failure status index at any stage across the iterative process once the process ceases. We denote indexes reflecting this failure status for bank $i$ across the whole process as $F_{C_i}$ and $F_{L_i}$ for the credit and liquidity criteria respectively which are given by:

$$F_{C_i} = \sum_{z=1}^{g} F_{C_{i,z}}$$  \hspace{1cm} (24) \\
$$F_{L_i} = \sum_{z=1}^{h} F_{L_{i,z}}$$  \hspace{1cm} (25)

There will be $n$ values of each of these indexes (one for each institution) which can be combined to form two $n$-dimensional vectors of zeros and ones which we denote as $F_C$ and $F_L$ for the capital and liquidity criteria respectively. The number of institutions which have a value of one for these indexes may be denoted as $m_C$ and $m_L$ respectively and these values are given by:

$$m_C = \sum_{z=1}^{g} \lambda_{C_{i,z}}$$  \hspace{1cm} (26) \\
$$m_L = \sum_{z=1}^{h} \lambda_{L_{i,z}}$$  \hspace{1cm} (27)
Step 8
Once the conditions identified in Step 7 for completion of the iterative procedure are met, and values computed for the $F_{ci}$’s and $F_{Li}$’s, $m_c$ and $m_L$, and the vectors $F_C$ and $F_L$ determined, the total number of institutional failures under the capital and liquidity criteria will be $(m_c+1)$ and $(m_L+1)$ respectively. The value of monetary losses due to the crisis can be calculated as:

$$L = A \cdot MD^t \cdot F - A \cdot MD \cdot F$$  \hspace{1cm} (28)$$

where $A_{von} = (1,1,\ldots,1)$, and $MD$ is the day’s payments position matrix as defined in (2) above.

Step 9
Steps 1 to 8 are repeated for each of the $n$ banks acting as the initially defaulting bank $d$. That is, $d$ progressively takes values $i = 1, 2, \ldots, n$.

4. Concluding Comments
Steps 1 to 9 outlined above entail $n \times q$ simulations for each of the credit failure and liquidity failure criteria producing $n \times q \times 2$ tests in all where $n$ is the number of institutions in system and $q$ is the number of days for which failures are simulated. The degree of contagion is measured for each simulation using:

- the value of monetary losses;
- the number of iterations required until no further knock-on effects are experienced;
- the default vector $F$ which defines the distribution of institution failures.

Humphrey (1986), Angelini, Maresca & Russo (1996), Kuussaari (1996), Northcott (2002) and Furfine (2003) all use some version of this methodology to evaluate the extent of payments system risk across a number of countries and it is hoped that this formal statement enhances the accessibility of other researchers to this literature.
References


