A Limited Stock Arbitrage of the Sydney Futures Exchange SPI Contract

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EXCHANGE'S SPI CONTRACT

Abstract

This paper examines the character of profitable arbitrage opportunities existing between the Australian share market and the Sydney Futures Exchange's share price index (SPI) contract. It first looks at the nature of the deviation of SPI futures prices from their theoretical values over the period 1988 to 1993. An adjustment cost model is offered as an explanation for these frequent deviations. The study details the construction of a feasible trading strategy designed to profit from SPI futures mispricing. To this end minimum variance portfolios are constructed from a limited number (ten) of Australian shares. A number of arbitrage strategies show a positive return after accounting for transaction costs. However the existence of profitable opportunities decreased markedly in the later years of the study. It is concluded that any inefficiencies, if they existed at all, were absent from 1992 onwards.

B.F.Hunt
5th September 1993
In theory there is a firm relationship between the spot or physical price of a share price index and the theoretical price of a futures contract written on that index. Specifically,

\[ F = S e^{(1 - d)t} \]  

(1)

Where,

- \( F \) is the theoretical futures price or futures fair value,
- \( S \) is the share price index level,
- \( i \) is the rate of interest, appropriate to an investment of length \( t \),
- \( d \) is the average dividend yield and
- \( t \) is the time to the maturity of the futures contract.

The theoretical underpinning of (1) is simple. If a person wishes to be long the index in \( t \) years time, they have a choice between buying the index now and bearing a cost of carry, \( (e^{(i - d)t} - 1) \), or buying a futures contract now and accepting delivery of the index later. In a world without transaction cost each choice ought to be equally expensive, hence equality (1).

It follows from the previous argument that any deviation in practice of the actual futures price, \( F \), from the theoretical price \( F_\star \), may indicate the presence of a profitable arbitrage opportunity.

This is certainly the case in a zero transaction cost world. At the maturity of a futures contract \( F \) must equal the spot price \( S \). Thus if there are no transaction costs and actual futures price, \( F \), is more than its theoretical value, \( F_\star \), then a strategy of selling a futures contract and buying the index will yield a riskless profit as the cost of buying and carrying the index will be less than the futures contract receipts at maturity. Similarly if \( F \) is less than \( F_\star \) a riskless profit is available to anyone who buys the futures contract and sells the index.

Notwithstanding the argument for a concurrence of the actual and theoretical futures price, research into share index futures markets has shown that from time to time there exists significant deviation between actual and theoretical prices. In this regard the Sydney Futures Exchange is no exception. For example a recent piece of work, Ip and Ng (1991), has provided evidence of frequent and significant deviation of actual Share Price Index futures (SPI) prices from their theoretical counterparts.

In a market without transaction costs any deviation from the theoretical price from the actual price of share price index futures is indicative of an unexploited profit opportunity and hence market inefficiency. However the same conclusion cannot necessarily be drawn from a market with significant transaction costs. If the cost of arbitrage transactions exceeds any profit from such transactions the theoretical and actual futures price deviation may persist even in an efficient market. Upon examination of the market structure, the existence of high transaction costs, as an explanation of the persistent deviation of SFE SPI futures prices away from their theoretical level, has some appeal.

The SPI futures contract is defined in terms of the Australian Stock Exchange's All Ordinaries Index, (AOI). The AOI is a broad based index covering more than 90% of stocks by value. More importantly, for potential arbitrageurs, the AOI covers more than
250 individual Australian stocks. While any SPI futures arbitrage involves the purchase or sale of a futures contract it also requires the sale or purchase of the index. In practice a transaction in the index is enacted by a transaction in each of the index stocks. For example for a person wishing to buy the index, as a complimentary transaction to an arbitrage sale of a SPI index, must simultaneously buy each of the stocks in the index in the proportion that those stocks are weighted in the index. The cost and impediments to purchase or sale of the AOI index are obvious. The high costs and various structural constraints dictate that strategy of buying or selling the AOI index is effectively infeasible.

Futures/physical arbitrage is undertaken through the formation of a hedge portfolio consisting of a position in futures and an opposite position in the physical asset. Unfortunately, a SFE share price index futures/physical arbitrage requires, for a perfect hedge, a portfolio consisting of a SPI futures contract and an opposite position in all of the numerous index stocks. As previously discussed, the construction of this hedge portfolio is effectively infeasible. At the heart of this paper is the construction of a feasible hedge portfolio from a much smaller subset of the AOI index stocks.

SFE, SPI Contract Specification

The SPI contract is one of the most successful contracts traded on the SFE. Table 1 below details the specifications of this cash settled contract.

<table>
<thead>
<tr>
<th>Table 1 ALL ORDINARIES SHARE PRICE INDEX FUTURES CONTRACT: SUMMARY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contract Unit</td>
</tr>
<tr>
<td>Cash Settlement Price</td>
</tr>
<tr>
<td>Mandatory Cash Settlement</td>
</tr>
<tr>
<td>Price Quotations</td>
</tr>
<tr>
<td>Contract Months</td>
</tr>
<tr>
<td>Termination of Trading</td>
</tr>
<tr>
<td>Settlement Day</td>
</tr>
</tbody>
</table>
Data

The data for this study was provided, on disk, by the SFE and the Australian Stock Exchange and Reuters. The data spans a 4 year period from 4th January 1988 to 26th February 1993. All prices are closing prices. While the physical All Ordinaries Index is continuous series, futures prices are discontinuous. For the purposes of this study we constructed a continuous near futures series SPI series. SPI prices were taken from the nearest to expiry futures contract up until there were less than 10 days to expiry. At this point the series was rolled into the next expiry contract. Estimates of dividend yield were obtained from the differences between the ASX’s All Ordinaries Accumulation index and the All Ordinaries Price index. Interest rates were approximated by rates on 90 bank accepted bills.

A plot of the AOI series over the research period is presented in Figure 1

![Figure 1: AOI Prices](image)

Relationship Between SPI Prices and Fair Futures Prices

Equation (1) was used to compute fair values for the futures index. In preliminary research we investigated the extent of deviation of the market SPI prices from their fair values. To facilitate this process we defined a percentage futures market premium over futures fair value, \( \pi \), as,

\[
\pi = \ln \left( \frac{F}{\bar{F}} \right)
\]

(2)

where,

- \( \pi \) is the percentage premium of the SPI price over fair value
- \( F \) is the SPI price and
- \( \bar{F} \) is the SPI fair value as implied from the AOI price, \( \bar{F} = S e^{(i - d) t} \).
Figure 2 Plots the futures premium, $\pi$ over the period

![Figure 2: SPI premium over Fair Value](image)

(04.01.1988 to 26.02.93)

One can see from Figure 2 the considerable deviation of SPI prices from fair value. Further insight into the extent and nature of this deviation can be obtained from Figure 3 which documents the distribution of the premium, $\pi$.

![Figure 3: Distribution of the SPI premium over Fair Value](image)

Figure 3 shows that negative premia are more numerous and more persistent than positive premia. This evidence supports the proposition that the sell shares/buy futures arbitrage is more difficult to enact than the buy shares/sell futures arbitrage.
Overlaid on the distribution of futures price premia is a plot of the probability of a next day premium reversal. This empirical distribution shows the proportion of times that say a positive premium of between 2% and 2.5% gets smaller, rather than larger, next day. The U shaped reversal curve clearly indicates a tendency for mispriced futures and physical prices to correct themselves. There are many candidate specifications for modelling financial time series. The observed tendency for mispriced futures prices to correct themselves suggests that some kind of mean reversion model is appropriate.

We propose a simple model of differential transaction costs to explain the existences of periods where futures price deviate from their fair values. This model also describes the dynamics by which futures prices return to the fair value. The model assumes that a trader transacts in two markets - the futures and the physical market. The trader faces the problem of minimising two types of cost; the cost of disequilibrium and transaction costs. Disequilibrium costs arise when prices in either market differ from the traders perceived correct market price. The existence of unexploited profit opportunities provides a rationale for these disequilibrium costs. It is assumed that the costs of disequilibrium are the same in both markets. In contrast, it is assumed that while transaction costs exist in both market they are greater for physical transactions than they are for futures transactions. Further it is assumed that all costs are quadratic functions.

The cost function for the trader can be expressed as,

$$ C = c_d (F - \bar{S} e^x)^2 + c_d (S - \bar{S})^2 + c_f (F - F_{-1})^2 + c_s (S - S_{-1})^2 $$

(3)

Where

- $C$ is the total cost,
- $\bar{S}$ is a stochastic variable representing the "correct" or equilibrium physical price,
- $F^*$ is the "correct" futures price, $F = \bar{S} e^x$,
- $S$ is the physical price,
- $F$ is the futures price,
- $c_d$ is the cost of disequilibrium, $c_d > 0,$
- $c_f, c_s$ are the adjustment costs in the futures and the physical market
- $x$ is the cost of carry, $x = (1 - d) t$

The traders task is to set $F$ and $S$ to minimise total costs. Solving this problem yields

$$ F = \left( \frac{c_d e^x}{c_d + c_f} \right) \bar{S} + \left( \frac{c_f}{c_d + c_f} \right) F_{-1} $$

(4)

$$ S = \left( \frac{c_d}{c_d + c_s} \right) \bar{S} + \left( \frac{c_s}{c_d + c_s} \right) S_{-1} $$

(5)

Multiplying (5) by $e^x$ and subtracting it from (4) produces
\[ F - \bar{F} = c_d \left( \frac{1}{c_d + c_f} - \frac{1}{c_d + c_s} \right) F^* - \frac{c_f}{c_d + c_f} F_{-1} + \frac{c_s}{c_d + c_s} \bar{F}_{-1} \]  

(6)

Where,

- \( F^* \) is the "correct" futures price, \( F = S^* e^X \)
- \( \bar{F} \) is the fair futures price implied by \( S \), i.e. \( \bar{F} = S e^X \)

It can be seen from (6) that as long as \( c_s > c_f \) then any change in \( S^* \), the equilibrium physical price, will result in a gap appearing between the futures price and the fair futures price. After the adjustment dynamics have settled down in the long run \( \bar{F} = F = F^* \) and thus \( F - \bar{F} = 0 \).

Unfortunately \( F^* \) is not observable so that any estimates of (6) will contain all of the attendant problems of omitted variables.

Table 2 contains regression results arising from an estimation of (6)

<table>
<thead>
<tr>
<th>Table 2: Regression Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable ( F - \bar{F} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constant</th>
<th>( F_{-1} )</th>
<th>( \bar{F}_{-1} )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.77126</td>
<td>0.812995</td>
<td>-0.814922</td>
<td>0.665946</td>
</tr>
<tr>
<td>(9.75)</td>
<td>(50.74)</td>
<td>(-49.76)</td>
<td></td>
</tr>
<tr>
<td>0.812322</td>
<td>-0.813314</td>
<td>(51.35)</td>
<td>0.665907</td>
</tr>
<tr>
<td>(51.02)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( t \) values in parenthesis

Feasible Arbitrage Profits

In theory any deviation of the SPI futures away from its fair value indicates the presence of an unexploited arbitrage opportunity. However, in practice the multiplicity of stocks in the AOI index makes it impractical to buy or sell the physical index as part of an arbitrage strategy. This paper investigates the use of a feasible hedge portfolio in a futures/spot share price index arbitrage. The feasible hedge strategy employs a ten stock portfolio as a proxy for the physical index. The ten stocks were selected on the basis of market value. Figure 4 shows the cumulative contribution of each stock to the AOI index and to the correlation with the index.
The specific steps taken in this research to examine the feasibility and profitability of employing a futures/spot share price index arbitrage strategy are,

1. the identification of historical periods where, absolute value of the futures premium, $\pi$ exceeds $x\%$

2. the opening of an arbitrage position. Specifically, if,

   \[
   \begin{align*}
   \pi &> x & \text{sell 1 SPI contract, "buy" stocks} \\
   \pi &< x & \text{buy 1 SPI contract, "sell" stocks}
   \end{align*}
   \]

3. the holding of any previously formed arbitrage position until such time as $\pi$ is less than $y\%$. At this time the arbitrage position is closed and profit on the arbitrage transaction is computed.
The feasible stock portfolio was constructed from a set of 10 liquid Australian stocks, upon which short-selling is permitted. The hedge portfolio was constructed with reference to the physical index. That is the weighting of each of the 10 stocks in the stock hedge portfolio was chosen to minimize the variance of the returns, over the previous 60 working days, of a portfolio containing each of the ten stocks and one unit of the index. Appendix A sets out the mathematics behind the creation of a short sales allowed hedge portfolio. As the minimum variance portfolio weights were constructed with reference to the physical index, it was necessary to adjust them for use in a futures arbitrage portfolio. To adjust for the difference in volatility between the future and the physical prices, the hedge portfolio weights were multiplied by the cost of carry factor, $e^r$.

Central to our research is the construction of hedge portfolios which are then tested for their ability to generate arbitrage profits. This test is in effect a test of market efficiency as the existence of consistent arbitrage profits is incompatible with an hypothesis of market efficiency. Of course the calculation of arbitrage profit depends crucially on assumptions as to the size of transaction costs. After consultation with a leading share and futures broker it was assumed that a futures transactions entails taking the wrong side of the futures bid/offer spread plus a fixed fee of $11.50. Further it was assumed that stock spread and commission cost were 0.40%.

Results

Although it is impractical to implement an arbitrage strategy involving the physical AOI index nevertheless we considered it a worthwhile exercise to compute the trading profit and loss which would have arisen had this strategy been feasible. The reason being that the results of this infeasible arbitrage strategy provide a benchmark against which we can measure the results of the implementation of the feasible stock arbitrage strategy. The arbitrage results are set out in Table 3.
<table>
<thead>
<tr>
<th>x%</th>
<th>y%</th>
<th>No of Transactions</th>
<th>Gross Profit</th>
<th>Net profit</th>
<th>Average Profit</th>
<th>Standard Deviation of Profit</th>
<th>Average duration of trade (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>462</td>
<td>772,465</td>
<td>201,969</td>
<td>437</td>
<td>1,640</td>
<td>25</td>
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<tr>
<td>2</td>
<td>0</td>
<td>167</td>
<td>384,230</td>
<td>179,063</td>
<td>1,072</td>
<td>1,780</td>
<td>33</td>
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<tr>
<td>2</td>
<td>1</td>
<td>167</td>
<td>425,717</td>
<td>219,951</td>
<td>1,317</td>
<td>1,238</td>
<td>14</td>
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<tr>
<td>3</td>
<td>0</td>
<td>46</td>
<td>169,975</td>
<td>115,453</td>
<td>2,510</td>
<td>1,779</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>46</td>
<td>170,409</td>
<td>115,930</td>
<td>2,520</td>
<td>1,362</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>46</td>
<td>110,272</td>
<td>56,073</td>
<td>1,219</td>
<td>1,150</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x%</th>
<th>y%</th>
<th>No of Transactions</th>
<th>Gross Profit</th>
<th>Net profit</th>
<th>Average Profit</th>
<th>Standard Deviation of Profit</th>
<th>Average duration of trade (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>462</td>
<td>400,414</td>
<td>(170,083)</td>
<td>(368)</td>
<td>2,148</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>167</td>
<td>215,054</td>
<td>9,887</td>
<td>59</td>
<td>2,093</td>
<td>33</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>167</td>
<td>339,971</td>
<td>134,205</td>
<td>804</td>
<td>1,486</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>46</td>
<td>115,824</td>
<td>61,302</td>
<td>1,333</td>
<td>1,959</td>
<td>33</td>
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<tr>
<td>3</td>
<td>1</td>
<td>46</td>
<td>148,673</td>
<td>94,195</td>
<td>2,048</td>
<td>1,377</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>46</td>
<td>79,547</td>
<td>25,348</td>
<td>251</td>
<td>1,605</td>
<td>8</td>
</tr>
</tbody>
</table>

(1) is the upper, open arbitrage threshold. (2) is the lower, close arbitrage threshold. (4) is the gross trading profit before transaction costs. (5) is profit after transaction costs. (6) is the average profit per arbitrage trade. (7) is the standard deviation of net profit per trade. (8) is the average duration of the arbitrage trade in days.

Conclusions

Many of the combinations of x and y produce profitable results over the period of investigation. Taken at face value these trading results would indicate a degree of inefficiency in the Australian futures and share markets. We would however make two points in respect of this conclusion. First, the trading profits are not generated without risk. Many of the arbitrage transactions result in a trading loss, even for those x and y which produce large overall profits. This can be seen from Figure 6 which details individual trade results for x=2%, y=1%.
The second point to be made about market efficiency is that there is less evidence of futures/physical mispricing, and hence profitable opportunities, in the later years of the study period than in the early years. The results would suggest that market inefficiency, if it existed, decreased over the period of the study. Indeed an argument can be made that the two markets are efficient from 1992 onwards.
Appendix A

The general problem of the minimisation of portfolio variance can be expressed as,

minimise $\sigma_p^2 = w' \Omega w$ subject to a number of constraints,

$B w = b$

Where

- $\sigma_p^2$ is portfolio variance
- $w$ is a $(n \times 1)$ vector of asset weights,
- $\Omega$ is a $(n \times n)$ covariance matrix of returns,
- $B$ is a $(p \times n)$ matrix of $p$ constraints and
- $b$ is a $(p \times 1)$ vector of constraints constants

In a hedge portfolio it is usual to have at least two constraints, namely

1. that portfolio expected return be $k$ percent and that
2. that one asset, that being "hedged", have unit weight

Incorporating these two constraints gives a definite structure to $B$ and $b$, i.e.,

$$B = \begin{pmatrix} r_1 & r_2 & \ldots & r_n \\ 1 & 1 & \ldots & 1 \end{pmatrix}$$

and

$$b = \begin{pmatrix} k \\ 1 \end{pmatrix}$$

The optimisation problem can be given a Lagrangian representation

$$L = 0.5 w' \Omega w + \lambda' (B w - b)$$

differentiating we produce,

$$\frac{\delta L}{\delta w} = \Omega w + b' \lambda = 0 \quad (1a)$$

$$\frac{\delta L}{\delta \lambda} = B w - b = 0 \quad (1b)$$

The set of equations (1) can, by partitioned inversion, be solved for a minimum variance set of weights, $w_k$, (see Goldberger (1964) p48),

$$w_k = \Omega^{-1} B' (B \Omega^{-1} B')^{-1} b$$

The expected return on $w^*$ is of course $k$. The variance of $w^*$ is given by,

$$\sigma_p^2 = b' (B \Omega^{-1} B')^{-1} b$$
Dropping the expected return constraint produces the more familiar absolute minimum variance hedge portfolio.

\[ w_{\min} = \Omega^{-1} \begin{bmatrix} 1 \end{bmatrix} / (s_1 \Omega^{-1} s_1^T)^{-1} \]
\[ = \Omega^{-1} s_1^T / \sigma_h^2 \]

where \( s_1 \) is a zero vector except with 1 in the first row and

\( \sigma_h^2 \) is the variance of the hedged asset