Earnings, Dividends and Returns: A Theoretical Model

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A THEORETICAL MODEL

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ABSTRACT

In this paper we develop a theoretical model to explain raw returns and abnormal returns. We show that returns can be explained in terms of levels and changes in earnings and dividends. This contrasts with extant empirical work which typically uses only a subset of these variables. The model also explains cross-sectional variation in coefficients on earnings variables in terms of company specific characteristics such as the firm's target payout ratio.

The model provides a theoretical explanation for the 'anomalous' results of Easton and Harris (1991) who demonstrate empirically that both earnings levels and changes jointly explain returns. Additionally, we suggest that single equation studies are likely to be of limited value in empirically investigating the relation between earnings and returns, particularly where such equations are estimated in cross section, and the coefficients are constrained to be constant over all firms.
1. INTRODUCTION

The relation between accounting earnings and returns has received considerable empirical attention in the accounting literature, but theoretical work has been limited. In the finance literature there has been extensive empirical investigation of the relation between dividends and returns. In this paper we provide a theoretical model which captures the effect on returns of both earnings and dividends. The model is derived using the discounted dividend equation for price, and empirically validated equations for the earnings process and the dividend process.

In their pioneering earnings return study, Ball and Brown (1968) demonstrated the positive correlation between earnings increases (decreases) and positive (negative) market-adjusted returns over the year preceding the announcement of a change in company earnings. They concluded that earnings contain information which is relevant to the formation of stock prices, although much of this information is incorporated in prices prior to the release of the earnings report. Since Ball and Brown there have been numerous empirical studies which have examined the relation between earnings and returns. The empirical work commonly uses the change in earnings (a proxy for unexpected earnings) as an explanator for some measure of abnormal returns, although raw returns have been used as the dependent variable in some cases. Lev (1989) critically reviews these empirical models and expresses concern about their low explanatory power. Although the earnings change variable has been found to be statistically significant, it is significant in explaining a relatively small percentage (typically less than 10%) of the total cross-sectional variation in returns. Reasons offered for the low explanatory power include the effect of cross-sectional differences across firms in any one year for which a model is estimated, measurement error associated with the earnings variable, difficulty in the choice of an optimal length of time for the window through which the relation between earnings and returns are viewed, and possible misspecification of the model.

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The misspecification issue is particularly significant, since there has been relatively little attempt to develop theoretical models that underpin the empirical work. The empirical research largely relies on the intuition that if earnings have information content relevant to prices, then unexpected changes in earnings should have power in explaining price revisions occurring around, or between, earnings announcement dates. In criticising this approach, Ohlson and Shroff (1992) suggest that it is overly heuristic, firstly in presuming that returns are a function of unexpected earnings, and secondly in the common empirical practice of presuming that unexpected earnings can be approximated by the earnings change. Ohlson (1991) provides a definition of earnings, and an earnings return model, which suggests that the appropriate explanator for returns is a current earnings level variable scaled through division by beginning of period price, rather than a similarly scaled earnings change variable. However, in Ohlson and Shroff (1992) it is argued that either earnings levels, or earnings changes, scaled by beginning period price, can be used as a measure of unexpected earnings. They further suggest that although either variable can be used to explain returns, the earnings level variable will be the better choice if it has the lower variance.

A recent empirical study by Easton and Harris (1991) provides support for the view that indeed either earnings levels, or earnings changes, can be used in explaining returns. However, their results also indicate that more cross-sectional variation in returns is explained when both the earnings level and earnings change variables are included in the regression model. Other empirical work has considered dividend signalling and the joint information impact of dividend changes and earnings changes. This work suggests that dividend and earnings changes jointly explain more variation in returns around earnings and dividend announcement dates than either variable alone, (see Griffin (1976), Brown, Finn and Hancock (1977), Aharony and Swary (1980)).

In this paper we develop a theoretical model which is consistent with the above empirical results¹. Using the return identity and well accepted models for dividends and earnings,

¹ Note, however, that the theoretical model in this paper is concerned with a longer return window than that typically employed in dividend signalling studies.
we show that returns, and abnormal returns, may be expressed as functions of levels and changes in earnings, jointly with levels and changes in dividends. An important feature of the model is that the earnings concept employed is accounting earnings as reported by the firm. This is significant because extant empirical work largely makes use of reported earnings, while theoretical work such as that of Ohlson (1991), uses a definition of earnings which is not necessarily coincident with reported earnings. Indeed, an attractive feature of the model is that it can be expressed in a form where all the variables are observable.

The plan of the paper is as follows. In the next section we define the components of total return after which, in section three, we determine the relationship between returns, dividends and earnings. Finally, in section four we explore some implications that our model has for empirical research and we sound a note of caution about the conduct and interpretation of such work.

2. THE COMPONENTS OF TOTAL RETURN

The Return Identity
Our model relies on the following identity which exists between total return, capital gain, and dividend yield

\[ R_t = \frac{\Delta P_t}{P_{t-1}} + \frac{D_t}{P_{t-1}} \]  

...(2.1)

where

- \( P_t \) = price at time \( t \)
- \( \Delta P_t \) = change in price measured at time \( t \) (ie. \( P_t - P_{t-1} \))
- \( D_t \) = dividend per share received at time \( t \)
- \( R_t \) = realised return measured at time \( t \)

\(^2\) All variables used in this equation and in the subsequent analysis are firm specific, but for notational convenience the subscript \( i \), representing the \( i \)th firm has been omitted from all equations.
In the subsequent analysis we assume that the starting point for measuring returns is the ex-dividend price in the prior period. We also assume that shares go ex-dividend and the dividend is paid at the same time as earnings and dividends are announced. Thus the ex-dividend price movement and dividend and earnings announcement effects occur simultaneously. Of course, in practice these events involve a time lag, and therefore additional information can have an impact on prices in the period between the dividend and earnings announcements and the ex-dividend date. However, this does not affect the main theoretical point of our paper, that returns can be explained by a function which contains both levels and changes in earnings and dividends.¹

**Dividends**

Dividends are assumed to be generated by the Lintner (1956) model. The Lintner model was chosen because it has support from grounded theory, and in empirical testing has remained dominant for over three decades. It has proved to be a robust model using accounting earnings to explain dividends at the micro level, with quarterly and annual data, and across countries, it has also been successful in explaining dividends at the macro level. (see Lintner (1956), Fama and Babiak (1968), Laub (1972), de Donker (1972), Chateau (1979), Shevlin (1982)).

In the Lintner model, management link dividends to earnings via a target payout ratio. However, they do not fully adjust dividends to the target payout level as earnings change, instead the adjustment is gradual. This is because management are strongly averse to cutting dividends, and prefer if possible to gradually increase them over time. Evidence in support of managements’ aversion to dividend cuts and desire to increase dividends over

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¹ The lag however is important in empirical work. For example, consider that the starting point for measuring the return is either the cum-div price at the prior earnings announcement, or the subsequent ex-div price, and that the end point for measuring the return is either the cum-div price at the current earnings announcement, or the subsequent ex-div price. In this case there are four possible return measurements depending upon the particular combination of cum-div and ex-div prices chosen. Each has slightly different components of dividends within the return intervals, and being measured over different intervals the returns are subject to potentially different information sets. The precise specification of the earnings return relationship therefore depends upon which of the return measures is to be explained.
time is provided by Smith (1971) and De-Angelo and De-Angelo (1990).\(^4\)

In the Lintner dividend generating process, represented by equation (2.2) below, we incorporate an additive, zero mean error term. The error term represents a disturbance, or shock, to the dividend generating process. For example, a special dividend may be declared,\(^5\) management may temporarily decide on a speed of adjustment which differs from normal, or there may be a constraint on current dividends such as a restrictive debt covenant. The dividend shock may contain significant information. If, for example, the dividend adjustment is slower than usual this may reflect management concern about sustaining the current level of reported earnings.

The Lintner model is

\[ D_t = a + cTY_t + (1-c)D_{t-1} + e_t \]  \hspace{1cm} \text{(2.2)}

where

- \( a \) = a constant \( (a \geq 0) \)
- \( c \) = a speed of adjustment coefficient
- \( T \) = the target payout ratio
- \( Y_t \) = earnings per share at time \( t \)
- \( e_t \) = a zero mean error term

Assuming that investors' dividend expectations follow the dividend generating process, then taking expectations of both sides of equation (2.2) results in

\[ E_{t-k}[D_t] = a + cT E_{t-k}[Y_t] + (1-c) E_{t-k}[D_{t-1}] \]  \hspace{1cm} \text{(2.3)}

\(^4\) While managers are averse to dividend cuts, the model does not preclude a temporary state where firms have an actual dividend payout of zero. However, with the exception of growth stocks, it is doubtful that firms would have a target payout of zero. In the case of growth stocks, the target payout may initially be set to zero, but it is expected to change later. We do not model such cases, but they could be accommodated by an extension of the growth version of the model (see Section 3) in order to allow for expected periodic change in the target payout ratio.

\(^5\) Brickley (1983) provides empirical evidence that special dividends are associated with a persistent increase in dividends and stock prices. However, the impact of the special dividend is less than the impact of an increase in the regular dividend. We do not attempt to model this differential price response.
where \( E_{t-k}[D_t] \) = the dividend expectation for time \( t \) made at time \( (t-k) \)

\( E_{t-k}[Y_t] \) = the earnings expectation for time \( t \) made at time \( (t-k) \).

For \( k = 1 \), \( E_{t-1}[D_{t-1}] = D_{t-1} \)

This follows because the expected dividend for time \( (t-1) \), with expectations formed at time \( (t-1) \), is simply the observed dividend at that time. Equation (2.3) can be therefore rewritten as:

\[
E_{t-1}[D_t] = a + cT E_{t-1}[Y_t] + (1-c)D_{t-1} \quad \cdots(2.4)
\]

**Earnings**

We make no explicit assumption about the earnings generating process, but we do assume that earnings expectations are formed by using the random walk model. The earnings process is a process with many degrees of freedom and such processes often appear to follow a random walk. There is also a long pedigree of empirical evidence starting with Little (1962) and continuing through Ball and Watts (1972), Watts and Leftwich (1977) and Whittred (1978), which suggests that annual earnings are well approximated by a random walk. Indeed, the assumption that the expectations model for earnings is a random walk underlies the common empirical practice, in earnings return research, of using earnings changes as a measure of unexpected earnings.

The random walk is written as:

\[
Y_t = Y_{t-1} + \varepsilon_t \quad \cdots(2.5)
\]

Where \( \varepsilon_t \) = a zero mean random error term

which implies
\[ E_{t-1}[Y_t] = Y_{t-1} \] ...(2.6)

From (2.5) we can write

\[ Y_{t+1} = Y_t + \epsilon_{t+1} = Y_{t-1} + \epsilon_{t+1} + \epsilon_t \]

and thus

\[ E_{t-1}[Y_{t+1}] = Y_{t-1} \]

Thus under the assumption that earnings expectations are based on a naive random walk

\[ E_{t-1}[Y_{t+j}] = Y_{t-1}, \; j \geq 0 \] ...(2.7)

Equations (2.1) to (2.7) provide the required components to express returns as a function of dividends and earnings.

**Choice of Models**

The choice of the Lintner model for dividends and the random walk for earnings was motivated by the extensive empirical work which suggests that these models provide reasonable descriptions of observed behaviour. However, we have deliberately structured our analysis so that the reader can substitute alternative processes and trace their effects. We have found the results presented in this paper to be quite robust, but not invariant to alternative linear processes for earnings and dividends. For example, if the dividend process and dividend expectations are assumed to follow a random walk then returns can

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6 While these models have strong empirical support they do not apply to all firms. For example, the Lintner model will provide poor dividend forecasts for firms with constant dividends. While the random walk model provides inappropriate long term earnings forecasts for firms with current negative earnings, but which are expected to survive.
be expressed solely as a function of levels and changes in dividends.\textsuperscript{7}

Although the random walk in earnings has empirical support, it does pose some problems as an expectations model. With a random walk model, profit expectations are independent of the level of profit retention and investment. The implication is that investors assess the potential future earnings from profit retention at zero, until those earnings actually start to be realised.\textsuperscript{8} Given firms with a target payout of less than 100\%, and a positive return on investment on average, it seems reasonable that investors would expect some earnings growth in the longer term.\textsuperscript{9} Indeed, the empirical results of Albrecht, Lookabill and McKeown (1977), suggest that growth in the investment base does lead to growth in earnings such that the earnings time series is best characterised as a random walk with drift. In a later section therefore, we briefly consider the impact of including earnings growth in the earnings expectations equation.

3. **RETURNS EARNINGS AND DIVIDENDS**

**Capital Gain**

The dividend valuation model can be written as:

\[ P_t = \sum_{j=1}^{\infty} \frac{E_t[D_{t+j}]}{(1+r)^j} \]  \hspace{2cm} \text{...(3.1)}

where: \( r \) = the required rate of return for the stock

\textsuperscript{7} A random walk in dividend expectations may not be an extreme assumption. Empirical evidence, Fama and Babiak (1968) and Shevlin (1982), suggests that a naive no change forecast for dividends performs little worse than forecasts using the Lintner model.

\textsuperscript{8} Management can signal increased future cash flows, and by implication increased earnings, by increasing current dividends. However, increasing both current profit retention and current dividends leads to a requirement for external financing. In turn the external financing has signalling consequences which may be detrimental to value maximisation (see Myers and Majluf, 1984).

\textsuperscript{9} Ohlson (1991) addresses this problem by assuming that retained profits are reinvested at the required rate of return. The problem with this approach is the implicit assumption that all investments have zero net present value. Additionally, when the return on investment equals the required return, valuation can be carried out as though the firm has a 100\% payout policy. Dividend policy and dividend signals are therefore largely irrelevant in such a setting. See Brealey and Myers (1984, pp 53-54) for a simple explanation of this approach to valuation.
Equation (3.1) can be derived by re-arrangement of equation (2.1), taking expectations, and applying recursive substitution for dividends. Thus there is consistency between the return identity and the valuation equation. Consistent with our return measurement assumption in section 2, equation (3.1) prices stocks ex-dividend. For simplicity we have followed a common practice of assuming that the required rate of return \( r \) is constant for all periods. Thus the term structure of interest rates is assumed to be flat and required returns are assumed not to change over time.\(^{10}\)

The dividend expectation term included in the summation sign on the right hand side of equation (3.1) can be rewritten using equation (2.3) as follows:

\[
E_t [D_{t,j}] = a + cT E_t [Y_{t,j}] + (1-c) E_t [D_{t,j-1}]
\]

Using (2.7), to substitute for expected earnings gives,

\[
E_t [D_{t,j}] = a + cTY_t + (1-c)E_t [D_{t,j-1}]
\]  
...(3.2)

In Appendix 1 we show that (3.2) can be rewritten as

\[
E_t [D_{t,j}] = \left[ \frac{1-(1-c)Y_t}{c} \right] (a + cTY_t) + (1-c)Y_t D_t
\]  
...(3.3)

Substituting (3.3) into the dividend valuation model, (3.1), results in

\[
P_t = \sum_{j=1}^{\infty} \frac{\left[ \frac{1-(1-c)Y_t}{c} \right] (a + cTY_t) + (1-c)Y_t D_t}{(1+r)^j}
\]  
...(3.4)

\(^{10}\) Relaxing these assumptions could allow other information variables into the model. Note that \( r \) may vary across firms.
In Appendix 2, by starting with (3.4) above, we determine the following expression for the difference in price in any one time period.

\[ P_t - P_{t-1} = \frac{cT(r+1)}{r(r+c)} \Delta Y_t + \frac{(1-c)}{(r+c)} \Delta D_t \]  ... (3.5)

where \( \Delta Y_t = Y_t - Y_{t-1} \)
= change in earnings measured at time \( t \)

and \( \Delta D_t = D_t - D_{t-1} \)
= dividend change at time \( t \).

After dividing (3.5) by \( P_{t-1} \), we derive (3.6) below which implies that the capital gains component of the ex-dividend return depends on earnings changes and dividend changes\(^{11}\). The dependency is linear and additive.

\[ \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{cT(r+1)}{r(r+c)} \frac{\Delta Y_t}{P_{t-1}} + \frac{(1-c)}{(r+c)} \frac{\Delta D_t}{P_{t-1}} \]  \( \ldots (3.6) \)

**Dividend Yield**

Dividend yield is given by \( \frac{D_t}{P_{t-1}} \). Using (2.2) we can express dividend yield as follows:

\[ \frac{D_t}{P_{t-1}} = a \frac{Y_t}{P_{t-1}} + cT \frac{\Delta Y_t}{P_{t-1}} + (1-c) \frac{\Delta D_{t-1}}{P_{t-1}} + \frac{e_t}{P_{t-1}} \]  \( \ldots (3.7) \)

This implies that the dividend yield component of the ex-dividend return can be viewed as a function of levels in earnings and lagged dividends.

Substituting (3.6) and (3.7) into the return identity (2.1) gives rise to the following:

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\(^{11}\) Note, however, that via the Lintner model, the change in dividends (\( \Delta D \)) in (3.6) could be re-expressed in terms of levels of earnings and lagged dividends.
Given (i) the dividend generating function (2.2) and the dividend expectations, (2.3), (ii) the earnings expectations, (2.7), and (iii) a constant discount rate, then realised returns as defined in equation (2.1) are related to earnings and dividends by the following linear, additive specification.

\[ R_t = \alpha_0 \frac{1}{P_{t-1}} + \alpha_1 \frac{\Delta Y_t}{P_{t-1}} + \alpha_2 \frac{Y_t}{P_{t-1}} + \alpha_3 \frac{\Delta D_t}{P_{t-1}} + \alpha_4 \frac{D_{t-1}}{P_{t-1}} + \frac{e_t}{P_{t-1}} \]  \quad (3.8)

where

\[ \alpha_0 = a \]
\[ \alpha_1 = \frac{cT(r+1)}{r(r+c)} \]
\[ \alpha_2 = cT \]
\[ \alpha_3 = \frac{(1-c)}{(r+c)} \]
\[ \alpha_4 = (1-c) \]

Clearly, in this model returns are a function of earnings changes, earnings levels, dividend changes, the beginning of period dividend, a constant component of dividends, and a shock component of current dividends, all deflated by the beginning of period price. In interpreting equation (3.8) it is important to recognise that \( e_t \) is not the error term for this equation. It is a variable which represents the dividend shock, and is derived from equation (2.2). Note, also, that \( e_t \) is the only variable in equation (3.8) that cannot be directly observed. However, it can be estimated, and as shown in equation (4.1) below it is possible to transform equation (3.8) to a form which consists only of observable variables.

From the Lintner model, (equation (2.2)), the term \( a \) was a non-negative constant. After division by \( P_{t-1} \), \( \frac{a}{P_{t-1}} \) is no longer constant over time, but can be viewed as a component of dividend yield which is always non-negative. A positive value for \( a \) is consistent with De Angelo and De Angelo (1990) who show that management are very
reluctant to omit dividends even in the face of a sustained decline in earnings. However, this does not imply that all firms will have a positive dividend yield at all times. For example, the positive effect of the constant, $a$, may eventually be offset by a large and continuing decline in earnings $Y_n$ or a negative term for the dividend shock, $e$, while for some firms $a$ might be set to zero.

In equation (3.8) returns are a function of dividend policy variables. This result appears inconsistent with Miller and Modigliani’s (1961) dividend irrelevance proposition. However, as suggested by Miller and Modigliani, the dividend irrelevance proposition does not hold in the presence of dividend signalling. With dividend signalling, expectations of future cash flows depend upon observed dividends. In our model the choice of dividend policy parameters and the announcement of current dividends affects investors’ expectation of future cash flows. This result arises directly from the dividend expectation equation (2.4).

**Abnormal Returns**

Much of the work which has examined the relationship between returns and earnings and/or dividends has focussed on abnormal returns rather than raw returns. In order to examine abnormal returns a measure of expected returns is required. This is simply obtained by taking expectations, at time $t-1$ of equation (3.8). As shown in Appendix 3, the relationship for abnormal returns can then be derived from equation (3.8) and the expectations equation. The result is given in equation (3.9) below.

$$ R_t - E_{t-1}(R_t) = \frac{cT(r+1)^2}{r(r+c)} \frac{\Delta Y}{P_{t-1}} + \left(\frac{r+1}{r+c}\right) \frac{e_t}{P_{t-1}} $$

...(3.9)

This equation is consistent with the intuition that abnormal returns should be a function of unexpected changes in dividends and earnings. Given the random walk for earnings expectations then the change in earnings, $\Delta Y_n$, is the unexpected component in earnings. Given the Lintner model for dividend expectations, the shock in dividends, $e_n$, is the difference between the actual dividend and the expectation of dividends conditional on current earnings.
However, the shock on dividends can be re-expressed, using equation (2.2), in terms of the level of current and lagged dividends, a constant, and the level of earnings. Using equation (2.2) to substitute for \( e_t \) in equation (3.9), (see Appendix 4), gives:

\[
R_t - E_t(R_t) = \beta_o \frac{1}{P_{t-1}} + \beta_1 \frac{\Delta Y_t}{P_{t-1}} + \beta_2 \frac{Y_t}{P_{t-1}} + \beta_3 \frac{\Delta D_t}{P_{t-1}} + \beta_4 \frac{D_{t-1}}{P_{t-1}} \quad \ldots (3.10)
\]

where:

\[
\beta_o = \frac{-a(r+1)}{r+c}
\]

\[
\beta_1 = \frac{cT(r+1)^2}{r(r+c)}
\]

\[
\beta_2 = \frac{-cT(r+1)}{r+c}
\]

\[
\beta_3 = \frac{r+1}{r+c}
\]

\[
\beta_4 = \frac{c(r+1)}{r+c}
\]

Thus abnormal returns can also be explained in terms of the observable variables: levels and changes in earnings and dividends.

**Growth**

In section two we introduced the possibility of including earnings growth in the earnings expectation model. This is achieved by replacing the earnings expectation process equation (2.5) with equation (3.11) below, where a compound growth process is assumed and the expected earnings growth rate \( g \) is constant for all \( t \).
\[ Y_{t+1} = (1+g)Y_t + \epsilon_{t+1} \]  ...(3.11)

As shown in Appendix 5 the resulting returns equation is:

\[ R_t = \gamma_0 \frac{1}{P_{t-1}} + \gamma_1 \frac{\Delta Y_t}{P_{t-1}} + \gamma_2 \frac{Y_t}{P_{t-1}} + \gamma_3 \frac{\Delta D_t}{P_{t-1}} + \gamma_4 \frac{D_{t-1}}{P_{t-1}} + \frac{\epsilon_t}{P_{t-1}} \]  ...(3.12)

where:

\[ \gamma_0 = a \]

\[ \gamma_1 = \frac{c}{(r-g)} \frac{T(1+g)(1+r)}{(r+g)(r+c)} \]

\[ \gamma_2 = cT \]

\[ \gamma_3 = \frac{(1-c)}{(r+c)} \]

\[ \gamma_4 = 1-c \]

In comparison with the random walk case (see equation (3.8)), the only difference is the co-efficient on the earnings change variable (\( \gamma_1 \)).

4. **IMPLICATIONS OF THE MODEL**

As mentioned in the introduction, earnings levels and changes have been viewed as competing variables in explaining returns. Thus, surprise might be expressed about the empirical results of Easton and Harris (1991) which showed that levels and changes in earnings were both statistically significant in an earnings return regression. Perhaps more surprising was their result that both levels and changes in earnings were statistically significant when regressed against a measure of abnormal returns. However, given
equation (3.8) or equation (3.12), these are exactly the results to be expected\textsuperscript{12}.

Studies of the joint impact of dividend and earnings announcements, such as Brown, Finn and Hancock (1977), find that earnings and dividend changes jointly appear to contain more price relevant information than either variable alone. Again this is consistent with equation (3.8) which suggests that dividends and earnings can provide complementary signals.

In general, any subset of the variables drawn from the right hand side of equation (3.8) that are not perfectly collinear, would be expected to produce significant co-efficients when regressed against returns. Equation (3.8) therefore has the potential to encompass a large volume of the empirical work that examines the relation between earnings and/or dividends and returns. It also suggests a wider set of variables should be considered than has traditionally been the case. These include not only levels and changes in earnings and dividends but also the inverse price term \( \left( \frac{1}{P_t} \right) \) and the dividend shock term \( \left( \frac{e_t}{P_t} \right) \).

However, the issue of perfect collinearity must be considered as the change in dividends, \( \Delta D_t \), in equation (3.8) is, via the Lintner model, perfectly collinear with earnings levels, \( Y_t \), lagged dividends, \( D_{t-1} \), and the shock on dividends, \( e_t \). While this does not affect the theoretical validity of equation (3.8), it does pose a problem for direct empirical estimation.

We show in Appendix 6 that with some manipulation equation (3.8) can be rewritten in a form which is not perfectly collinear. The resulting equation is:

\[
R_t = \frac{(r+1)}{(r+c)} \frac{\Delta D_t}{P_{t-1}} + \frac{cT(r+1)}{r(r+c)} \frac{\Delta Y_t}{P_{t-1}} + \frac{D_{t-1}}{P_{t-1}} \quad \ldots (4.1)
\]

\textsuperscript{12} In this section of the paper subsequent comments that relate to equation (3.8) can also be applied to equation (3.12).
Under the assumption of compound growth in earnings given by equation (3.11), and using the same approach as in Appendix 6, equation (3.12) can be rewritten as:

\[ R_t = \frac{(r+1)\Delta D_t}{(r+c)P_{t-1}} + \frac{cT(1+g)(1+r)}{(r-g)(r+c)} \frac{\Delta Y_t}{P_{t-1}} + \frac{D_{t-1}}{P_{t-1}} \quad \cdots (4.2) \]

The collinearity problem is resolved by eliminating the level of earnings and the dividend shock term from the equations. However, they are still implicit in the equations since earnings and the dividend shock are determinants of \( \Delta D_t \).

Alternatively, as shown in Appendix 7, recursive application of the dividend generating process (2.2) to the return equation (3.8) allows the replacement of the dividend terms \( \Delta D_t \) and \( D_{t+1} \), by current and lagged levels of earnings. The resulting equation is entirely a function of earnings variables and dividend shocks but is not very convenient for empirical work.\(^{13}\)

Equation (3.8) provides a convenient expression which could be useful in the analysis of empirical work. For example, it suggests that the co-efficients on the earnings variables are likely to vary in cross section. This adds support to Easton and Zmijewski's (1989) empirical result, that "earnings response coefficients" do show systematic variation in cross-section, and also adds weight to their warning about the mis-specification inherent in cross-sectional studies which constrain the coefficients on the earnings variables to be constant across companies.\(^{14}\) In our model, coefficients on earnings variables are expected to vary across companies according to company specific characteristics, including the target payout ratio, the speed of adjustment co-efficient, the growth rate, and the discount rate. In addition our analysis suggests that the co-efficients differ for earnings levels and earnings changes, and that these co-efficients may vary depending on the form

\(^{13}\) Conversely, equation (2.2) can be rearranged to express earnings as a function of dividend variables, and thus the earnings terms in (3.8) could be replaced with current and lagged dividend variables.

\(^{14}\) Note that Easton and Zmijewski (1989) use a shorter window in their empirical work than is implicit in our model. Cross sectional variation, however, is likely to be a feature common to both long and short windows.
of the model estimated. In empirical work care must also be taken to guard against specification bias due to omitted variables.

An attractive feature of modelling based on (3.8) is that the specification of realised, expected, and abnormal return, relations can be in terms of observable company specific financial variables. Potential uses of (3.8) extend beyond the earnings dividend return relation, to areas such as dividend policy and to asset pricing. For example, tests of asset pricing models usually use realised returns as a proxy for expected returns. A model of realised returns, such as (3.8), may be helpful both in interpreting the empirical results of such studies and in designing improved experiments. However, the earnings, dividends, and return relation are not uniquely defined by a single equation. First, as shown above, equation (3.8) can be restated in several forms. Second, it is conceivable that the same equation might be derived from a different set of assumed processes. The implication for future empirical work in the area of earnings, dividends, returns, seems to be that, rather than estimating single equation models, more attention should be directed to the estimation of simultaneous equation systems.

CONCLUSION

The theoretical model developed in this paper shows that both changes and levels of dividends and earnings can jointly explain realised returns and also abnormal returns. This theoretical result provides an explanation for the 'anomalous' empirical result of Easton and Harris (1991), where both earnings levels and earnings changes were found to explain equity returns. Realised returns may also be expressed in other ways. For example, in terms of lagged earnings and dividend shocks, or changes in earnings and levels and changes in dividends. Thus, rather than a unique single equation specification for realised returns, there are several equivalent single equation specifications.
Without a theoretical basis, the chance of misspecifying an empirical earnings return model by, for example, omission of explanatory variables, seems considerable. Equally, there seem to be strong prospects of a chance hit on a statistically significant result, since the variables used in empirical work are likely to either be drawn from, or correlate with, subsets of the theoretical explanatory variables identified in Equation (3.8).

From an empirical perspective the advantage of the theoretical model developed in this paper is that can be expressed in a form where all the independent variables are directly measurable. They are all observable company specific variables. The co-efficients of the model are also company specific depending upon factors such as the company’s target payout ratio. Because of this, model coefficients will vary in cross section. As a consequence, cross-sectional estimation equations that constrain the co-efficients to be the same across all firms are likely to be fundamentally mis-specified.

The lack of a unique single equation specification for realised returns, and the possibility that a single equation formulation might be derived from a potentially large number of alternative assumed processes, poses a problem for empirical testing using single equation methodology. A joint test of the return equation and the dividend and earnings processes, that comprise the model, would seem to require simultaneous equation estimation.
REFERENCES:


APPENDIX 1  EXPECTED DIVIDENDS

\[ E_t(D_{t+j}) = a + cT Y_t + (1-c)E_t(D_{t+j-1}) \]

\[ = a + cT Y_t + (1-c) \left\{ a + cT E_t(Y_{t+j-1}) + (1-c) E_t(D_{t+j-2}) \right\} \]

\[ = [1+(1-c)a + [1+(1-c)cT Y_t + (1-c)^2 E_t(D_{t+j-2}) \]

\[ = [1+(1-c)+(1-c)^3] [a+cT Y_t] + (1-c)^3 E_t(D_{t+j-3}) \]

\[ = [1 + (1-c) +...+(1-c)^{j-1}] [a+cT Y_t] + (1-c)^j D_t \]

\[ = \left[ \frac{1-(1-c)^j}{c} \right] [a+cT Y_t] + (1-c)^j D_t \]

APPENDIX 2  PRICES, CAPITAL GAIN AND EXPECTED DIVIDENDS

\[ P_t = \sum_{j=1}^{\infty} \frac{E_t(D_{t+j})}{(1+r)^j} \]

\[ = \sum_{j=1}^{\infty} \left[ \frac{1-(1-c)^j}{c} \right] [a+cT Y_t] + (1-c)^j D_t \]

\[ = \sum_{j=1}^{\infty} \left[ \frac{1-(1-c)^j}{c} \right] [a+cT Y_{t-1}] + (1-c)^j D_{t-1} \]

and \[ P_{t-1} = \sum_{j=1}^{\infty} \frac{E_{t-1}(D_{t+j-1})}{(1+r)^j} \]

\[ = \sum_{j=1}^{\infty} \left[ \frac{1-(1-c)^j}{c} \right] [a+cT Y_{t-1}] + (1-c)^j D_{t-1} \]

then \[ P_t - P_{t-1} = \sum_{j=1}^{\infty} \frac{1-(1-c)^j}{(1+r)^j} cT(Y_t-Y_{t-1}) + (1-c)^j D_t - D_{t-1} \]

\[ = \sum_{j=1}^{\infty} \frac{1-(1-c)^j}{(1+r)^j} T(Y_t-Y_{t-1}) + \sum_{j=1}^{\infty} \left( \frac{1-c}{1+r} \right)^j (D_t-D_{t-1}) \]
\[ \sum_{j=1}^{\infty} \left( \frac{1-c}{1+r} \right)^j = \left( \frac{1-c}{1+r} \right) \left[ 1 - \frac{(1-c)}{1+r} \right] = \frac{1-c}{r+c} \]

and \[ \sum_{j=1}^{\infty} \frac{1-(1-c)^j}{(1+r)^j} = \sum_{j=1}^{\infty} \frac{1}{(1+r)^j} - \sum_{j=1}^{\infty} \frac{(1-c)^j}{(1+r)^j} = \frac{1}{r} - \frac{(1-c)}{(r+c)} \]

\[ \therefore P_t - P_{t-1} = \left[ \frac{1}{r} - \frac{(1-c)}{(r+c)} \right] T(Y_t - Y_{t-1}) + \left( \frac{1-c}{r+c} \right) (D_t - D_{t-1}) \]

\[ P_t - P_{t-1} = \frac{c(r+1)}{r(r+c)} T \Delta Y_t + \left( \frac{1-c}{r+c} \right) \Delta D_t \]

and \[ \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{cT(r+1)}{r(r+c)} \frac{\Delta Y_t}{P_{t-1}} + \left( \frac{1-c}{r+c} \right) \frac{\Delta D_t}{P_{t-1}} \]

⇒ capital gains depend on earnings changes plus dividend changes.
**APPENDIX 3**  

**ABNORMAL RETURNS**

Abnormal Return  
= Realised Return - Expected Return  
= \( R_t - E_{t-1}(R_t) \)

Recall  
\[
R_t = \alpha_o \frac{1}{P_t} + \alpha_1 \frac{\Delta Y_t}{P_t} + \alpha_2 \frac{Y_t}{P_t} + \alpha_3 \frac{\Delta D_t}{P_t} + \alpha_4 \frac{D_t}{P_t} + \frac{e_t}{P_t}
\]

So  
\[
E_{t-1}(R_t) = \alpha_o \frac{1}{P_t} + \alpha_1 \frac{E_{t-1}(Y_t) - Y_t}{P_t} + \alpha_2 \frac{E_{t-1}(Y_t)}{P_t} + \alpha_3 \frac{E_{t-1}(D_t) - D_t}{P_t} + \alpha_4 \frac{D_t}{P_t}
\]

Then  
\[
R_t - E_{t-1}(R_t) = \alpha_1 \frac{Y_t - E_{t-1}(Y_t)}{P_t} + \alpha_2 \frac{Y_t - E_{t-1}(Y_t)}{P_t} + \alpha_3 \frac{D_t - E_{t-1}(D_t)}{P_t} + \frac{e_t}{P_t}
\]

Since \( E_{t-1}(Y_t) = Y_{t-1} \), then  
\[
R_t - E_{t-1}(R_t) = (\alpha_1 + \alpha_2) \frac{\Delta Y_t}{P_t} + \alpha_3 \frac{D_t - E_{t-1}(D_t)}{P_t} + \frac{e_t}{P_t}
\]

Now  
\[
\alpha_3[D_t - E_{t-1}(D_t)] = \frac{\alpha_3 \frac{cTY_t}{(1-c)D_t - e_t - a - cTY_{t-1} - (1-c)D_{t-1}}}{P_t}
\]

where \( \alpha_2 = cT \)

It follows that  
\[
R_t - E_{t-1}(R_t) = (\alpha_1 + \alpha_2 + \alpha_2 \alpha_3) \frac{\Delta Y_t}{P_t} + \frac{e_t}{P_t}
\]

where  
\[
\alpha_1 + \alpha_2 + \alpha_2 \alpha_3 = \frac{cT(r+1)}{r(r+c)} + cT + \frac{cT(1-c)}{(r+c)}
\]

\[
= \frac{cT(r+1)^2}{r(r+c)}
\]

\[
1 + \alpha_3 = 1 + \frac{(1-c)}{(r+c)}
\]

\[
= \frac{(r+1)}{(r+c)}
\]
Recall equation (3.9)

\[ R_t - E_{t-1}(R_t) = \frac{cT(r+1)^2}{r(r+c)} \frac{\Delta Y_t}{P_{t-1}} + \left( \frac{r+1}{r+c} \right) \frac{e_t}{P_{t-1}} \]

From equation (2.2) we have:

\[
e_t = D_t - a - cT Y_t - (1-c)D_{t-1} = -a - cT Y_t + \Delta D_t + cD_{t-1}
\]

Substituting the above expression for \( e_t \) into equation (3.9) results in

\[
R_t - E_{t-1}(R_t) = \left( \frac{r+1}{r+c} \right) \frac{a}{P_{t-1}} + \frac{cT(r+1)^2}{r(r+c)} \frac{\Delta Y_t}{P_{t-1}} - \frac{cT(r+1)}{(r+c)} \frac{Y_t}{P_{t-1}} + \left( \frac{r+1}{r+c} \right) \frac{\Delta D_t}{P_{t-1}} + \frac{c}{(r+c)} \frac{D_{t-1}}{P_{t-1}}
\]

\[
= \beta_o \frac{1}{P_{t-1}} + \beta_1 \frac{\Delta Y_t}{P_{t-1}} + \beta_2 \frac{Y_t}{P_{t-1}} + \beta_3 \frac{\Delta D_t}{P_{t-1}} + \beta_4 \frac{D_{t-1}}{P_{t-1}}
\]

where

\[
\beta_o = -a \left( \frac{r+1}{r+c} \right)
\]

\[
\beta_1 = \frac{cT(r+1)^2}{r(r+c)}
\]

\[
\beta_2 = -\frac{cT(r+1)}{(r+c)}
\]

\[
\beta_3 = \left( \frac{r+1}{r+c} \right)
\]

\[
\beta_4 = c \left( \frac{r+1}{r+c} \right)
\]
APPENDIX 5  RETURNS AND EARNINGS GROWTH

Consider an earnings generating process with constant growth, \( g \), such that

\[
Y_{t+1} = Y_t + gY_t + \epsilon_{t+1} = (1+g)Y_t + \epsilon_{t+1}
\]

Then

\[
Y_{t+j} = (1+g)^jY_t + \sum_{i=1}^{j} (1+g)^{j-i} \epsilon_{t+i},
\]

and earnings expectations are given by:

\[
E_t(Y_{t+j}) = (1+g)^jE_t(Y_t) = (1+g)^jY_t
\]

From APPENDIX 1 dividend expectations using the Lintner Model are given by:

\[
E_t(D_{t+j}) = a + cTE_t(Y_{t+j}) + (1-c)E_t(D_{t+j-1})
\]

Now

\[
E_t(D_{t+j-1}) = a + cTE_t(Y_{t+j-1}) + (1-c)E_t(D_{t+j-2})
\]

\[
\therefore E_t(D_{t+j}) = a + cTE_t(Y_{t+j}) + (1-c)[a+cTE_t(Y_{t+j-1}) + (1-c)E_t(D_{t+j-1})]
\]

\[
= a[1+(1-c)] + cT[E_t(Y_{t+j})+(1-c)E_t(Y_{t+j-1})] + (1-c)^2E_t(D_{t+j-2})
\]

\[
= a[1 + (1-c) + ... + (1-c)^{j-1}] + cT[E_t(Y_{t+j}) + (1-c)E_t(Y_{t+j-1}) +
\]

\[
+ (1-c)^{j-1}E_t(Y_{t+j})] + (1-c)^jD_t
\]
However

\[ E_t(Y_{t,t}) + (1-c)E_t(Y_{t+1,t}) + (1-c)^2E_t(Y_{t+2,t}) + \ldots + (1-c)^{i-1}E_t(Y_{t+i}) \]

\[ = (1+g)^iY_t + (1-c)(1+g)^{i-1}Y_t + (1-c)^2(1+g)^{i-2}Y_t + \ldots + (1-c)^{i-1}(1+g)Y_t \]

\[ = (1+g)((1+g)^{i-2} + (1-c)(1+g)^{i-3} + (1-c)^2(1+g)^{i-3} + \ldots + (1-c)^{i-2})Y_t \]

\[ = (1+g) \left( \frac{1 - (1-c)^i}{1+g} \right) Y_t \]

\[ = (1+g) \left[ \frac{(1+g)^i - (1-c)^i}{(g+c)} \right] Y_t \]

\[ \therefore E_t(D_{t,t}) = a \left[ \frac{1 - (1-c)^i}{c} \right] + cT(1+g) \left[ \frac{(1+g)^i - (1-c)^i}{(g+c)} \right] Y_t + (1-c)^i D_t \]

From APPENDIX 2 recall the dividend valuation model,

\[ P_t = \sum_{j=1}^{\infty} \frac{E_t(D_{t,j})}{(1+r)^j} \]

After incorporating an earnings generating process with constant growth, g, then

\[ P_t = \sum_{j=1}^{\infty} \frac{a \left[ \frac{1 - (1-c)^i}{c} \right] + cT(1+g) \left[ \frac{(1+g)^i - (1-c)^i}{(g+c)} \right] Y_t + (1-c)^i D_t}{(1+r)^j} \]

It follows that

\[ P_t - P_{t-1} = \sum_{j=1}^{\infty} \frac{cT(1+g) \left[ \frac{(1+g)^i - (1-c)^i}{(g+c)} \right] (Y_t - Y_{t-1}) + (1-c)^i D_t - D_{t-1}}{(1+r)^j} \]

Now \[ \frac{cT(1+g)}{(g+c)} \sum_{j=1}^{\infty} \frac{(1+g)^j - (1-c)^j}{(1+r)^j} \]
\[ \begin{align*}
&= \frac{cT(1+g)}{(g+c)} \left\{ \sum_{j=1}^{\infty} \left( \frac{1+g}{1+r} \right)^j \right. - \left. \sum_{j=1}^{\infty} \left( \frac{1-c}{1+r} \right)^j \right\} \\
&= \frac{cT(1+g)}{(g+c)} \left[ \frac{1+g}{r-g} - \frac{1-c}{1+r} \right] \\
&= \frac{cT(1+g)(1+r)}{(r-g)(r+c)}
\end{align*} \]

\[ P_t - P_{t-1} = \frac{cT(1+g)(1+r)}{(r-g)(r+c)} \Delta Y_t + \left( \frac{1-c}{r+c} \right) \Delta D_t \]

Dividing both sides by \( P_{t-1} \) results in the following expression which implies capital gains returns depend on earnings changes and dividend changes. The dependence is linear and additive.

\[ \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{cT(1+g)(1+r)}{(r-g)(r+c)} \frac{\Delta Y_t}{P_{t-1}} + \left( \frac{1-c}{r+c} \right) \frac{\Delta D_t}{P_{t-1}} \]

Dividend yield is given by

\[ \frac{D_t}{P_{t-1}} = \frac{a}{P_{t-1}} + cT \frac{Y_t}{P_{t-1}} + (1-c) \frac{D_{t-1}}{P_{t-1}} + \frac{e_t}{P_{t-1}} \]

Recall the identity (2.1),

\[ R_t = \frac{\Delta P_t}{P_{t-1}} + \frac{D_t}{P_{t-1}} \]

After substitution

\[ R_t = \gamma_o \frac{1}{P_{t-1}} + \gamma_1 \frac{\Delta T_t}{P_{t-1}} + \gamma_2 \frac{Y_t}{P_{t-1}} + \gamma_3 \frac{\Delta D_t}{P_{t-1}} + \gamma_4 \frac{D_{t-1}}{P_{t-1}} + \frac{e_t}{P_{t-1}} \]

where

\[ \begin{align*}
\gamma_o &= a \\
\gamma_1 &= \frac{cT(1+g)(1+r)}{(r-g)(r+c)} \\
\gamma_2 &= cT \\
\gamma_3 &= \left( \frac{1-c}{r+c} \right) \\
\gamma_4 &= (1-c)
\end{align*} \]
APPENDIX 6  RETURNS COLLINEARITY AND OBSERVABLES

Recall equation (3.5) where

$$\Delta P_t = \frac{cT(r+1)}{r(r+c)} \Delta Y_t + \left(\frac{1-c}{r+c}\right) \Delta D_t$$

Now $\Delta D_t$

$$= D_t - D_{t-1}$$

$$= a + cTY_t + (1-c)D_{t-1} + e_t - D_{t-1}$$

$$= a + cTY_t - cD_{t-1} + e_t$$

and $\left(\frac{1-c}{r+c}\right) \Delta D_t$

$$= \left(\frac{1-c}{r+c}\right) [a + cTY_t - cD_{t-1} + e_t]$$

$$= a \left(\frac{1-c}{r+c}\right) + cT \left(\frac{1-c}{r+c}\right) Y_t - c \left(\frac{1-c}{r+c}\right) D_{t-1} + \left(\frac{1-c}{r+c}\right) e_t$$

From equation (3.8) after multiplying throughout by $P_{t-1}$, we have

$$R_t, P_{t-1} = \Delta P_t + D_t$$

$$= a + \frac{cT(r+1)}{r(r+c)} \Delta Y_t + cTY_t + \left(\frac{1-c}{r+c}\right) \Delta D_t +$$

$$+ (1-c)D_{t-1} + e_t$$

$$= a \left[1 + \left(\frac{1-c}{r+c}\right)\right] + \frac{cT(r+1)}{r(r+c)} \Delta Y_t + cT \left[1 + \left(\frac{1-c}{r+c}\right)\right] Y_t +$$

$$+ [1 - (1-c) \left(\frac{1-c}{r+c}\right)] D_{t-1} + \left[1 + \left(\frac{1-c}{r+c}\right)\right] e_t$$

$$= a \left(\frac{r+1}{r+c}\right) + \frac{cT(r+1)}{r(r+c)} \Delta Y_t + cT \left(\frac{r+1}{r+c}\right) Y_t +$$

$$+ D_{t-1} - c \left(\frac{r+1}{r+c}\right) D_{t-1} + \left(\frac{r+1}{r+c}\right) e_t$$

$$= \left(\frac{r+1}{r+c}\right) \left[a + cTY_t - cD_{t-1} + e_t + \frac{cT}{r} \Delta Y_t\right] + D_{t-1}$$

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\[
\Delta P_t + D_t = \left(\frac{r+1}{r+c}\right) \left[ \Delta D_t + \frac{c^T}{r} \Delta Y_t \right] + D_{t-1}
\]

and \( R_t \) = \[
\left(\frac{r+1}{r+c}\right) \frac{\Delta D_t}{P_{t-1}} + \frac{cT(r+1)}{r(r+c)} \frac{\Delta Y_t}{P_{t-1}} + \frac{D_{t-1}}{P_{t-1}}
\]

APPENDIX 7 \hspace{1cm} \text{DIVIDENDS IN TERMS OF LAGGED EARNINGS}

Recall (2.2)
\[ D_t = a + cTY_t + (1-c)D_{t-1} + e_t \]

Then
\[
\Delta D_t = D_t - D_{t-1} = a + cTY_t - cD_{t-1} + e_t
\]
\[
= a + cTY_t - c [a + cTY_{t-1} + (1-c)D_{t-2} + e_{t-1}] + e_t
\]
\[
= a[1-c] + cT[Y_t - cY_{t-1}] - c(1-c)D_{t-2} + (e_t - ce_{t-1})
\]
\[
= a[1-c-c(1-c)] + cT[Y_t - cY_{t-1} - c(1-c)Y_{t-2}] -
\]
\[
- c(1-c)^2D_{t-3} + [e_t - ce_{t-1} - c(1-c)e_{t-2}]
\]
\[
= a[1-c-c(1-c)-c(1-c)^2-...-c(1-c)^{n-1}] +
\]
\[
+ cT[Y_t - cY_{t-1} - c(1-c)Y_{t-2} - c(1-c)^2Y_{t-3} -...-c(1-c)^{n-1}Y_{t-n}] +
\]
\[
+ [e_t - ce_{t-1} - c(1-c)e_{t-2} - c(1-c)^2e_{t-3} -...-c(1-c)^{n-1}e_{t-n}]
\]
\[
= a + cTY_t + e_t - \sum_{j=1}^{n} c(1-c)^{j-1}[a + cTY_{t-j} + e_{t-j}]
\]

Thus the dividend term \( \Delta D_t \) can be replaced by current and lagged levels of earnings and dividend shocks. A similar process can be used to express \( D_{t-1} \) in terms of past levels of earnings and dividend shocks.