Modelling the Equity Risk Premium in the Long Term

Ian Davidson
John Okunev
Mohammad Tahir

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by

Ian Davidson
University of Warwick, Coventry, UK

John Okunev
University of Technology, Sydney, Australia

and

Mohammad Tahir
Australian National University, Canberra, Australia

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1.0 Introduction

There has been considerable interest in recent times in studying the nature of the random process generating the long term risk premium. Papers by Copeland (1982), Fama and French (1988), Finnerty and Leistikow (1993), Reichenstein and Rich (1993), Kairys (1993) and Boudoukh, Richardson and Smith (1993) have all adopted different approaches in attempting to study the risk premium.

Copeland (1982) models ex post holding period returns as composed of an ex ante expected return plus a component of unanticipated return, which may be negative or positive depending upon whether investors under- or overestimate expected returns. Using annual observations on Standard and Poor's (S&P) composite stock index and corporate index for the period 1926-1978, he concludes that the holding period return spread overestimated the true market risk premium.

Fama and French (1988) argue that the market risk premium tends to revert slowly to its mean, with reversion often taking a few years. Finnerty and Leistikow (1993) model ex post risk premium of various asset classes as a mean reverting process, and find that the risk premium is mean reverting for some asset classes towards a time dependent long term mean for the period 1926-1989. Their model fails to detect mean reversion in the equity risk premium. Finnerty and Leistikow's conclusions have, however, been questioned by Ibbotson and Lummer (1994) on the grounds that they are not supported by their empirical evidence.

Reichenstein and Rich (1993) adopt an ex ante approach and model the long horizon S&P stock returns in terms of the earnings price ratio, dividend yield and the value line forecasts of the risk premium. Their results are similar to those of Fama and French (1988) and Campbell and
Shiller (1988) in terms of improved explanatory power with longer term investment horizons. Their study focuses on the more recent time frame of 1968-1990 and shows that risk premium based on forecasts instead of historical data, provides more consistent predictions of stock returns than either dividend yield or earnings price ratio.

In Kairys' (1993) analysis, the risk premium is modelled in terms of lagged commercial paper rate. His analysis covers the period 1836-1990, and the empirical results support the view that the lagged commercial paper rate is significant in forecasting the risk premium. Boudoukh, Richardson and Smith (1994) develop tests of inequality restrictions implied by conditional asset pricing models. As an application, they test whether the ex ante risk premium is always positive. Using annual data on aggregate US stock returns, inflation, long and short rates of interest, and dividend yields over three time periods (1802-1990, 1802-1896, 1897-1990), they report reliable evidence that the ex ante risk premium is negative in some states of the world; these states are related to periods of high expected inflation and especially to downward sloping term structure.

In this paper we present a different approach to those outlined above. We model the ex ante risk premium implied from the information contained in the share price. We commence by modelling the stochastic nature of the dividend series and based upon these results we derive the appropriate functional form of the share price. A similar approach has been adopted by Campbell and Kyle (1993). From the formulation of the dividend and share price equations we are able to infer the expected future risk premium. We then go on to model the ex ante risk premium as a mean reverting Ornstein Uhlenbeck process with a time dependent mean. Our model shows clearly that the ex ante equity risk premium is mean reverting towards a stable long term mean of 3.5% for the period 1871-1986 and 4.55% for the period 1926-1986. These results are substantially different
from the ex post risk premium where the average realised risk premium for the period 1871-1986 is 5.7 % whilst for the period 1926-1986 it is 7.9%. The lower ex ante risk premium may assist in solving the so called equity premium puzzle (see Mehra and Prescott, 1985)

We also investigate whether the ex post risk premium reverts to the ex ante risk premium by using a naive model (viz. ex post risk premium is equal to the ex ante risk premium plus a forecast error) and a general model based on an Ornstein Uhlenbeck process. Our results suggest that the general model offers superior explanatory power when compared to the naive model.

The format of the paper follows. Section 2 outlines the modelling of the dividend and share price dynamics. Section 3 describes how the ex ante risk premium is derived and also investigates the relationship between the ex ante and ex post risk premiums. Section 4 provides some empirical evidence on the dynamic behaviour of the ex ante and ex post risk premiums, and section 5 concludes the paper.

2.0 Modelling Dividends and the Share Price

We commence by modelling dividends as a mean reverting Ornstein Uhlenbeck process and also incorporate mean reversion towards a time varying mean. This approach has also been adopted by Shiller (1981) and Campbell and Kyle (1993). However they model detrended dividends as an Ornstein Uhlenbeck process. This is equivalent to dividing dividends by the long term growth of dividends, which in our formulation means dividing dividends by \(\exp(kDT)\). In the continuous time framework the instantaneous change in dividends may be described by the following process:¹

¹ The continuous time framework has been applied to model dividends, see for example Merton (1973) and Campbell and Kyle (1993).
\[ dD(t) = \beta_D \left( \frac{\alpha_D}{\beta_D} e^{\gamma_D t} - D(t) \right) dt + \sigma_D \delta w_D(t) \]  

where

- \( dD(t) \) is the instantaneous change in dividends at time \( t \);
- \( D(t) \) represents dividends at time \( t \);
- \( \beta_D \) represents the speed of adjustment coefficient;
- \((\alpha_D/\beta_D)\exp(\kappa_D t)\) represents the long term mean which grows or decays exponentially at rate \( \exp(\kappa_D) \);
- \( \sigma_D^2 \) is the variance of \( dD(t) \) per unit time; and
- \( w_D(t) \) is a Wiener process with mean zero and unit variance.

Under this formulation dividends can be allowed to grow in times of inflation or decay in times of deflation. This formulation also allows dividends to be mean reverting, which is consistent with literature (see Lintner (1956), Shiller (1981), Marsh and Merton (1987) and Fama and French (1988)). The model of equation (1) captures various possible combinations of mean reversion and also permits a random walk as a special case. For example for \( k_D=0 \) and \( \beta_D>0 \) this represents mean reversion of dividends towards a stable long term mean. On the other hand if \( k_D \neq 0 \) and \( \beta_D > 0 \), then this represents mean reversion about a time varying long term mean. In the case where \( \beta_D=0 \), then dividends are represented by a random walk. The model can also be viewed as capturing the Marsh and Merton (1987) suggestion that the fundamental value of dividends is dependent upon long term sustainable earnings if one regards the long term mean of dividends as the same as the long term mean of earnings.
In order to operationalize equation (1), we need to determine its discrete time version.\(^2\)

This is given by the following expression:

\[
\Delta D(t) = \frac{\alpha_D}{(k_D + \beta_D)} e^{k_D} (e^{k_D} - e^{\beta_D}) + (e^{\beta_D} - 1) D(t) \\
+ e^{\beta_D} \int_t^{t+1} \sigma_D e^{\beta_D} dW_D(s)
\]

(2)

From equation (2) we note that changes in dividends are dependent upon the long term mean of dividends and also on lagged dividends. Since equation (2) is a non-linear function of time it is appropriate to use non-linear maximum likelihood regression to estimate the parameters \(\alpha_D, \beta_D\) and \(k_D\).

Based upon the assumption that dividends can be described by equation (1) we now proceed to formulate the corresponding functional form of the share price. It is well established in the literature that the share price represents the discounted value of expected future dividends. For the sake of simplicity we assume that the term structure is flat and that \(D(t)\) represents the level of dividends at time \(t\). Therefore, the discounted value of expected future dividends is given by:

\[
P(t) = E \left[ e^{(r+s-i)D(s)} ds \right]
\]

(3)

where \(P(t)\) is the share price at time \(t\), \(i\) is the risk adjusted discount rate (cost of capital) applied to random dividends \(D(s)\) at time \(s\), and \(E\) is the expectation operator.

Substituting for \(D(s)\) from equation (1) the solution of equation (3) is given by:

\(^2\) The discrete time approximation is obtained by substituting \(t+1\) into the solution of equation (1) and then subtracting \(D(t)\) from \(D(t+1)\). This gives the correct discrete time approximation to equation (1).
\[ P(t) = \frac{\alpha_D e^{b_D t}}{(i - k_D)(i + \beta_D)} + \frac{D(t)}{(i + \beta_D)} \]  \hspace{1cm} (4)

Given that the parameters \( \alpha_D, \beta_D \) and \( k_D \) can be determined from the dividend process, and \( P(t) \) and \( D(t) \) are known at time \( t \), one can infer the cost of capital \( i \). If we assume that the cost of capital is the sum of the risk premium and the risk free rate (that is, \( i = C(t) + RF(t) \), where \( C(t) \) is the expected risk premium and \( RF(t) \) is the risk free rate at time \( t \)), then the anticipated risk premium can be determined. This risk premium represents the market's expectation of the future risk premium and may not be the same as the realised risk premium. In the next section we attempt to model the risk premium dynamics.

### 3.0 Modelling the Risk Premium

Previous researchers have found that the realised risk premium may have negative values, and that it appears to oscillate about some long term mean (see Fama and French (1988)). It is unclear whether the long term mean is stationary or is time dependent. This form of behaviour appears to be a type of mean reverting process about a long term mean which may or may not be stationary. We now proceed to attempt to model this type of behaviour in a formal manner.

We commence by modelling the ex ante risk premium as a mean reverting elastic random walk towards a time varying mean. We adopt the Ornstein Uhlenbeck process to model the time varying behaviour of the ex ante risk premium. The question arises why should we adopt this process when there are numerous other types of processes which could be adopted. We justify this selection on the basis that the Ornstein Uhlenbeck process is one of the few processes which permit
negative values.\textsuperscript{3} As mentioned previously it is unclear whether the long term mean is stationary. We therefore incorporate mean reversion towards a time varying mean into the model. This allows us to test the hypothesis whether the equilibrium long term risk premium is time dependent. We propose that the instantaneous change in the risk premium be modelled by the following process:\textsuperscript{4}

\begin{equation}
dC(t) = \beta \left( \frac{\alpha}{\beta} e^{\kappa t} - C(t) \right) dt + \sigma dw(t)
\end{equation}

where dC(t) is the instantaneous change in the ex ante risk premium at time t; 
(\alpha/\beta)e^{\kappa t} represents the long term mean which may grow or decay at a rate of e^{\kappa t}; 
\beta is the speed of adjustment coefficient; 
C(t) is the ex ante risk premium at time t; 
\sigma^2 is the variance per unit time of dC(t); and 
w(t) is a Wiener process with mean zero and unit variance.

The formulation of equation (5) is similar to that of the dividend process already mentioned.

The discrete time version of equation (5) is given by:

\textsuperscript{3} The additive Brownian motion could also be adopted, but this process is a special case of the Ornstein Uhlenbeck process. For a discussion of different types of diffusion processes see Chan, Karolyi, Longstaff and Sanders (1992).

\textsuperscript{4} Finnerty and Leistikow (1993) have used a linear time term in their analysis. The exponential term approximates to the linear form for small values of \( k \).
\[ \Delta C(t) = \frac{\alpha}{(k + \beta)} e^{\beta t} (e^{k(t+\Delta t)} - e^{k(t+\Delta t)}) + (e^{\beta(t+\Delta t)} - 1) C(t) \]
\[ + e^{\beta(t+\Delta t)} \int_{t}^{t+\Delta t} \sigma e^{\beta s} dw(s) \]  

(6)

Since regression equation (6) is a non-linear function of time and \( C(t) \), we use non-linear maximum likelihood regression to estimate the parameters \( \alpha \), \( k \) and \( \beta \).

If \( k=0 \) the time dependence vanishes and equation (6) reduces to the ordinary least squares regression equation as the variance of \( \Delta C(t) \) can be shown to be constant. That is, the long term equilibrium risk premium is \( \mu = \alpha/\beta \). In the linear case equation (6) reduces to the following regression equation:

\[ \Delta C(t) = \frac{\alpha}{\beta} (1 - e^{\beta(t+\Delta t)}) + (e^{\beta(t+\Delta t)} - 1) C(t) \]
\[ + e^{\beta(t+\Delta t)} \int_{t}^{t+\Delta t} \sigma e^{\beta s} dw(s) \]  

(7)

From the empirical section that follows it is evident that the ex ante risk premium is stochastic and it is of interest to model the dynamic behaviour of the ex post risk premium towards the expected risk premium. One possible scenario is to assume that the realised risk premium equals the expected risk premium plus a forecast error. We refer to this as the naive model. The naive model has been used extensively in modelling expected inflation (see Mishkin, 1992). Alternatively, one could model the movements in the ex post risk premium towards a stochastic ex ante risk premium by the following formulation:
\[ dE(t) = \beta_1 [C(t) - E(t)] dt + \sigma_1 dz_t \] (8)

where

- \( dE(t) \) is the instantaneous change in the ex post risk premium;
- \( C(t) \) is the ex ante risk premium at time \( t \);
- \( E(t) \) is the ex post risk premium at time \( t \);
- \( \beta_1 \) is the speed of adjustment of the ex post risk premium towards the ex ante risk premium;
- \( \sigma_1 \) is the standard deviation per unit time of \( dE(t) \); and
- \( z_t(t) \) is a standard Wiener process.

This approach suggests that movements in the ex post risk premium revert towards the ex ante risk premium which is stochastic and whose movements are described by equation (5). The greater the difference between the ex ante and the ex post risk premium the greater the restoring force back towards equilibrium. \( \beta_1 \) is the speed of adjustment coefficient which determines how quickly the ex post risk premium reverts towards the ex ante risk premium. The extent of mean reversion, therefore, depends upon two factors, viz. the difference between the ex post and ex ante risk premiums, and the speed of adjustment coefficient \( \beta_1 \). If \( \beta_1 \) is small then movements back towards equilibrium may be quite slow even though there may be a large difference between the ex post and ex ante risk premiums.

The solution to equation (8) is given by.\(^5\)

\[ \text{The mathematical workings are available from the authors upon request.} \]
\[
E(t) = \frac{\beta_1 C(t)}{(\beta_1 - \beta)} - \frac{\beta \mu (1 - e^{-\beta t})}{(\beta_1 - \beta)} \\
- \frac{\beta_1 e^{-\beta t}}{(\beta_1 - \beta)} \int_0^t e^{\beta r} \sigma_1 dz + e^{-\beta t} \int_0^t e^{\beta r} \sigma_1 dz_1
\]  

(9)

From equation (9) it is evident that the ex post risk premium equals a multiple of the ex ante risk premium together with a time dependent term and the difference between the accumulation of stochastic shocks to the ex post and the ex ante risk premium. Therefore at any point in time it is likely that the two will not be equal. Also note that the shock terms are not identically and independently distributed (as the residuals are time dependent) and therefore assuming that the ex post risk premium equals the expected risk premium plus a forecast error may be misspecified.

Since residuals are time dependent, direct estimation of equation (9) may lead to biased results. To overcome this difficulty we estimate equation (9) in terms of first differences. In the first difference formulation it can be shown that the residuals are stationary. The correct discrete time approximation is given by:  

\[
\Delta E(t) = a_0 + a_1 \Delta C(t) + a_1 E(t) + a_3 C(t) + e_1(t)
\]  

(10)

where

\[
a_0 = \frac{\beta \mu (e^{\beta t} - 1)}{(\beta_1 - \beta)}
\]

The mathematical workings can be obtained from the authors.
\[ a_1 = \frac{\beta_1}{(\beta_1 - \beta)} \]

\[ a_2 = e^{\beta_1} - 1 \]

\[ a_3 = \frac{\beta_1}{(\beta_1 - \beta)}(e^{\beta_1} - 1) \]

and \( e(t) \) is a disturbance term which is homoscedastic and serially independent.

If \( a_2 = a_3 = 0 \), then equation (10) reduces to the naive regression in the first difference form. Thus the naive model is nested within the general model put forward in equation (10). It is also interesting to note that equation (9) represents the long term equilibrium relationship, and equation (10) represents the short term partial adjustment model.

In the next section we estimate the ex ante and the ex post risk premium and investigate the dynamic relationship between the two.

### 4.0 Empirical Results

To test the model empirically we propose to use long term data. It is essential to use as long a time frame as possible in determining the long term characteristics of any financial variable.\(^7\) The data are annual observations for the period 1871-1986 of the S&P composite price index, together with the corresponding dividend series accruing to the S&P index for each year. The risk

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\(^7\) Perron (1988) has pointed out that it is the span of data, not the frequency which is important when examining the long term characteristics of financial variables.
free rate is the commercial paper rate. We work in terms of real variables and adjust nominal variables by dividing by the corresponding producer price index at time \( t \).\(^8\)

Table 1 presents summary results of the average return (inclusive of dividends) and standard deviation of the S&P index, risk premium and commercial paper rates for the periods 1926-1986 and 1871-1986. We partition the data set into the two subperiods to illustrate how patterns of behaviour change with differing sample periods. Also other researchers have examined the risk premium from the depression years to the eighties.\(^9\) The average return on the S&P index is similar for the two periods, and is 9.2\% for 1926-1986 and 8.4\% for the entire sample. However, the risk premium varies considerably between the two periods. For 1926-1986 the average risk premium was 7.9\%, whilst for the entire period the average was 5.7\%. One of the reasons for this disparity is that real interest rates averaged only 1.3\% for 1926-1986, whilst for 1871-1986 they averaged 2.7\%.

<Table 1 about here>

Table 2 presents estimates of the parameter \( \alpha_d \), \( k_d \), and \( \beta_d \) for the dividend model for the periods 1871-1986 and 1926-1986. It is evident that the parameter estimates for the two periods are similar. The growth rate of dividends is approximately 1.15\% per year. This figure appears to be a plausible estimate of the growth rate as the average growth rate is approximately 1.2\%.

<Table 2 about here>

Furthermore, dividends appear to be mean reverting at a rate of 17\% per year. This result is in accordance with the findings of Lintner (1956) and Fama and French (1988) who found evidence of mean reversion of the dividend series. Using the parameter estimates from the dividend model and given that the share price and the dividends are known at time \( t \), the implied risk premium was

\(^8\) We have chosen to work in terms of real variables. We conducted the same analysis using nominal variables and obtained similar results.

\(^9\) See Fama and French (1988)
determined by using equation (4). The average risk premiums for 1871-1986 and 1926-1986 were 3.5% and 4.55% respectively. It is interesting to note that these values are substantially lower than the average realised risk premium displayed from Table 1 and suggest that the market may underestimate the future risk premium.

The average values of the ex ante risk premium may not be very meaningful if the long term mean is time dependent. We now proceed to test the hypothesis whether the long term ex ante mean risk premium is time dependent. We do this by recourse to equation (6). Since equation (6) is a non linear function of time we use nonlinear maximum likelihood regression to estimate the parameters $\alpha$, $k$, $\beta$. Table 3 presents the results of this estimation procedure. It is apparent from

<Table 3 about here>

Table 3 that $k$ is not significantly different from zero for both sample periods, and therefore we can reject the hypothesis that the equilibrium long term risk premium is time dependent. This implies that the long term risk premium is stable, and consequently we can efficiently estimate the parameters by using the OLS regression. Table 4 presents summary results of the OLS estimation.

<Table 4 about here>

Consistent with the maximum likelihood results the risk premium is significantly mean reverting and the speed of adjustment coefficient is essentially the same under the two procedures. Recall that $\gamma_0$ and $\gamma_1$ are biased estimates of the long term mean and speed of adjustment coefficient. The transformations necessary to calculate the true values of $\alpha$ and $\beta$ are given in equation (7). The long term mean calculated from the regression coefficient is 3.5% for the entire period and 4.55% for the period 1926-1986. Two of the important assumptions of the Ornstein Uhlenbeck process are that increments in the risk premium have constant variance and are serially independent. Tests of heteroscedasticity using an Arch test, and serial correlation via T test revealed no evidence of heteroscedasticity or serial dependence for either time period. The results relating to homoscedasticity and serial correlation seem to conform quite well with the model proposed in
equation (5). In applying the Dickey Fuller (1981) test to equation (7) we can reject the hypothesis of a unit root and sustain that C(t) is stationary.

In summary the model seems to capture the time varying characteristics of the risk premium quite well. The ex ante risk premium appears to be mean reverting towards a stable long term mean, and is homoscedastic and serially independent. The empirical results also indicate the sensitivity of the results with respect to the time frame of the sample, and suggest that when examining long term characteristics of a process it is advisable to use long time spans (preferably in excess of 100 years). This conclusion has also been borne out in other areas of the financial literature.\(^{10}\) Our results suggest that in the long term the ex ante risk premium is mean reverting towards a long term mean between 3.5% and 4.55%. Unlike the results of Finnerty and Leistikow (1993) we find no evidence that the long term mean is time dependent. The difference in the results may be due to the fact that Finnerty and Leistikow (1993) model realised risk premiums whereas we have modelled the ex ante risk premium.

Figure 1 displays the dynamic behaviour of both ex ante and ex post risk premiums. It is immediately apparent that the ex post risk premium is more volatile than the ex ante risk premium and ex post risk premium appears to be mean reverting towards the ex ante risk premium. Both series exhibit negative values. Using equation (10) we test whether the ex post risk premium reverts to the ex ante risk premium. Table 5 presents summary results of the parameter estimates for the two sample periods. The first regression is of the form \(\Delta E(t) = a_0 + a_1 \Delta C(t) + e(t)\) which is the first difference equation of the naive model (viz. the realised risk premium equals the expected risk premium plus a forecast error). The results of this regression are then compared with the model proposed in equation (10). It is immediately apparent that mean reversion model of equation (10) is superior to the naive model. In the shorter sample period the explanatory power of the general model is nearly ten times greater than that of the naive model. All variables are significant at the 5% significance level.

\(^{10}\) See for example Lothian and Taylor (1995). They reach similar conclusions in modelling real exchange rates.
level. Tests of heteroscedasticity (via Arch and Breusch Pagan tests) and serial correlation (via T test and Lagrange Multiplier tests) revealed no evidence of either. Similar results hold for the 1926-1986 sample period.\textsuperscript{11}

It is apparent that the speed of adjustment coefficient $\beta_1$ is significantly different from zero and supports the view that the ex post risk premium reverts fairly quickly back towards the ex ante risk premium. These results differ from those of Fama and French (1988) who found that the risk premium reverts slowly. The difference between the two results may be due to the different approaches used in modelling the risk premium dynamics.

Equation (10) can be reformulated to provide a forecasting equation which may provide superior forecasting ability to that of the naive model. The future realised risk premium is given by:

$$E(t+1) = \alpha_0 + \alpha_1 \Delta C(t) + (1+\alpha_2)E(t) + \alpha_3 C(t) + e_i(t)$$  \hspace{1cm} (11)

To implement equation (11) at time $t$ one need to forecast the change in the ex ante risk premium. This can be obtained from equation (6).

5.0 Conclusion

In this paper we model the ex ante risk premium implied in the information contained in the share price. This contrasts with earlier studies which have dealt mainly with the ex post risk premium. We commence by modelling the dividend series and derive the appropriate functional form of the share price. From the formulation of the dividend and the share price equations we are able to infer the expected risk premium. We then model the ex ante risk premium as a mean reverting Ornstein Uhlenbeck process with a time varying mean. Our results suggest that the ex ante risk premium is mean reverting towards a stable long term mean of 3.5% for the period 1871-1986 and 4.55% for the subperiod 1926-1986. These findings are substantially different from the ex

\textsuperscript{11} We obtained similar estimates of $\alpha$, and explanatory power using the price level equations of the naive model. As expected the constant term should not be significantly different from zero in the first difference equation.

We also investigate whether the ex post risk premium reverts to the ex ante risk premium. We do so by a naive model and a general model derived from a mean reverting process. We conclude that the explanatory power of the general model is superior to that of the naive model for both sample periods, and it provides a possible approach to forecasting future realised risk premiums.
References


Table 1

Summary Statistics of Average Returns and Standard Deviations For Differing Sample Periods

<table>
<thead>
<tr>
<th></th>
<th>Period 1926-1986</th>
<th>Period 1871-1986</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Return</strong></td>
<td>.092</td>
<td>.084</td>
</tr>
<tr>
<td><strong>Risk Premium</strong></td>
<td>.079</td>
<td>.057</td>
</tr>
<tr>
<td><strong>Com/Paper Rate</strong></td>
<td>.013</td>
<td>.027</td>
</tr>
<tr>
<td><strong>Average Return</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>.20</td>
<td>.177</td>
</tr>
<tr>
<td></td>
<td>.204</td>
<td>.183</td>
</tr>
<tr>
<td></td>
<td>.085</td>
<td>.093</td>
</tr>
</tbody>
</table>

Table 2

Results of Non Linear Maximum Likelihood Estimation of Dividends Using Equation (2)

<table>
<thead>
<tr>
<th></th>
<th>Period 1926-1986</th>
<th>Period 1871-1986</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_D$</td>
<td>0.00131</td>
<td>0.0013</td>
</tr>
<tr>
<td>$\beta_D$</td>
<td>0.1726</td>
<td>0.1723</td>
</tr>
<tr>
<td>$k_D$</td>
<td>0.0115</td>
<td>0.01153</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.08</td>
<td></td>
</tr>
</tbody>
</table>

(1.86)

(2.17)

(2.81)

(2.91)

(3.02)

(5.77)

1 Figures in parentheses are asymptotic t statistics
Table 3
Results of Non Linear Maximum Likelihood Estimation of the Ex Ante Risk Premium Using Equation (6)

<table>
<thead>
<tr>
<th></th>
<th>Period 1926-1986</th>
<th></th>
<th></th>
<th></th>
<th>Period 1871-1986</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>( \beta )</td>
<td>( k )</td>
<td>( R^2 )</td>
<td>( \alpha )</td>
<td>( \beta )</td>
<td>( k )</td>
<td>( R^2 )</td>
</tr>
<tr>
<td>0.0962</td>
<td>0.66</td>
<td>-0.0137</td>
<td>0.24</td>
<td>0.00213</td>
<td>0.835</td>
<td>0.0053</td>
<td>0.28</td>
</tr>
<tr>
<td>(0.68)²</td>
<td>(3.19)</td>
<td>(-0.75)</td>
<td></td>
<td>(1.27)</td>
<td>(4.35)</td>
<td>(0.58)</td>
<td></td>
</tr>
</tbody>
</table>

¹ Figures in parentheses are asymptotic t statistics

Table 4
Results of OLS Estimation of the Ex Ante Risk Premium Using the Following
\[ \Delta C(t) = \gamma_0 + \gamma_1 C(t) + e(t) \]

<table>
<thead>
<tr>
<th></th>
<th>Period 1926-1986</th>
<th></th>
<th></th>
<th></th>
<th>Period 1871-1986</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_0 )</td>
<td>( \gamma_1 )</td>
<td>( \beta )</td>
<td>( \alpha/\beta )</td>
<td>( R^2 )</td>
<td>( \gamma_0 )</td>
<td>( \gamma_1 )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>0.0214</td>
<td>-0.47</td>
<td>0.63</td>
<td>0.455</td>
<td>0.23</td>
<td>0.02</td>
<td>-0.56</td>
<td>0.82</td>
</tr>
<tr>
<td>(1.98)¹</td>
<td>(-4.35)</td>
<td></td>
<td></td>
<td></td>
<td>(2.26)</td>
<td>(-6.64)</td>
<td></td>
</tr>
</tbody>
</table>

¹ Figures in parenthesis are t statistics.
Table 5

Estimation of the Relationship between Ex post and Ex ante Risk Premium Using:

\[ \Delta E(t) = a_0 + a_1 \Delta C(t) + a_2 E(t) + a_3 C(t) + e_1(t) \]

| \(a_0\)  | \(a_1\)  | \(a_2\)  | \(a_3\)  | \(R^2\)  | \(a_0\)  | \(a_1\)  | \(a_2\)  | \(a_3\)  | \(R^2\)  |
|------|------|------|------|------|------|------|------|------|------|------|
| 0.0047 | 0.669 | 0.047 | 0.0019 | 0.781 | 0.10 |
| (0.133) | (1.74) | (0.088) | (3.59) | | | |
| 0.049 | 0.694 | -0.948 | 0.577 | 0.49 | 0.814 | -0.956 | 0.498 | 0.53 |
| (1.66) | (2.13) | (-7.21) | (1.71) | (2.09) | (4.35) | (-10.05) | (2.33) | |