Monetary and Fiscal Policy Under Nonlinear Exchange Rate Dynamics

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Abstract

A nonlinear exchange rate model based on the famous Dornbusch (1976) overshooting model is modified to allow for explicit consideration of the sources of supply and demand in the foreign exchange market along the lines suggested by Kouri (1983). Imperfect substitutability between domestic and foreign assets and finite speed of adjustment are introduced into the foreign exchange market. Portfolio considerations dictate that the function describing the fraction of wealth domestic residents desire to hold in foreign assets be nonlinear. The exchange rate dynamics are governed by a set of nonlinear differential equations which exhibit limit cycle behaviour under perfect foresight. A number of fiscal and monetary policies are examined within the framework of the nonlinear model and compared with results obtained in the traditional linear mode of analysis.

1. INTRODUCTION

There is now a vast literature on the role of monetary and fiscal policy as well as a range of intervention rules within models of flexible exchange rates. Some of the more prominent contributions to this literature include Branson and Buit (1983), Blundell-Wignall and Masson (1985), Mussa (1985), Frenkel and Rodriguez (1982) and Bini Smaghi (1989).

Virtually all analysis of flexible exchange rate dynamics is carried out within the modelling framework of the Dornbusch (1976) overshooting model. Dornbusch assumes that commodity markets adjust sluggishly whilst asset markets clear instantaneously and allows for rational expectations. The dynamics of this model, and all similar models, are saddle-point in nature. In order that exchange rates and prices can settle onto non-divergent paths it is necessary to allow one of the variables to jump instantaneously from its initial value to a point on the stable manifold of the saddle-point. Since the commodity market is assumed to adjust sluggishly it is the exchange rate which is allowed to make an instantaneous
jump, and therefore exhibits the so-called overshooting behaviour. For example, a monetary expansion, in the Dornbusch model, induces an immediate depreciation of the currency in excess of its equilibrium value, which is then approached asymptotically.

The technique of allowing one of the economic variables to adjust instantaneously so as to obtain non-divergent dynamic paths has become known as the jump-variable technique, and it is regarded as unsatisfactory by a number of authors because the economic model generating the saddle-point behaviour contains no endogenous mechanism that allows the jump to happen. This is indeed a feature of many dynamic rational expectations models. George and Oxley (1985) argue that the saddle-point dynamic behaviour is unsatisfactory as it yields structurally unstable models. They suggest that a nonlinear dynamic analysis is required.

In a series of papers starting with Chiarella (1986), the author has attempted to avoid the imposition of arbitrary jumps in dynamic rational expectations models by allowing for nonlinear mechanisms which the economic logic of the underlying models would impose, once dynamic paths start to diverge from the model's static equilibrium point. Central to this nonlinear analysis is the realization that the assumption that a certain market clears instantaneously (e.g. the asset market in the Dornbusch model) has very subtle implications for the stability analysis of the model at hand. In particular the phase plane divides into regions of fast motion and slow motion, which when combined with the nonlinear features of the model can generate quite complicated dynamics such as limit cycles or chaotic motion. The existence of a region of fast motion will still lead to some variables making sudden jumps, but these are no longer arbitrarily imposed and occur in a manner different to what one would expect from the traditional jump-variable technique. A good general discussion on the subtleties involved when regions of fast motion and slow motion need to be analysed is contained in Chapter 10 of Andronov, Vitt and Chaikin (1966).

In order to analyse rational expectations models from the nonlinear perspective the economic theorists' main task is to identify the appropriate nonlinear mechanism in the model at hand. This task may not always be performed as easily as it is stated. As far as exchange rate models of the Dornbusch type are concerned the need for a nonlinear perspective is imposed not only because of stability considerations, but also because empirical studies of exchange rate dynamics indicate behaviour at variance with that predicted by traditional models; see, for example, the discussion by Dornbusch and Frankel (1987). A nonlinear exchange rate model which does not require the imposition of arbitrary jumps and which also allows exchange rates to be more variable than is possible with traditional models, has been constructed in Chiarella (1990). There explicit consideration of the sources of supply and demand in the foreign exchange
market, along the lines suggested by Khouri (1983), is introduced into the Dornbusch model. Imperfect substitutability between domestic and foreign assets and finite speed of adjustment are introduced into the foreign exchange market. Portfolio considerations dictate that the function describing the fraction of wealth domestic residents desire to hold in foreign assets be nonlinear. The rational expectations assumption on the rate of exchange depreciation in the deterministic framework adopted reduces to the perfect foresight assumption. However, initially adaptive expectations are posited with perfect foresight being treated as the limiting case when the time lag in the formation of expectations tends to zero. This round-aboutness in the treatment of expectations turns out to be productive as it allows us to see the relationship between the slow and fast manifolds of the model, which has hitherto been hidden when perfect foresight expectations are assumed from the outset. The resulting nonlinear model, in the perfect foresight limit, exhibits a stable limit cycle for expectations, the exchange rate, commodity prices and interest rates, and these stable paths are approached without the imposition of any arbitrary jumps.

In section 2 we review the dynamic behaviour of a simplified version of the nonlinear exchange rate dynamics model. In particular we allow the commodity market to adjust instantaneously. Given the key role that sluggish adjustment of the commodity market plays in the original Dornbusch model this may seem a rather drastic assumption. However our model also allows for finite speed of adjustment in the asset market, which renders the dynamics of the linearised version of the model to still be of the Dornbusch type. Indeed Frenkel and Rodriguez (1982) analyse a model which allows for finite speed of adjustment in both goods and asset markets and find that the key parameters determining the extent of overshooting or undershooting of the exchange rate are the speed of adjustment in asset markets, the interest elasticity of the demand for money, and the effects of relative prices on the balance of trade. The speed of adjustment in the goods market does not seem to be a key parameter.

In section 3 we consider the equilibrium point of the nonlinear model and analyse comparative static effects of changes in the fiscal and monetary parameters. In section 4 we give a phase plane analysis of the nonlinear dynamics under adaptive expectations. In section 5 we show how to obtain the dynamic picture in the perfect foresight limit. In section 6 we analyse the effect on the perfect foresight dynamics of changes in the fiscal and monetary parameters and make a comparison with the impact of such changes within the framework of the standard linear model. In section 7 we make some concluding remarks.

2. A NONLINEAR MODEL OF EXCHANGE RATE DYNAMICS

Our basic economic setting is that of Dornbusch (1976), however in order to introduce the appropriate nonlinearity into the model we incorporate the portfolio
approach to the modelling of the foreign exchange market as developed by Kouri (1983). Our exposition of the model follows the notation and approach of Gray and Turnovsky (1979) which shows very clearly the saddle-point structure of the Dornbusch model under perfect myopic foresight.

Let

\[ R^* = \text{foreign (nominal) rate of interest (assumed exogenous),} \]
\[ R = \text{domestic (nominal) rate of interest,} \]
\[ E = \text{logarithm of the current exchange rate (units of domestic currency per unit of foreign currency),} \]
\[ X = \text{expected rate of exchange depreciation,} \]
\[ M = \text{logarithm of the domestic nominal money supply (assumed exogenous),} \]
\[ P = \text{logarithm of the domestic price level,} \]
\[ Y = \text{logarithm of domestic real output (assumed to be fixed).} \]

Consider first of all the money market where we make the standard assumption that the demand for real balances is an increasing function of output and a decreasing function of the nominal interest rate, i.e.

\[ M - P = \alpha_1 Y + \alpha_2 R, \quad (\alpha_1 > 0, \alpha_2 < 0). \]  \hspace{1cm} (1)

Next consider the goods market where demand, \( D \), is assumed to be an increasing function of output \( Y \), relative price \( (E - P) \), and a decreasing function of domestic nominal interest rate \( R \), i.e.

\[ D = \beta_0 + \beta_1 Y + \beta_2 R + \beta_3 (E - P), \]
\[ (0 < \beta_1 < 1, \beta_2 < 0, \beta_3 > 0), \]  \hspace{1cm} (2)

where \( \beta_0 \) is a shift parameter incorporating a fiscal policy parameter. Assuming for the moment sluggish price adjustment in the goods market according to which the domestic price adjusts to excess demand, so that

\[ \dot{P} = \rho [\beta_0 + (\beta_1 - 1)Y + \beta_2 R + \beta_3 (E - P)], \quad (\rho > 0). \]  \hspace{1cm} (3)

At a later point in the analysis we shall specialise (3) to the limiting case of instantaneous price adjustment i.e. \( \rho \to \infty \).
The essential feature of our nonlinear model is the manner in which the foreign exchange market is modelled. We adopt the portfolio approach of Kouri (1983), in which domestic residents hold some fraction of domestic marketable wealth in foreign assets. This fraction depends on the relative expected return and risk characteristics of domestic and foreign assets, which in the deterministic framework we are adopting is measured by the real interest differential $R^* + X - R$. Thus taking the prices of foreign assets as exogenous in foreign currency and the stock of domestic marketable wealth as exogenous in domestic currency then the stock demand for foreign assets may be written

$$\tilde{F}^d = \bar{\gamma} (R^* + X - R) V/s,$$

(4)

where $V$ stands for the value of domestic marketable wealth in domestic currency, $s$ is the exchange rate (\(\ln s = E\)) and $\bar{\gamma}$ is the fraction of V domestic residents desire to hold in foreign assets. We assume that this fraction is an increasing function of the real interest differential $R^* + X - R$ and is bounded above and below as $R^* + X - R$ tends to large positive or negative values, i.e.

$$\bar{\gamma}' (R^* + X - R) > 0,$$

(5a)

and

$$\bar{\gamma}_i < \bar{\gamma} < \bar{\gamma}_u$$

(5b)

The upper and lower limits $\bar{\gamma}_u$ and $\bar{\gamma}_i$ arise from the assumption that domestic residents will hold domestic marketable wealth neither entirely in foreign assets nor entirely in domestic assets as the interest differential $R^* + X - R$ assumes large values. Rather some fixed proportion of wealth will be held in domestic assets and some in foreign assets, perhaps to satisfy transactions demand. It is also possible that the central bank can change the relationship between $\bar{\gamma}_u$ and $\bar{\gamma}_i$.
by intervention in the foreign exchange market.

Taking logarithms of (4), the log of domestic demand for foreign assets can be written

\[ \log \tilde{F}^d = f(R^* + X - R) - E + \log V, \]  

(6)

where \( f = \log \tilde{f}, f' > 0 \) and \( f_i < f < f_u \) \( (f_i = \log \tilde{f}_i, f_u = \log \tilde{f}_u) \). The general shape of \( f \), displaying the essential features of positive slope and upper and lower bounds is shown in figure 1. This demand function, unlike the one implicit in the Dornbusch class of models, allows for imperfect substitutability between foreign and domestic interest bearing assets.

The implicit demand function of the Dornbusch model may be thought of as the step-like limiting function obtained when the middle section of the function \( f \) tends to infinite slope (see the dotted curve in figure 3). In this limit demand for the foreign asset is indeterminate between \( f_u \) and \( f_i \) with the interest parity condition \( R^* + X - R = 0 \) being satisfied for all time.

The other element of the foreign exchange market which we include in the model that we analyse below is the notion that there is gradual rather than instantaneous adjustment of portfolios of foreign and domestic interest bearing assets. Empirical evidence of significant adjustment lags in portfolios has been presented by Kearney and MacDonald (1985) in their estimation of a structural model of the sterling-dollar exchange rate. Theoretical arguments favouring gradual portfolio adjustment include; transactions costs, the view that in portfolio balance models money serves as a buffer asset or signalling device, and; the view that in an aggregate model aggregate quantities may adjust with lags due to difference in timing by individual agents who may be maintaining continuous portfolio equilibrium (see e.g. Davis and Lewis (1977)). The possible presence of controls on international capital movements would also lead to gradual portfolio adjustment. The effect of gradual portfolio adjustment on traditional models of exchange rate dynamics is analysed by Karacaoglu and Ursprung (1988).

In order to capture the gradual portfolio adjustment we assume that the exchange rate adjusts to excess demand for foreign exchange according to

\[ \tau_s \dot{E} = f(R^* + X - R) - E + \log V - \bar{S}, \]  

(7)

where \( \tau_s > 0 \) and \( \bar{S} \) is the fixed supply of foreign exchange.

The parameter \( \tau_s \) is the time lag with which the exchange rate adjusts to excess demand. In practice we would expect \( \tau_s \) to be small but not zero. It is more traditional to formulate dynamic exchange rate models in terms of speed of adjustment parameters, \( 1/\tau_s \), in this case, however we prefer the time lag
interpretation since in our subsequent analysis we need to concentrate on the
behaviour of the dynamics as certain time lags tend to zero. It should be noted
that we are concentrating on the special case, also considered by Kouri, in which
foreign residents do not hold domestic assets and there is a fixed supply of
foreign exchange. (The assumption of fixed supply of foreign exchange is not as
restrictive as it may appear. It is equally possible to model the demand of foreign
residents for domestic interest bearing assets as a function of the interest
differential. This function will also have upper and lower bounds for similar
reasons that the function \( \hat{Y} \) does. When the resulting supply function is introduced
into equation (7) the \( f(\cdot) - \bar{S} \) term is replaced by an aggregate excess demand
function still having the same shape as \( f \). The condition \( R^* + X - R = 0 \) of the
standard Dornbusch model is obtained when the demand function \( f \) tends to the
perfect substitutability limit discussed earlier (where \( f \) becomes a step function).

The final element of our model is the mechanism for the formation of the
expectations variable \( X \). We shall assume that expectations are formed according
to the adaptive expectations scheme

\[
\tau_X \dot{X} = \dot{E} - X, \tag{8}
\]

where \( \tau_X > 0 \) is the time lag in the formation of expectations. In the limiting case \( \tau_X = 0 \) the conventional perfect foresight mechanism

\[ X = \dot{E}, \tag{9} \]

is obtained.

It is generally argued that in a world of rational economic agents only the perfect
foresight mechanism (9) is relevant. However a major part of the contribution in
Chiarella (1990) is to show that when proper account is taken of the nonlinear
nature of the demand function discussed above, the exchange rate dynamics are
qualitatively the same whether \( \tau_X = 0 \) or \( \tau_X \) is positive and small.

We now turn to the analysis of the exchange rate model in which the exchange
rate adjusts to excess demand according to (7) and exchange rate expectations
are formed according to (8).

The dynamics of the exchange rate model can be expressed by the differential
equations

\[
\tau_e \dot{E} = f(R^* + X - R) - E + \log V - \bar{S}, \tag{10a}
\]

\[
\tau_X \dot{X} = \dot{E} - X, \tag{10b}
\]
\[ \dot{\rho} = p[\beta_0 + (\beta_1 - 1)Y + \beta_2 R + \beta_3 (E - P)]. \]  

(10c)

3. **ANALYSIS OF THE EQUILIBRIUM POINT OF THE MODEL**

The equilibrium point of this system \((\bar{E}, \bar{X}, \bar{P})\) is given by

\[ \bar{X} = 0, \]  

(13a)

with \(\bar{E}\) and \(\bar{P}\) being determined by the simultaneous set of equations

\[ (\beta_2 + \alpha_2 \beta_3)P/\alpha_2 - \beta_3 E = \beta_0 + (\beta_1 - 1)Y + \beta_2 (M - \alpha_1 Y)/\alpha_2, \]  

(13b)

and

\[ f(R^* - (M - \alpha_1 Y)/\alpha_2 + P/\alpha_2) - E = S. \]  

(13c)

We may use (11) to calculate \(R\), the corresponding equilibrium value of the domestic interest rate.

The graphical determination of \(\bar{P}\) and \(\bar{E}\) is shown in figure 2. The comparative statics of this equilibrium point are fairly straightforward. We are particularly interested in how the equilibrium is effected by changes in the monetary variable \(M\), the fiscal variable \(\beta_0\) and the foreign asset reserves \(f_u\) and \(f_r\).

Consider first of all an increase in \(M\). This raises the \(\dot{\rho} = 0\) line whilst the \(\dot{E} = 0\) curve moves outwards, so that \(\bar{P}\) increases whilst the effect on \(\bar{E}\) is not immediately clear. However comparative statics calculations in this instance yield

\[ d\bar{P} = \frac{\beta_3 f'(\bar{\rho}) - \beta_2}{\beta_3 f'(\bar{\rho}) - \beta_2 - \alpha_2 \beta_3} dM, \]  

(14a)

and

\[ d\bar{E} = \frac{\beta_3 f'(\bar{\rho})}{\beta_3 f'(\bar{\rho}) - \beta_2 - \alpha_2 \beta_3} dM, \]  

(14b)

where \(f'(\bar{\rho})\) is the derivative of \(f\) evaluated at the equilibrium point. Recalling the signs of the various coefficients \((\alpha_2 < 0, \beta_2 < 0, \beta_3 > 0, f' > 0)\) we see that

\[ 0 < d\bar{P} \leq dM, \]  

(15a)

\[ 0 < d\bar{E} \leq dM. \]  

(15b)
Observe that in the limit \( f'(*) \to \infty \) (see the dashed line in figure 2) when holdings of foreign assets adjust instantaneously to a change in sign in the real interest differential \((R^* + X - R)\) we obtain the classical result

\[
dP = dE = dM. \tag{15}\]

However when we allow, \( 0 < f'(*) < \infty \), which accounts for the degree of sensitivity of the relative demand for foreign assets at the equilibrium value of the real interest differential \((R^* \cdot R)\), then the exchange rate and goods price change less than the increase in money. This effect comes about principally because a change in \( M \) changes the equilibrium real interest differential (this quantity remains at \( 0 \) in the traditional Dornbusch model), which in turn through its effect on relative holdings of foreign and domestic assets affects the equilibrium exchange rate and the equilibrium domestic goods price.

The effect of a fiscal expansion, \( d\beta_0 > 0 \), is also easily calculated, and we find that

\[
dP = \frac{\alpha_2}{\beta_3 f'(*) - \beta_2 - \alpha_2 \beta_3} d\beta_0 > 0, \tag{16a}\]

and

\[
dE = \frac{f'(*)}{\beta_3 f'(*) - \beta_2 - \alpha_2 \beta_3} d\beta_0 > 0. \tag{16b}\]

Finally, consider the effect of changes to \( f_u \) and \( f_r \), the upper and lower bounds on the fraction of domestic wealth held in foreign assets.

An increase in \( f_u \) moves the right hand asymptote in figure 2 further to the right. If we assume that the schedule \( E = 0 \) also moves uniformly to the right then

\[
\frac{dE}{df_u} > 0 \quad \text{and} \quad \frac{dP}{df_u} > 0. \tag{17a}\]

Similarly

\[
\frac{dE}{df_r} > 0 \quad \text{and} \quad \frac{dP}{df_r} > 0. \tag{17b}\]

Changes in \( f_u \) and \( f_r \) could conceivably occur as a result of central bank intervention in the foreign exchange market.
4. ANALYSIS OF THE DYNAMICS

We define the deviation variables

\[ e = E - \bar{E}, \quad x = X - \bar{X}, \quad p = P - \bar{P} \quad \text{and} \quad r = R - \bar{R}, \]  

in terms of which the differential system (12) may be written

\[ \tau \dot{e} = \phi(x + p/\alpha_2) - e, \]  
\[ \tau \dot{x} = \phi(x + p/\alpha_2) - e - \tau x, \]  
\[ \dot{p} = p - (\beta_2 + \alpha_2 \beta_3) p/\alpha_2 + \beta_3 e, \]

where the function \( \phi \) is defined according to

\[ \phi(z) = f(n + z) - f(n), \]

and

\[ n = R^* - (M - \alpha_1 \gamma)/\alpha_2 + P/\alpha_2. \]

![Figure 3 The Two-dimensional Degenerate Saddle Point](image-url)
In Chiarella (1990) we have analysed the three dimensional nonlinear system (19) and shown how it may generate limit cycle behaviour for the expectations time lag $\tau_\varepsilon$ sufficiently small. We also showed in that paper how the use of the jump-variable technique may be avoided, since the limit cycles are stable. However in order to get a qualitative feel for how changes in the policy parameters $M$, $\beta_0$, $f$, and $f_i$ affect the dynamics behaviour of the model we need to consider the two dimensional special case which is obtained when we allow $\rho \to \infty$, so that there is instantaneous price adjustment in the goods market. This still leaves a finite speed of adjustment in the asset market, so that the linearised version of the model still displays the Dornbusch overshooting behaviour. It is not possible to adopt the alternative approach of allowing finite speed of adjustment in the goods market and infinite adjustment in the asset market (i.e. let $\tau_\varepsilon \to 0$). Following this path would reduce (19) to a single differential equation, which is not able to generate limit cycle behaviour. Furthermore in this case the foreign asset demand collapses to the step function shown in figure 1 and becomes indeterminate.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.pdf}
\caption{Phase diagram for the instantaneous price adjustment model}
\end{figure}

Allowing $\rho \to \infty$ in equation (19c) we obtain

\[ p = \alpha_2 \Theta e, \tag{21} \]
where

\[ \Theta = \beta_3 / (\beta_2 + \alpha_2 \beta_3) < 0. \]

The differential equations (19a,b) for e and x then reduced to

\[
\tau_e \dot{e} = \phi(x + \Theta e) - e, \tag{22a}
\]

\[
\tau_e \tau_x \dot{x} = \phi(x + \Theta e) - e - \tau_e x. \tag{22b}
\]

Note that for x and e large the nonlinear term \( \overline{\phi}(x + \Theta e) \) saturates (i.e. \( \phi \) is close to its upper and lower limit). Thus far from the origin e and x are governed by the set of linear differential equations

\[
\tau_e \dot{e} = \overline{\phi} - e, \tag{22a'}
\]

\[
\tau_e \tau_x \dot{x} = \overline{\phi} - e - \tau_e x. \tag{22b'}
\]

which has all trajectories moving back towards the origin. Here \( \phi \) is either the maximum or minimum value of \( \phi \). We find that the characteristic equation for the eigenvalues of equation (22) is

\[
\tau_x \lambda^2 + (\tau_x \delta_4 + \delta_2) \lambda + \delta_4 = 0, \tag{23}
\]

where

\[
\delta_4 = 1 - \beta_3 f(n)/(\beta_2 + \alpha_2 \beta_3) > 0.
\]

![Figure 5 Determination of the turning points of \( \psi \)](image)

![Figure 6 The limit cycle in the perfect foresight limit](image)

The sign of \( \delta_4 \) indicates that the sign distribution of the roots of (23) is \((+,-)\) or \((-,-)\). Since the coefficient of \( \lambda \) in (23) (i.e. \( \tau_x \delta_4 + \delta_2 \)) is minus the sum of the roots, the sign distribution changes from \((-,-)\) to \((+,-)\) as \( \tau_x \) tends towards zero. Thus as
\[ \tau_x^* = -\delta_x/\delta_4, \] (24)

the dynamic behaviour of the nonlinear system (22) switches from asymptotic stability towards the origin to a stable limit cycle as shown in figure 4.

The geometric analysis of the nonlinear differential system (22) is considerably simplified if the nonlinear term can be made to appear in just one differential equation. This end is achieved by transforming from the co-ordinates \((e, x)\) to the new set of co-ordinates \((y, z)\) where

\[ y = \Theta e + x, \] (25a)
\[ z = e - \tau_x x. \] (25b)

Note that \(y = x - r\) is the deviation of the interest differential or the risk premium. Under this transformation the differential system (22) becomes

\[ \tau_x \tau_e \dot{y} = \phi(y)/\gamma - (\tau_x + \tau_e - \Theta \gamma \tau_x \tau_e) y - (1 - \Theta \gamma \tau_e) z, \] (26a)
\[ \dot{z} = \gamma y - \Theta \gamma z, \] (26b)

where \[ \gamma = 1/(1 + \Theta \tau_e). \]

In order to construct the phase plane for the nonlinear differential system (26) and to analyse its behaviour as we vary various parameters in the model we need to understand the nature of the curve obtained by setting \(y = 0\), viz.

\[ z = \psi(y) = (\phi(y) - \xi y)/\gamma(1 - \Theta \gamma \tau_e), \] (27)

where \[ \xi = \gamma(\tau_x + \tau_e - \Theta \gamma \tau_x \tau_e). \]

Since \(\Theta < 0\), the sign of the parameter \(\gamma\) is ambiguous, however by referring to equation (24) it is a straightforward matter to show that the bifurcation value of \(\tau_x\) satisfies
$1 + \Theta \tau^*_x > 0.$  \hfill (28)

Figure 7 Monetary and Fiscal Policy Dynamics - Traditional Analysis

Hence since the major object of our analysis is to understand the behaviour of the dynamics in the limit cycle region, where $\tau_x < \tau^*_x$ we consider only the case $\gamma > 0$. It follows that $\xi > 0$. In fact it is easy to show that $\xi$ is a monotonic increasing positive function of $\tau_x$ for $0 \leq \tau_x < -1/\Theta$. Recalling the definition of $\phi$ (equation 19d) we note that

$$\phi(+\infty) = f_u - f(n) > 0,$$

and

$$\phi(-\infty) = f_l - f(n) < 0.$$  \hfill (29)

Thus the graph of $\psi(y)$ is asymptotic to $\phi(-\infty) - \xi y$ as $y \to -\infty$ and asymptotic to $\phi(+\infty) - \xi y$ as $y \to +\infty$.

The other significant feature of the graph of $\psi(y)$ for our purposes is to determine whether $\psi(y)$ declines monotonically between these two asymptotes or whether it has turning points. The slope of $\psi(y)$ is given by
\( \psi'(y) = (\phi'(y) - \xi)/\gamma(1-\Theta \gamma \tau_x). \)

The graph of \( \phi'(y) \) is sketched in figure 6 with various values of \( \xi \) displayed. For illustrative purposes we assume that \( \phi(y) \) attains its maximum at \( y = 0 \), however this assumption is not crucial for the results which follow.

Let \( \tilde{\tau}_x \) denote the value of \( \tau_x \) for which \( \xi = \phi'(0) = f'(n) \). Then see from figure 5 that for \( \tau_x > \tilde{\tau}_x \), \( \phi'(y) < \xi \) for all \( y \) and hence \( \psi'(y) < 0 \) for all \( y \) and the graph of \( \psi \) descends monotonically between the two asymptotes as shown in figure 7a.

On the other hand for \( \tilde{\tau}_x > \tau_x > 0 \), the horizontal line \( \xi \) intersects \( \phi'(y) \) at two points (the two points \( y_u, y_i \) in figure 6) so that \( \psi(y) \) will exhibit a minimum then a maximum as \( y \) increases from \(-\infty\) to \(+\infty\), as displayed in figure 7b. The relationship of \( \tau_x \) to the bifurcation value \( \tau^*_x \) is of interest. Writing \( \xi(\tau_x) \) to indicate the functional dependence of \( \xi \) on \( \tau_x \) it can be shown (see Chiarella (1990), footnote 1) that \( \xi(\tau^*_x) < f'(n) \), which, given the monotonic increasing nature of \( \xi(\tau_x) \), indicates that \( \tau^*_x > \tau_x \). Hence the limit cycle emerges in figure 4a.

We complete the phase plane in figure 4 by putting in the line \( \dot{z} = 0 \) (i.e. \( z = y/\Theta \)), the direction arrows and displaying typical trajectories. In figure 4a for the case \( \tau_x > \tau^*_x \) when the origin is asymptotically stable and in figure 4b for \( \tau_x < \tau^*_x \). The stable limit cycle appears in the latter case due to the fact that; (i) the origin is locally unstable, (ii) for large \( y \) and \( z \) trajectories are moving back towards the centre, since we have shown earlier, \( e \) and \( x \) are (see equation 22) and, (iii) there are no other equilibrium points. The foregoing reasoning assumes that only one limit cycle occurs. It remains to prove rigorously that more than one limit cycle will not occur, this is done (in Chiarella 1986) by showing that the differential system under consideration can be transformed into Lienard’s equation which has a unique limit cycle.

The traditional mode of analysis based on linearisation of (22) can only handle the case \( \tau_x < \tau^*_x \), when the origin is asymptotically unstable, by use of the jump-variable technique to force \( y \) and \( z \) to jump instantaneously to the origin from any arbitrary initial point.

Before proceeding to analyse how changes in policy parameters affect the dynamic behaviour of the model, we discuss the perfect foresight limit.

5. THE PERFECT FORESIGHT LIMIT \( \tau \rightarrow 0^+ \)

To analyse the perfect foresight limit we rewrite equation (26) in the form

\[
\dot{y} = \frac{(1-\Theta \gamma \tau_x)(\psi(y) - z)}{\tau_y - \tau_x}.
\]

(31)
and make the following observations:

(a) For \( \tau_x = 0 \) and \( \psi(y) \neq z \), \( \dot{y} \) is large with \( \dot{y} \) becoming infinite in the limit \( \tau_x \rightarrow 0 \). Hence in regions of the phase plane where \( \psi(y) \neq z \), \( y \) tends to move infinitely rapidly as \( \tau_x \rightarrow 0 \); we denote such a region as the fast region.

(b) For \( \tau_x = 0 \) and \( \psi(y) = z \), \( \dot{y} \) tends to the limit 0/0 and hence is not determinate from equation (31). Rather the dynamics are governed by the differential equation for \( z \), equation (26b). Regions of the phase plane where motion is governed by equation (26b) we shall denote the slow manifold.

The fast and slow regions are displayed in figure 6. When drawing the direction arrows we use a double arrowhead to indicate that motion in the direction indicated is infinitely rapid. Now consider how motion evolves from the initial point Q in the fast region in figure 6. The variable \( y \) moves instantaneously horizontally to the point N on the slow manifold \( z = \psi(y) \). Motion is then along the slow manifold under the influence of equation (30b). When the point C is reached, \( y \) jumps instantaneously horizontally across to A on the opposite branch of the slow manifold. Motion is then down to A under the influence of (30b), then instantaneously across to B and from where the cycle repeats itself. The limit cycle in the perfect foresight limit is \( A \rightarrow B \rightarrow C \rightarrow D \rightarrow A \rightarrow \ldots \).

A similar pattern is observed from the initial point Q' (i.e. \( Q' \rightarrow N' \rightarrow A \rightarrow B \rightarrow C \rightarrow D \rightarrow A \rightarrow \ldots \)). It is important to realize, in the context of figure 6, that the traditional linear analysis focuses on a neighbourhood around 0 in which the dynamics appear totally unstable. The model can only be stabilized by allowing the jump \( O' \rightarrow 0 \) (see figure 3).

6. THE DYNAMICS OF MONETARY AND FISCAL POLICY CHANGES

The effect of monetary and fiscal policy changes can be worked out by considering how changes in M and \( \beta_0 \) alter the phase diagram in figure 6. Before undertaking this analysis we need, for comparison purposes, to consider how changes in M and \( \beta_0 \) affect the dynamics of the traditional linear model.

The linear (Dornbusch) model is obtained by setting \( \tau_x = 0 \) in equation (26), \( \phi'(\cdot) = \infty \), and \( 0 = x - r = y \). Since \( z = e \) when \( \tau_x = 0 \), the Dornbusch model in this case is the degenerate one

\[ \dot{e} = -\Theta e. \]  

(32)
As illustrated in figure 3, this model may be stabilized only by allowing an initial jump in \( e \) to the equilibrium value at the origin.

We have seen in section 3 that \( dM > 0 \) leads to \( dE > 0 \), which means that the equilibrium for \( e \) is moved from 0 to a positive value. In order to maintain stability, \( e \) must jump immediately to the new equilibrium value, as displayed in figure 7a.

Similarly the change in fiscal policy \( d\beta_0 > 0 \) implies \( dE < 0 \) and \( e \) moves down immediately to the new equilibrium as displayed in figure 7b.

These are the classical results of the Dornbusch style of analysis. Immediate jump in exchange rate in response to monetary and fiscal changes. We do not see the overshooting of the new equilibrium in the present case because of the instantaneous clearing of the goods market (i.e. \( \rho \to \infty \)).

Next consider the effect of monetary and fiscal policy changes in the nonlinear perfect foresight model.

\[ Z(= \varepsilon) \]

\[ y(= \Theta \varepsilon + x) \]

\[ y(= \Theta \varepsilon + x) \]

\[ y(= \Theta \varepsilon + x) \]

Figure 8 The Phase Diagram; \( d\beta_0 > 0 \)

An increase in the fiscal variable, \( d\beta_0 > 0 \), will lead to a lower equilibrium exchange rate, \( dE < 0 \). This change has the effect of moving the origin of the phase plane from 0 to 0', with coordinates (\( \Theta \) dE, dE), as shown in figure 8. Changes in the fiscal policy parameter \( \beta_0 \) have no effect on the shape and
properties of the slow manifold which is moved downward and to the right. These changes are shown in figure 8, where dashed (solid) lines represent axes and slow manifold prior to (after) the change in fiscal policy. The exchange rate fluctuations now occur around a higher equilibrium point, but still with the same amplitude. If the change in fiscal policy occurs when the system is at the point I on the old cycle then there is an immediate move to point N' on the new cycle. The effect on the time path for \( e \) will be a sharp change in direction towards the fluctuation of same amplitude around a lower level. The expectations variable \( x \) will undergo a discontinuous jump to a higher point on its cycle in this case. Such a discontinuous change is in keeping with the perfect foresight assumption; agents will anticipate the change in direction of \( e(t) \) brought about by the fiscal policy change.

Since the equilibrium \( x(0) \) is unaffected by any parameter change, and since the amplitude of the \( e \) and \( y \) fluctuations are the same before and after the fiscal policy change, the limit cycle for \( x \) remains the same.

![Figure 9 The Time Path for e; \( d\beta > 0 \)](image1)

![Figure 10 The Time Path for x; \( d\beta > 0 \)](image2)

The effect of a change in monetary policy requires a little more analysis since the slow-manifold, via the money demand function, is affected by changes in the money stock \( M \).

In equation (19e) let \( n_0 \) (\( n_1 \)) denote the value of \( n \) before (after) an increase in the money stock. Note that \( n_1 > n_0 \). Let \( \phi_0(z) \), \( \phi_1(z) \) denote the corresponding functions \( \phi(z) \) as defined in equation (19d).

Simple geometric reasoning reveals that the function \( \phi_1(z) \) has the same qualitative shape as \( \phi_0(z) \) but will be at a lower level and pushed somewhat to the left. The two functions are displayed in figure 11. In drawing these two graphs it is useful to bear in mind that \( \phi_1'(0) = f(n_1) < \phi_0'(0) = f(n_0) \).

The graphs of \( \phi'(y) \) for \( n_0 \), \( n_1 \) are then displayed in figure 12 where we see that
the effect of \( dM > 0 \) (i.e. \( dn > 0 \)) is to move the graph of \( \phi'(y) \) to the left. Thus the turning points of the function \( \psi(y) \) (equation 27), namely \( y_u \) and \( y_l \), are displaced uniformly to the left, as illustrated in figure 12, which shows how figure 5 is effected by \( dM > 0 \). For the purposes of determining the effect of \( dM > 0 \) on the amplitude of the fluctuations we note that

\[
y_u' - y_l' = y_u - y_l. \tag{33}
\]

In fact we show in appendix 1 that

\[
\frac{\partial y_u}{\partial n} = \frac{\partial y_l}{\partial n} = -1. \tag{34}
\]

Referring to the properties of the function \( \psi(y) \) discussed just prior to equation (30) we observe that the asymptotes are displaced uniformly to the left as a result of \( dM > 0 \).

Letting \( z_u = \psi(y_u) \), \( z_l = \psi(y_l) \) denote the maximum and minimum values of \( \psi \) respectively, we show in appendix 1 that

\[
\frac{\partial z_u}{\partial n} = \frac{\partial z_l}{\partial n} < 0. \tag{35}
\]

It follows from (35) that the amplitude of the fluctuation is unaffected by a change in the monetary policy variable.

We are now in a position to sketch in figure 13 the effect on the function \( \psi \) of \( dM > 0 \).
Figure 13 shows only part of the effect of $dM > 0$ on the phase plane. Since $dM > 0$ implies $dE > 0$, the origin of the fluctuation will shift up and to the left along the line $z = 0$ (i.e. $z = y/\theta$).

To determine whether the net effect of these changes is to move the fluctuation above or below its previous level we need to consider $\Delta z_u / \Delta M$.

![Figure 13 The Effect of $dM > 0$ on the Function $\psi$](image1)

![Figure 14 The Effect of $d\gamma < 0$ on the Turning Points of $\psi$](image2)

The change $\Delta z_u$ consists of two components. The first component is the decrease due to the change in $\psi(y)$ brought about by the change in the demand for foreign exchange function, $f$, through the impact on the interest differential of a change in $M$. Denote this change by $\Delta z_u^{(1)}$. We show in appendix 1 that

$$\Delta z_u^{(1)} = \frac{f'(n) - \xi}{\alpha_\gamma (1 - \Theta \gamma c)} \Delta M. \quad (36)$$

The second component is the increase due to the increase in the equilibrium exchange rate $E$. We denote this component by $\Delta z_u^{(2)}$, and since it equals $dE$ in the perfect foresight limit case (i.e. $\tau_x = 0$) we have from equation (14b)

$$\Delta z_u^{(2)} = \frac{\Theta f'(n)}{\Theta f'(n) - 1} \Delta M. \quad (37)$$

The total change in $z_u$ is then given by

$$\Delta z_u = \Delta z_u^{(1)} + \Delta z_u^{(2)},$$

which on setting $\xi = \gamma = 1$ when $\tau_x = 0$, becomes
\[ \Delta z_u = \left[ \frac{\tau_e - f'(n)}{\alpha_2 (\Theta r_e - 1)} \right] + \left[ \frac{\Theta f'(n)}{\Theta f'(n) - 1} \right] \Delta M. \] (38)

It is not possible to give a definite sign to \( \Delta z_u \), and which of the two effects is dominant in any particular situation would be an empirical question. The broad qualitative effects on the time path for \( e \) and \( x \) will be the same as in figure 9 and 10 for the change in fiscal policy. The exchange rate will experience a sudden change in direction as the fluctuation (amplitude unchanged) moves to a higher or lower level. The expectations variable \( x \) will experience a sudden jump to a higher point on its cycle.

\[ z = \psi_0(y) \]
\[ z = \psi_1(y) \]

Figure 15 The Effect of \( df_u < 0 \) on the Phase Plane

We have seen that fiscal and monetary changes cannot effect the amplitude of exchange rate fluctuations. Rather they can only effect the level around which these fluctuations occur. One might like to enquire as to which parameters could effect the amplitude of exchange rate fluctuations.

Analysis of the model indicates that changes to \( f_u \) and \( f_i \) can effect the amplitude of the fluctuations. We recall that \( f_u \) and \( f_i \) relate to the maximum and minimum fractions of domestic marketable wealth that domestic residents desire to hold in
foreign assets. These fractions depend on the relative costs and benefits to
domestic residents from holding the two currencies. Unlike changing the monetary
and fiscal parameters it is not as simple a matter for authorities to change these
values which really depend on microeconomics factors. They could be altered by
changes in tax regimes or by changes in anti-inflation policies.

Consider for example a decrease in $f_u$. It is a relatively simple matter to show that
the graph of $\psi'(y)$ is lowered as shown in figure 14. The turning points of the
function $\psi(y)$ are both moved closer to the origin as are both of its asymptotes.
Since $\psi, \psi'(0) < \psi_0, \psi'(0)$ it follows that the net effect of the decrease in $f_u$ is to
"shrink" the function $\psi(y)$ as illustrated in figure 15. As a consequence the
amplitude of the exchange-rate fluctuations is reduced. At the same time we know
from equation (17a) that the equilibrium exchange-rate level is moved down.

7. CONCLUSION

We have analysed the effect of monetary and fiscal policy changes within a
simple two dimensional nonlinear model of exchange rate dynamics whose key
features are: imperfect substitutability between domestic and foreign assets, a
nonlinear foreign asset demand function, finite speed of adjustment in the asset
market and perfect foresight. The nonlinear dynamics approach obviates the need
to impose the arbitrary jumps on the exchange rate required by the traditional
jump-variable technique. Rather the exchange rate movements are attracted to a
stable limit cycle whose amplitude depends on the parameters of the model. We
find that traditional fiscal and monetary policy changes, by their effect on the
equilibrium exchange rate are able to move the location of the centre of the
fluctuation. It is not possible however to effect the amplitude of the fluctuations
with such policies. We have briefly discussed policies which can affect the
amplitude of the fluctuations, such as microeconomic policies which could alter the
bounds of the foreign asset demand function.

Ideally we would like to extend the type of analysis undertaken here to the more
complete three-dimensional nonlinear model of exchange rate dynamics which
allows for finite speed of adjustment in both the goods and asset markets. Such
an undertaking seems impossible by sole use of analytical techniques and it
seems that computer simulations would be required. We leave this task for future
research.

REFERENCES


APPENDIX 1

Note first of all that \( y \) is a solution of
\[
f'(n + y_u) = \xi. \tag{A1}
\]
Differentiating both sides with respect to \( n \) we find that
\[
f''(n + y_u) (1 + \frac{\partial y_u}{\partial n}) = \frac{\partial \xi}{\partial n} = 0, \tag{A2}
\]
from which the result
\[
\frac{\partial y_u}{\partial n} = -1. \tag{A3}
\]
follows. Similarly
\[
\frac{\partial y_l}{\partial n} = -1. \tag{A4}
\]
By definition
\[
z_u = [f(y_u + n) - f(n) - \xi y_u]/\gamma(1-\theta \gamma c),
\]
and
\[
z_l = [f(y_l + n) - f(n) - \xi y_l]/\gamma(1-\theta \gamma c).
\]
Differentiating these last two expressions with respect to \( n \) and using (A3) and (A4) we find that
\[
\frac{\partial z_u}{\partial n} = \frac{\partial z_l}{\partial n} = \frac{\xi \cdot f'(n)}{\gamma(1-\theta \gamma c)} < 0. \tag{A5}
\]
The last inequality follows since we have \( \xi < f'(n) \) in the region of limit cycle motion.

It follows from (A5) that
\[ \frac{\partial}{\partial n} [z_u - z_l] = 0. \tag{A6} \]

Noting from equation (19e) that
\[ \frac{\partial n}{\partial M} = \frac{1}{\alpha_2}, \]

it follows from (A5) that
\[ \frac{\partial z_u}{\partial M} = \frac{\partial z_l}{\partial M} = \frac{-\xi' f'(n))}{\gamma_2 \gamma (1 - \theta \gamma r_w)}. \tag{A7} \]