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DOES THE PROCESS OF SPATIAL AGGREGATION OF U.K. UNEMPLOYMENT RATE SERIES SERVE TO INDUCE OR REMOVE EVIDENCE OF ASYMMETRY IN THE BUSINESS CYCLE?

by

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ABSTRACT

Asymmetry in the business cycle has been recognised as a nonlinear phenomenon by a number of recent studies which have examined unemployment rate series in North America, Europe and Australia. While many of these studies hypothesise that linear modelling techniques are inadequate for modelling aggregate unemployment rate series, other studies provide evidence to the contrary. Further conflicting results emerge from studies which attempt to answer the questions as to whether nonlinearities in the disaggregated data are driving nonlinear structure in the aggregated series or, whether nonlinearities are being masked by the disaggregation process?

Using U.K. aggregate and regional unemployment rate data, this study seeks an answer to these questions. Analysing the dynamics of the appropriate best-fitting linear and nonlinear models indicates whether nonlinearities are present in the aggregate series and whether the process of aggregation across spatial market series serves to induce or remove any evidence of nonlinearity in the aggregate series.

INTRODUCTION

The recent study of Peat and Stevenson (1996) provides empirical evidence of business cycle asymmetry using data drawn from the Australian labour market.[1] While they find evidence of cyclical asymmetry in the aggregate unemployment rate series using a specific test for asymmetry as well as tests based on time series techniques, they only find similar evidence in one of the six regional unemployment rate series studied. Their results for aggregate unemployment rate data provides empirical validation for those theoretical business cycle models, like the four-regime Keynesian type model of Day and Lin (1992), which can produce asymmetric cycles as a result of nonlinear or regime-switching specification.

Linear time series models with symmetrically distributed random shocks cannot generate output with asymmetric fluctuations. However, nonlinear or piecewise-linear models are capable of producing complex outcomes like limit cycles, jumps and discontinuities which are typical of economic behaviour. As a consequence, asymmetry in the business cycle has been recognised as a nonlinear phenomenon by recent studies using postwar unemployment rate series. For the U.S., Rothman (1991) was the first to identify asymmetric evidence within a Markov chain context. Strong evidence of nonlinear behaviour was found by Luukkonen and Teräsvirta (1991), as well as by Teräsvirta and Anderson (1992), when they tested thirteen OECD unemployment rate series for linearity against nonlinear alternatives which included the smooth threshold autoregressive (STAR) class. Using U.S. monthly aggregate and sectoral unemployment rates, Ham and Sayers (1990) found strong evidence of nonlinearity of the self-exciting threshold autoregressive (SETAR) type, while Rothman (1992) used an exponential autoregressive (EAR) model, as introduced by Ozaki (1980), to reveal a better fitting model to the aggregate data than models from the ARIMA (p,d,q) class.

Not all studies find departure from linearity in favour of a nonlinear model for either or both the aggregate or spatially disaggregated unemployment rate series. For the U.K., Jones, Manning and Stevenson (1994) find that while they cannot reject a linear autoregressive model in favour of a two-regime SETAR model as the best-fitting model for the aggregate unemployment rate series, the opposite was the case for most of the ten regional unemployment rate series fitted. Evidence of no departure from linearity in either

1 Reference to further recent papers concerned with business cycle asymmetry can be found in the introduction section of Acemoglu and Scott (1994).

the aggregate or spatially disaggregated unemployment rate series has been found by Frank and Stengos (1988) and Frank, Sayers and Stengos (1989) for Canadian data, while Frank, Stengos and Sayers[2] find similar results for U.S. disaggregated (state) unemployment.

Best-fitting nonlinear models for the aggregate unemployment rate series in Australia and the U.S. support the notion of an asymmetrical business cycle. However, while a nonlinear model was the best-fitting model for the aggregate series, it was not always the case for the spatially disaggregated data. Alternatively, a linear aggregate model and nonlinear spatially disaggregated models were the best-fitting varieties for the U.K. unemployment rate series. The superiority of the linear model over a nonlinear alternative for the U.K. may have been influenced by the fact that only one nonlinear alternative (SETAR) was tested against the linear model. What needs to be determined is whether the linear model for the U.K. aggregate unemployment rate series is superior to a number of nonlinear alternatives.

Using U.K. aggregate and regional unemployment rate series, this paper explores asymmetry between ascending and descending phases of the business cycle by using statistical methods designed to test directly for asymmetry as well as to distinguish between linear and nonlinear processes. As a consequence, further evidence will be advanced as to the adequacy of linear modelling techniques for the aggregate unemployment rate series. Further, this study will hopefully provide answers to the questions of whether nonlinearities in the disaggregated data are driving nonlinear structure in the aggregated series or, whether nonlinearities in the aggregate are being masked by the disaggregation process.

A description of the data used in our study is given in the following section. Section 3 involves a discussion of the testing procedures used for determining evidence of asymmetric behaviour, while in Section 4 we are concerned with reporting our test results. In Section 5 we model each of the time series of United Kingdom aggregate and regional unemployment rates with the appropriate models indicated by our test results for nonlinearity. Further, we analyse the residuals for independent and identically (i.i.d.) behaviour, and comment on the ability of the various models to generate cyclical behaviour. Section 6 contains our concluding remarks.

[2] Cited in Ham and Sayers (1990)

2. DATA

The data used in this study are seasonally adjusted United Kingdom aggregate unemployment rates from 1971(1) through to 1994(8), and regional unemployment rates from 1974(4) to 1994(8)[3].

Stationarity of the data is a requirement for both our testing procedure and the fitting of linear models. A unit root in the data introduces difficulties in the use of standard statistical techniques designed for these purposes. In this study we applied two tests for the presence of a unit root in our data.

The first of the stationarity tests used was the robust, classical Augmented Dickey-Fuller (1981) test. This test controls for remaining serial correlation in the residuals by ensuring they are white noise.[4] The second test was proposed by Sims (1988) who used Bayesian methods to show that the classical tests have low power and, as such, give excessive weight to the unit root hypothesis.[5] He proposed a prior distribution which assigns a probability of (1-α) to the presence of a unit root. We calculate the marginal prior probability of the unit root null, (1-α*), such that on an expost basis we force the Sims criterion to accept the null hypothesis.[6]

Table 1 contains the results of our stationarity tests. For all seasonally-adjusted series, using the Augmented Dickey-Fuller (ADF) test we accept the null of a unit root. Using the Sims test, we note that the prior of a unit root is never greater than 0.01 and, as with the ADF test, we accept the null of a unit root.[7] After first-differencing, we rejected the unit root for

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[3] The data were obtained from the National On-Line Manpower Information System (NOMIS) located at the University of Durham.

[4] To determine the lag length required to remove any remaining serial correlation we subjectively chose an artifically long lag length and sequentially reduced the lag until the Durbin-h statistic revealed significant autocorrelation. The lag length prior to this lag then became the appropriate lag length required to remove further serial correlation.

[5] Low power indicates an inability of the classical tests to reject a false null hypothesis, the consequence being an inability to distinguish between a unit root and near unit root process.

[6] A value of (1-α*) near one implies the sample information dictates the need for a large prior weight on the unit root null in order to force the Sims criterion to favour a unit root. A value near zero implies favouring the hypothesis of the existence of a unit root.

[7] With both the ADF and Sims tests the presence of a unit root is tested using three models comprising a trend and drift term (Model 1), a drift term only (Model 2) and neither trend nor drift terms (Model 3). On a sequential basis starting with Model 1 we continue testing until we establish evidence of no presence of a unit root at which point we stop. If no such evidence is encountered we conclude the existence of a unit root.

Table 1 Stationarity Test Results For The Raw and First-Differenced, Seasonally-Adjusted Monthly United Kingdom Aggregate And Regional Unemployment Rate Data

Region	Series	n	Model	Augmented Dickey Fuller Statistic	(p-value)	Lag to Remove Serial Correlation	Sims Test (1-α*)
Great Britain	GB	260	3	0.209	(0.881)	22	0.006
	DGB	260	2	-3.286	(0.016)	21	1.000
East Anglia	EA	240	3	-0.546	(0.566)	3	0.010
	DEA	240	3	-2.753	(0.006)	2	1.000
East Midlands	EM	237	3	-0.468	(0.609)	6	0.007
	DEM	237	3	-2.394	(0.016)	5	1.000
North	NT	238	3	-0.286	(0.704)	5	0.006
	DNT	238	3	-2.358	(0.017)	4	1.000
North West	NW	221	3	-0.167	(0.757)	22	0.007
	DNW	221	3	-2.398	(0.016)	21	1.000
Scotland	SC	238	3	-0.291	(0.701)	5	0.006
	DSC	238	3	-2.243	(0.023)	4	1.000
South East	SE	221	3	0.290	(0.899)	22	0.006
	DSE	221	2	-3.364	(0.013)	21	0.999
South West	SW	239	3	0.799	(0.421)	4	0.009
	DSW	239	3	-2.264	(0.022)	3	1.000
Wales	WA	219	3	-0.048	(0.804)	24	0.009
	DWA	219	2	-2.902	(0.046)	23	1.000
West	WM	221	3	-0.127	(0.774)	22	0.010
Midlands	DWM	221	3	-2.655	(0.008)	21	0.999
Yorkshire and	YH	221	3	-0.053	(0.802)	22	0.007
Humberside	DYH	221	3	-2.567	(0.010)	21	1.000

Key: n = number of observations
D** = the first difference of the ** series

all series (denoted by D as the first letter in the identifier) using both tests. In the remainder of the paper we worked with the first-difference of the seasonally-adjusted series.

3. TESTS FOR ASYMMETRY OF UNITED KINGDOM UNEMPLOYMENT RATES.

The testing procedure we use to study possible asymmetry of the first-difference of the seasonally-adjusted aggregate and regional United Kingdom unemployment rates comprises a specific test for the presence of asymmetries based on skewness statistics as well as tests based on time series techniques. In the latter case, the hypothesis of linearity is tested against specific or general forms of nonlinearity, most which are capable of representing processes which can generate asymmetric cycles.

3.1 A Specific Test Based on Skewness Statistics

DeLong and Summers (1986) test for asymmetry of a time-series by testing the null hypothesis of zero skewness. If m_2 and m_3 are the second and third centred moments of a stationary time series, then the test statistic is given by

$$SK = \frac{m_3}{\sqrt{m_2^3}}$$

Under the null hypothesis, the estimate of the skewness coefficient, $\hat{SK}$, is normally distributed.[8]

3.2 Tests Based on Time Series Techniques

Following Luukkonen and Teräsvirta (1991), assume that a white noise process$\{e_t\}$ and a time series $\{Y_t\}$ are observed at t = 0,1,2,....

Suppose the following relationship holds:

$$h(Y_t, Y_{t-1}, Y_{t-2}, \ldots) = e_t \qquad (1)$$

[8] We followed the Monte Carlo approach of DeLong and Summers (1986). Firstly, we fitted an AR model to our data to filter out serial correlation. We then calculated the $S\hat{K}$ coefficient from the fitted values. the variance of the SK coefficient was found by simulation whereby $\hat{\sigma}_{SK}$ was calculated from 300 simulated series using the fitted AR model. The z-score of $\hat{SK}$ was calculated in the usual way with $z = \dfrac{S\hat{K} - 0}{\hat{\sigma}_{SK} / \sqrt{n}}$.

where h is a given function. If the process is stationary and ergodic, then the relationship can be approximated by a linear autoregressive model if

$$\sum_{j=0}^{p} h_j Y_{t-j} = e_t, \quad p < \infty \tag{2}$$

and the roots of $\sum_{j=0}^{p} h_j Z^j = 0$ lie outside the unit circle.

As Mittnik and Niu (1993) point out, if the linear model given by (2) is to be retained and asymmetric cycles are to be accounted for, then a non-symmetric error distribution is necessary. If we want the error distribution to be symmetric (possibly normal), and our model to generate asymmetric cycles, then (2) is no longer appropriate. They categorise the sources of asymmetric behaviour by viewing the variable as output from a stochastic dynamic system as depicted in Figure 1.

<u>Figure 1</u> <u>Components of a Stochastic Dynamic System</u>

Noise/Shocks $\rightarrow$ [Economic System/ Transmission Mechanism] $\rightarrow$ Observed Variables

e_t + $g(e_{t-1}, e_{t-2}, ...)$ = Y_t

If the system is represented by

$$Y_t = g(e_{t-1}, e_{t-2}, \ldots) + e_t,$$

where Y_t denotes the variable of interest, e_t is an innovation or noise-input sequence and the function g represents the economic system or transmission mechanism, then asymmetry of economic and financial data, Y_t, can be viewed as being caused by different combinations of the transmission mechanism, g, and input noise, e_t. The four categories that explain the symmetrical nature of the observed variables from the above dynamic system are summarised in Table 2 below.

<u>Table 2</u> <u>Explanations for Asymmetry of Time Series Output</u>

Category	Noise Input	Transmission Mechanism	Output
1	Symmetric	Linear	Symmetric
2	Asymmetric	Linear	Asymmetric
3	Symmetric	Nonlinear	Asymmetric
4	Asymmetric	Nonlinear	Asymmetric

Asymmetries in time series output can result from either a linear or nonlinear transmission mechanism with either symmetrically or asymmetrically innovations. Our testing procedure involves determination of the superiority of models from the nonlinear autoregressive class as opposed to the linear alternative for modelling the first-differenced aggregate and regional U.K. unemployment rate series. Conclusions from our nonlinear time series tests provide the first stage in determining from which of the above categories the most suitable model is likely to come. The second stage comprises tests of the residuals from the fitted models which provide evidence of the symmetrical nature of the noise and, therefore, a clue to the appropriate functional form of the underlying stochastic dynamic system.

The tests for which we report results include tests of the linear model given by (2) against various alternative specifications of h in (1). While the possible set of alternative specifications is large, we focus on a sufficiently small subset of nonlinear autoregressive models comprising the exponential autoregressive (EAR), exponential smooth threshold autoregressive (ESTAR), logistic smooth threshold autoregressive (LSTAR), self-exciting threshold autoregressive (SETAR), and autoregressive conditionally heteroskedastic (ARCH) models.[9]

Our tests against specific nonlinear specifications can be classified as to whether the data is arranged or not; which is tantamount to spotting threshold nonlinearity versus some other form.

Recently, Peat and Stevenson (1995) developed tests of nonlinearity of the EAR and STAR types using the idea of regressing predictive residuals against AR and "nuisance

[9] Another popular nonlinear alternative to the linear model, which is often reported in the literature, is the bilinear model. A major shortcoming of the bilinear model for use in this study is its inability to capture certain nonlinear characteristics like limit-cycle behaviour. As we are interested in identifying cyclical behaviour in our models, the bilinear model was seen as inappropriate.

parameters" but when the data is unarranged. The number of parameters included generally coincided with the order of the AR model which was determined by minimisation using the Akaike Information (AIC). EAR, and STAR nonlinearity tests were reserved for data that is ordered in the time domain; that is, a time series.

Both the Tsay (1989) and (1991) tests use the idea of an arranged autoregression to tranform a threshold autoregressive (TAR) model into a regular change-point regression problem. A recursive estimation procedure[10] is then followed which results in the sequential estimation of a linear model for a fixed-length, but moving window, across the arranged data set. An indication of threshold nonlinearity results if stability of each of the coefficients over part of the data set is then followed by a smooth transition to another stable set. The Tsay (1989) test (TAR-F) involves regressing the predictive residuals on the AR model used in the recursive estimation procedure, with an F-statistic being formed using the new residuals as well as the predictive residuals. The statistical significance of this statistic indicated SETAR type nonlinearity. A further test of threshold autoregression employed in this study was the Chan-Tong Test [see Chan (1990)]. Detection of threshold nonlinearity is based on a likelihood ratio test that compares the performance of a linear autoregressive model of order p against a nonlinear alternative The class of threshold models deployed as the nonlinear alternative are piece-wise linear approximations to the general nonlinear model and include the SETAR specification.

A further test for detecting a specific form of nonlinearity used in this study is the Luukkonen et al. (1988a) S_3 test. It is a general test for STAR nonlinearity. It does not differentiate between the two types of STAR nonlinearity, namely, ESTAR and LSTAR.[11]

As a test for ARCH effects we subjected the residuals from the linear AR(p) models of all series to the McLeod-Li test. McLeod-Li (1983) proposed a portmanteau statistic based on squared residuals that has potential use in detecting nonlinearity, possibility in the direction of bilinearity. Its asymptotic null distribution is χ_m^2 if the true innovations are independent. m, the order of the autocorrelation function, is generally taken to be the square root of the sample size. Luukkonen et al. (1988b) suggest that the McLeod-Li test has power when testing against ARCH-type alternatives.

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[10] For a full discussion of "arranged autoregression" and recursive or "local estimation" see Cao and Tsay (1992).

[11] See Teräsvirta (1994) for details of the ESTAR and LSTAR specifications.

Finally, we use the BDS test of Brock, Dechert and Schienkman (1987) to test the null hypothesis that the variable of interest is independent and identically distributed (i.i.d.). Not only are the residuals of the various linear and nonlinear models tested for i.i.d., but so are the first-differenced, seasonally adjusted series themselves. The latter can be seen as residuals after the original series are passed through a first-differenced filter. As the BDS is a powerful test for nonlinear dependence we include it as a general nonlinearity test.

4. ASYMMETRY TEST RESULTS

4.1 Test Based on Skewness Statistics

Table 3 contains the results of the DeLong and Summers (1986) test for asymmetry. For the first-differenced, seasonally-adjusted aggregate and regional United Kingdom unemployment rate series we note that $S\hat{K}$, the estimate of SK, is significantly different at the 5% level of significance for all series except DNT and DSC. With the exception of these two series we conclude that asymmetry is present in all of the series.

These results are consistent with those of Pfann (1992) who employed the DeLong and Summers asymmetry test when examining quarterly U.S. employment series which included aggregate employment and the disaggregated employment series for white males, white females, nonwhite males, nonwhite females, professionals and non-farm labourers. While the evidence was not uniform over all series, he found overall evidence in support of the presence of asymmetry in the first-differenced time series. That result is similar to the result reported here in Column 3 of Table 3.

Table 3 Results of the Asymmetry Test Based on the Skewness Statistic for the First-Differenced, Seasonally-Adjusted Aggregate and Regional United Kingdom Unemployment Rate Series.

Series	$\hat{SK}$	Var(SK)	Z
DGB	0.320	0.320	9.799**
DEA	0.320	6.448	2.183**
DEM	0.339	5.242.	2.567**
DNT	0.289	9.947	1.585
DNW	0.442	0.473	11.119**
DSC	0.021	5.972	0.149
DSE	0.477	2.952	4.811**
DSW	0.233	0.610	5.166**
DWA	0.557	9.983	3.054**
DWM	0.632	3.664	5.719**
DYH	0.371	4.545	3.011**

4.2 Tests Based on Time Series Techniques

In Table 4 we report the results for the BDS test run on the transformed aggregate and regional series. We saw the transformed series as residuals after the original seasonally-adjusted series are passed through a first-difference filter. If the first-difference filter is an appropriate model for the original series then the transformed series should be i.i.d. For the first-differenced series there is statistically significant evidence (at the 1% level of significance) against the hypothesis of i.i.d. behaviour. Quantiles of the BDS statistic for a finite sample of 250, based on 5000 replications of normal random variables, are given in Table 5.

In order to provide a benchmark for comparing nonlinear autoregressive models, a linear autoregressive model of order p was fitted to all the series. The maximum order of the autoregressive process, p, draws on the information criterion proposed by Akaike (1974) for the identification of parsimonious models.[12] The chosen linear AR(p) models for the first-

[12] See Tong (1990) for an excellent treatment of the derivation of Akaike's Information Criterion, defined as

AIC(m) = -2 ln(maximised likelihood) + 2(no. of independently adjusted parameters) and where, for our purposes, m is the number of lags included in the autoregressive model. This reference also contains important cautionary remarks on the use of the AIC for preliminary model identification.
Enders (1995) provides an alternative formulation for the AIC.
It is calculated as:

AIC(m) = Tln (residual sum of squares) + 2n

where n = number of parameters estimated, and
T = number of usable observations

The normalised criteria reported in Table 6, namely MAIC1 and MAIC2, are both AIC measures divided by T.

Table 4 BDS Statistics For First-Differenced Aggregate And Regional U.K. Unemployment Rate Series

Series	β	m			
		2	3	4	5
DGB	1.0	29.76	34.91	40.09	46.68
	1.3	36.09	47.34	61.80	82.54
DEA	1.0	21.25	26.29	30.64	35.57
	1.3	27.07	36.48	47.49	62.16
DEM	1.0	17.00	20.58	23.56	26.34
	1.3	19.83	26.17	33.43	42.09
DNT	1.0	14.77	17.02	18.46	20.28
	1.3	16.01	19.24	21.75	25.18
DNW	1.0	21.78	27.30	32.19	38.17
	1.3	22.95	31.00	8.21	49.71
DSC	1.0	19.76	22.98	26.88	30.88
	1.3	20.00	25.01	31.86	42.09
DSE	1.0	28.67	34.38	40.64	48.98
	1.3	34.11	47.24	66.77	97.27
DSW	1.0	22.89	26.68	30.66	34.77
	1.3	30.36	39.31	53.18	71.15
DWA	1.0	16.50	19.62	21.69	23.73
	1.3	16.90	20.72	23.52	26.59
DWM	1.0	21.63	26.42	30.99	36.73
	1.3	33.56	33.56	45.91	64.62
DYH	1.0	23.98	23.98	26.97	30.35
	1.3	27.50	27.50	32.64	40.25

Note: m = embedding dimension; e is a measure of variability of the series and
e = $\left(\frac{\hat{\sigma}}{r}\right)^{\beta} r$, where $\hat{\sigma}$ is the estimated standard deviation and r is the spread.

Table 5 Quantiles of the BDS Statistic

Normal Random Variables Based on 5000 Replications
[Source: Brock, Hsieh and LeBaron (1991), Appendix C]

β	n	m				m			
		2	3	4	5	2	3	4	5
		Upper 5%				Lower 5%			
1.0	250	1.86	1.91	1.98	2.10	-1.88	-1.87	-1.89	-1.89
1.3	250	2.35	2.59	3.02	3.88	-2.24	-2.47	-2.87	-3.49
		Upper 1%				Lower 1%			
1.0	250	2.79	2.92	2.96	3.06	-2.52	-2.47	-2.50	-2.52
1.3	250	3.71	4.04	4.85	6.44	-3.05	-3.38	-4.06	-4.86

n = number of observations
m = embedding dimension
e = measure of variability of the series
$\left(\frac{\hat{\sigma}}{r}\right)^{\beta} r$, where $\hat{\sigma}$ is the estimated standard deviation and r is the spread

difference of the aggregate and regional unemployment rates for the U.K. are given in Table 6. AR(p) models were fitted as they form the basis of likelihood ratio statistics which are used to test for autoregressive nonlinearity. An underlying assumption of the likelihood ratio test is the existence of white noise residuals in the AR(p) process. As a consequence,
we include for each series the value of the Ljung-Box Q statistic which is a test statistic for white noise when compared with the chi-square distribution.

From the Ljung-Box Q statistics (L-B) in Table 6, we conclude that white noise in the residuals cannot be rejected at 5% level of significance for all series. Their autocorrelation functions were also characterised by a lack of obvious remaining serial correlation.

Table 6 Summary Statistics For Linear AR(p) Models For The First-Differenced, Seasonally-Adjusted Monthly Aggregate And Regional United Kingdom Unemployment Rates

Series	n	p	MAIC1	MAIC2	L-B(df)	M-L(df)	L-M
DGB	280	3	-2.72700	-5.56488	19.07(17)	22.19(17)	-0.20
DEA	241	3	-2.21804	-5.05592	9.07(16)	61.92(16)**	1.68*
DEM	241	3	-2.23776	-5.07564	23.52(16	22.20(16)	0.93
DNT	239	5	-1.70236	-4.54023	12.28(16)	36.94(16)**	1.16
DNW	237	7	-2.37234	-5.21022	16.53(16)	13.19(16)	-0.18
DSC	241	3	-2.27561	-5.11349	16.08(16)	13.23(16)	-0.51
DSE	241	3	-2.77321	-5.61109	11.91(16)	25.75(16)	0.72
DSW	240	4	-2.10729	-4.94516	7.82(16)	26.30(16)	0.95
DWA	242	2	-1.81053	-4.64841	14.40(16)	24.16(16)	2.13**
DWM	242	2	-2.21692	-5.05479	17.55(16)	13.10(16)	0.40
DYH	241	3	-2.27155	-5.10943	23.96(16)	23.03(16)	0.32

Key: n = Number of Observations
p = Autoregressive Order; AR(p)
MAIC1 = Minimum AIC1
MAIC2 = Minimum AIC2
L-B = Ljung-Box Q Statistic For Residuals $\begin{cases} \sim \chi^2(17;0.05) = 27.59 \\ \sim \chi^2(17;0.01) = 31.99 \\ \sim \chi^2(16;0.05) = 26.30 \\ \sim \chi^2(16;0.01) = 33.41 \end{cases}$
M-L = McLeod-Li Statistics For Residuals $\sim\chi^2$ as for L-B
L-M = Lin-Mudholkar Statistic For Residuals [distributed ~N(0,1)]
* = Significance at the 0.05 level
** = Significance at the 0.01 level

Our test for ARCH effects was confined to the McLeod-Li test. Significance is noted for two regional series DEA and DNT. Apart from evidence of conditional heteroscedasticity, this statistical significance could be indicating a possible bilinear specification.[13] All other series appear clean of ARCH effects.

Uncorrelatedness plus normality imply independence. The Lin-Mudholkar test is a test of the normal distribution of a data series and is based on the Gaussian assumption of independence of the sample mean and variance. The Lin-Mudholkar (L-M) statistic will be asymptotically standard normal under the null hypothesis that the residuals from the AR(p) models are Gaussian. From Table 6 we observe that two series, namely DEA and DWA, are non-Gaussian according to the L-M test at the 5% level of significance and the 1% level respectively. The residuals from all the remaining autoregressive models are Gaussian white noise according to the L-M and L-B tests and, as such, conform to i.i.d. behaviour. A separate test of the i.i.d. nature of the residuals from our AR(p) models is the BDS test. Table 7 contains the BDS statistics for the residuals from linear AR(p) models fitted to the first-differenced, seasonally adjusted monthly aggregate and regional U.K. unemployment rate series. Using the quantiles for the BDS statistic in Table 4, the residuals for all series except DEA appear to be i.i.d., including those of DWA whose residuals were previously found to be non-Gaussian by the L-M test. We conclude that only the residuals from the linear autoregressive model fitted to DEA are non i.i.d.

Linearity of the transmission mechanism along with symmetry of the residuals (as implied by their Gaussian nature) should result in symmetrical output. Recall that this scenario was depicted as Category 1 in Table 2. Category 2 which had both asymmetric noise and output also allowed for a linear transmission mechanism. From the DeLong and Summers asymmetry test results reported in Table 3, along with the i.i.d. or non i.i.d. nature of their residuals, only the DEA, DNT and DSC series emerge as candidates to be modelled by a linear autoregressive process. In the case of the DEA series, its output and residuals are both asymmetric so it could fall into either Category 2 or Category 4. DNT and DSC have both symmetric output and residuals and therefore could be modelled by a linear transmission mechanism
(i.e. Category 1).

[13] The work of Weiss(1986) argued that bilinearity may be misspecified as ARCH.

Table 7 BDS Statistics For Residuals From Linear AR(p) Models Fitted To The First-Differenced, Seasonally-Adjusted Monthly Aggregate And Regional U.K. Unemployment Rate Series.

Series	β	m			
		2	3	4	5
DGB	1.0	1.92	1.99	1.89	2.01
	1.3	1.80	1.71	1.04	1.03
DEA	1.0	3.70	4.04	3.60	2.93
	1.3	4.46	4.44	3.83	3.03
DEM	1.0	1.38	1.78	0.91	0.41
	1.3	0.85	1.46	0.46	-0.31
DNT	1.0	0.97	0.50	0.68	0.64
	1.3	0.48	-0.11	-0.06	-0.03
DNW	1.0	0.02	0.44	0.49	0.43
	1.3	0.07	0.09	0.14	0.46
DSC	1.0	1.85	1.64	2.16	2.62
	1.3	1.50	1.49	1.13	0.89
DSE	1.0	2.02	2.81	2.74	2.85
	1.3	1.58	2.11	2.27	2.38
DSW	1.0	2.01	2.67	2.77	2.48
	1.3	1.54	2.55	2.15	2.02
DWA	1.0	1.45	1.38	1.36	1.29
	1.3	1.38	0.92	1.21	1.08
DWM	1.0	2.48	2.22	1.74	0.85
	1.3	2.37	1.97	1.26	0.05
DYH	1.0	2.58	1.70	1.36	0.99
	1.3	2.01	1.23	0.61	-0.19

Note: m = embedding dimension; e is a measure of variability of the series, and
e = $\left(\frac{\hat{\sigma}}{r}\right)^{\beta} r$, where $\hat{\sigma}$ is the estimated standard deviation and r is the spread

With symmetric residuals (noise) and asymmetric output, all the other series should be candidates to be modelled by a nonlinear transmission mechanism. As a further tests of this proposition we resort to our tests based on time series techniques, along with the Chan-Tong threshold test. Table 8 contains the results of the time series based tests with significance being judged to be a p-value less than 0.10. Table 9 contains the results of the Chan-Tong test.

For the aggregate series, DGB, our battery of time series tests seem to indicate potential STAR and SETAR nonlinearity. STAR nonlinearity is suggested by the S_3 and CN tests but not by the Peat-Stevenson (1995) ESTAR and LSTAR tests. The CN tests incorporates the cumulative normal distribution function as the smooth ramping function which transfers the data between different regimes. Unlike the ESTAR and LSTAR tests it does not require a scaling parameter, γ, to be input to the test.[14] The scaling parameter will affect the EAR test also. It would seem we have used an inappropriate value for γ in the case of the EAR and ESTAR tests (i.e. one divided by the variance of the lagged series) and for the LSTAR test (i.e. one divided by the standard deviation of lagged series). Even though these scaling factors are suggested by Teräsvirta (1994), a more complete study of the appropriate values to use in the Peat-Stevenson (1995) tests needs further consideration. Teräsvirta (1994) claims that the EAR model belongs to the more general STAR class of models. In the light of the above discussion, we concluded that the EAR model could be a candidate among potential nonlinear specifications for the transformed aggregate series. The Chan-Tong test (Table 9) does not suggest SETAR nonlinearity for the aggregate series.

A similar picture emerged for the regional series with potential EAR, ESTAR, LSTAR and SETAR nonlinearity suggested by the time series tests for all series excepting DNT and DYH. Again, the Chan-Tong tests suggested against the SETAR model for all the regional series.

On the balance of evidence suggested by our battery of tests for asymmetrical behaviour, we chose to fit EAR, ESTAR, LSTAR and SETAR models to the aggregate and regional transformed U.K. unemployment series.

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[14] For the formulae for EAR, ESTAR and LSTAR models and the role played by the scaling parameter, γ, see equations (3), (4) and (5) in Section 5 of the paper.

Table 8 Summary of Significance Test Results (p-values) Of Nonlinearity Tests

Region	p	EAR	S3	CN	ESTAR	LSTAR	SETAR
DGB	1	0.535	0.023	0.072	0.154	0.824	0.055
	2	0.516	0.187	0.190	0.219	0.724	0.203
	3	0.615	0.221	0.300	0.262	0.772	0.395
DEA	1	0.397	0.018	0.097	0.409	0.698	0.003
	2	0.200	0.343	0.443	0.804	0.359	0.127
	3	0.543	0.692	0.559	0.673	0.709	0.170
DEM	1	0.431	0.037	0.144	0.118	0.734	0.057
	2	0.712	0.221	0.290	0.215	0.878	0.209
	3	0.727	0.273	0.422	0.339	0.860	0.468
DNT	1	0.679	0.460	0.587	0.613	0.918	0.304
	2	0.771	0.933	0.937	0.894	0.914	0.684
	3	0.781	0.803	0.861	0.807	0.897	0.817
	4	0.840	0.413	0.715	0.683	0.922	0.631
	5	0.589	0.210	0.234	0.306	0.713	0.343
DNW	1	0.966	0.815	0.908	0.959	0.999	0.587
	2	0.296	0.903	0.855	0.963	0.488	0.916
	3	0.606	0.993	0.912	0.969	0.765	0.881
	4	0.755	0.974	0.754	0.808	0.863	0.703
	5	0.395	0.211	0.008	0.014	0.522	0.042
	6	0.536	0.242	0.019	0.030	0.653	0.032
	7	0.425	0.511	0.066	0.104	0.533	0.098
DSC	1	0.139	0.577	0.377	0.913	0.335	0.710
	2	0.004	0.758	0.446	0.847	0.011	0.919
	3	0.034	0.471	0.766	0.961	0.070	0.939
DSE	1	0.716	0.000	0.001	0.350	0.936	0.005
	2	0.662	0.003	0.020	0.492	0.843	0.016
	3	0.773	0.027	0.077	0.367	0.891	0.062
DSW	1	0.163	0.015	0.045	0.773	0.378	0.027
	2	0.301	0.014	0.059	0.107	0.494	0.240
	3	0.651	0.099	0.030	0.036	0.802	0.062
	4	0.638	0.274	0.052	0.048	0.771	0.079
DWA	1	0.030	0.000	0.001	0.000	0.095	0.001
	2	0.060	0.028	0.072	0.060	0.131	0.094
DWM	1	0.787	0.028	0.078	0.333	0.964	0.051
	2	0.679	0.495	0.433	0.739	0.856	0.362
DYH	1	0.475	0.193	0.315	0.280	0.775	0.264
	2	0.735	0.874	0.711	0.662	0.893	0.668
	3	0.943	0.767	0.599	0.576	0.983	0.498

Table 9 Likelihood Ratio Statistic (Chan-Tong Test) for SETAR Models Of First-Differenced Seasonally-Adjusted , Monthly Aggregate And Regional United Kingdom Unemployment Rates

Series	d	λ_d	Series	d	λ_d	Series	d	λ_d
DGB	1	6.51	DNW	1	14.68	DSW	1	16.61
(p=3)	2	10.93	(p=7)	2	8.28	(p=4)	2	9.66
	3	7.16		3	11.57		3	12.41
DEA	1	4.81		4	13.91		4	5.27
(p=3)	2	8.22		5	14.45	DWA	1	5.30
	3	10.16		6	12.15	(p=2)	2	8.23
DEM	1	7.98		7	14.34	DWM	1	4.73
(p=3)	2	5.57	DSC	1	9.20	(p=2)	2	6.36
	3	11.74	(p=3)	2	11.77	DYH	1	5.29
DNT	1	10.16		3	9.31	(p=3)	2	5.37
(p=5)	2	10.03	DSE	1	7.86		3	8.26
	3	17.36	(p=3)	2	9.28			
	4	13.49		3	8.58			
	5	13.74						

λ (5%) and λ(1%) are approximate upper percentage points for the Likelihood Ratio Tests for SETAR models as detailed in Chan (1990)

p	λ(5%)	λ(1%)
2	13.38	17.60
3	15.42	19.83
4	17.33	21.93
5	19.16	23.93
7	22.65	27.71

5. DYNAMICS OF THE AGGREGATE AND REGIONAL UNITED KINGDOM UNEMPLOYMENT RATES

We fitted nonlinear models of the types suggested by our testing procedures to the transformed United Kingdom unemployment rate series. The benchmark for comparing the various nonlinear models fitted to the different series were the corresponding linear autoregressive models, the summary statistics of which were given in Table 6.

Nonlinear models of the EAR and STAR types were fitted to the aggregate and most of the regional series. Equations (3), (4) and (5) depict the EAR, LSTAR and ESTAR specifications.[15]

EAR: $$Y_t = \sum_{j=1}^{p}[\alpha_j + \beta_j \exp(-\gamma Y_{t-1}^2 / \sigma_Y^2)]Y_{t-j} + \varepsilon_t \tag{3}$$

where $\varepsilon_t \sim$ NID $(0, \sigma_\varepsilon^2)$and $\gamma > 0$,

LSTAR: $$Y_t = \sum_{j=1}^{p}(\alpha_j + \beta_j[1 + \exp\{-\gamma(Y_{t-1} - c)/\sigma_y\}]^{-1})Y_{t-j} + \varepsilon_t \tag{4}$$

where $\varepsilon_t \sim$ NID $(0, \sigma_\varepsilon^2)$and $\gamma > 0$,

ESTAR: $$Y_t = \sum_{j=1}^{p}(\alpha_j + \beta_j[1 - \exp\{-\gamma(Y_{t-1} - c)^2 / \sigma_Y^2\}]Y_{t-j} + \varepsilon_t \tag{5}$$

where $\varepsilon_t \sim$ NID $(0,\sigma_\varepsilon^2)$ and $\gamma > 0$.

5.1 Best-Fitting Models

Table 10 contains the summary statistics for the best-fitting models.

For the aggregate series, DGB, the EAR model with lag 3 resulted in the best fit on the basis of Akaike's criterion (MAIC). This model was superior to the linear autoregressive model as

[15] With no drift term (i.e. constant term) present in any of our best-fitting AR(p) models, we elected to exclude them from the nonlinear alternative.

Table 10 Summary Statistics For Best-Fitting Models For The First-Differenced, Seasonally-Adjusted Monthly Aggregate And Regional United Kingdom Unemployment Rates

Series	Model	n	p	MAIC1	MAIC2	L-B(d.f.)	M-L(d.f.)	L-M
DGB	EAR	280	3	-2.73731	5.57519	18.01(17)	18.50(17)	-0.47
DEA	LINEAR	241	3	-2.20859	-5.04647	9.06(16)	61.92(16)**	1.68*
DEM	LSTAR	241	3	-2.23858	-5.07646	24.66(16)	20.52(16)	1.07
DNT	LINEAR	239	5	-1.69316	-4.53104	12.28(16)	36.94(16)**	1.16
DNW	LSTAR	237	7	-2.38734	-5.22522	22.74(16)	13.29(16)	-0.63
DSC	EAR	241	3	-2.30286	-5.14074	14.72(16)	13.48(16)	-0.53
DSE	ESTAR	241	3	-2.77954	5.61741	10.56(16)	19.10(16)	0.45
DSW	LSTAR	240	4	-2.13178	-4.96966	6.96(16)	25.42(16)	0.80
DWA	ESTAR	242	2	-1.82058	-4.65845	12.34(16)	13.08(16)	2.15**
DWM	LINEAR	242	2	-2.21110	-5.04898	17.55(16)	13.10(16)	0.40
DYH	EAR	241	3	-2.30045	-5.13832	32.19(16)*	28.77(16)	-0.55

Key:
n = Number of Observations
p = Autoregressive Order; AR(p)
MAIC1 = Minimum AIC1
MAIC2 = Minimum AIC2

L-B = Ljung-Box Q Statistic For Residuals $\begin{cases} \sim \chi^2(17;0.05) = 27.59 \\ \sim \chi^2(17;0.01) = 31.99 \\ \sim \chi^2(16;0.05) = 26.30 \\ \sim \chi^2(16;0.01) = 33.41 \end{cases}$

M-L = McLeod-Li Statistic For Residuals $\sim\chi^2$ as for L-B
L-M = Lin-Mudholkar Statistic For Residuals [~N(0,1)]
* = Significance at the 0.05 level
** = Significance at the 0.01 level

judged by the MAIC.[16]. From Table 11 of the BDS statistics for residuals from the best-fitting models, it is clear that the residuals from the EAR model for the aggregate series are i.i.d. This is confirmed by the statistically insignificant Ljung-Box (L-B) and Lin-Mudholkar (L-M) statistics in Table 10. According to the categories for explain asymmetry of time series output given in Table 2, we conclude that the aggregate series, DGB, belongs to category 3 where the transmission mechanism is best modelled by a nonlinear function.

For the regional series were found a combination of nonlinear and linear models to be the best-fitting.

According to a combination of the DeLong and Summers asymmetry test (Table 3), the best-fitting transmission mechanism (Table 10), and the BDS test of i.i.d. residuals from the best-fitting models (Table 11), the DEM, DNW, DSE, DSW, DWA and DYH series all appear to fit Category 3 in Table 2.

We concluded that DSC most likely belonged to category 3 on the basis of a best-fitting nonlinear model and i.i.d. residuals. The fact that the output from the DSC series was judged to be symmetric by the DeLong and Summers test and, as such, contradicts the notion of a nonlinear transmission mechanism and i.i.d. residuals, may be explained by the economy of Scotland. Both geographically and economically it is a large, diverse region. Some of its traditional heavy industry, like ship building, are in decline and contributing to increased unemployment. On the other hand, industries like tourism and oil are in a growth phase and are adding to employment, albeit mainly outside the heavy industrial centres. At a regional level it could be that unemployment levels in the different industries are out of phase with each other and making the aggregate data more symmetric. This explanation is consistent with the study of Neftci and McNevin (1986) who found that while major capital goods industries for the U.S. exhibit significant asymmetry, individual capital goods series are likely to be out of phase with one another making the aggregate data more symmetric.

[16] We are mindful of Tong's reservations of using the MAIC as a means of comparison between classes of nonlinearity [see Tong (1990), page 288]. However, given that we are using the same method of estimation for both the linear and nonlinear models, based on the same algorithm, we assume the approximate AIC which results from using the conditional rather than exact likelihood function is relatively uniform throughout and across the classes.

Table 11 U.K. BDS Statistics For Residuals From the Best-Fitting Models From the First-Differenced, Seasonally -Adjusted Monthly Aggregate And Regional Unemployment Rate Series

Series	β	m			
		2	3	4	5
DGB	1.0	1.48	1.47	1.37	1.51
	1.3	1.46	0.73	-0.11	0.73
DEA	1.0	3.70	4.04	3.60	2.93
	1.3	4.46	4.44	3.83	3.03
DEM	1.0	0.94	1.52	0.73	0.14
	1.3	0.48	1.34	0.36	-0.36
DNT	1.0	0.97	0.50	0.68	0.64
	1.3	0.48	-0.11	-0.06	-0.03
DNW	1.0	0.33	0.30	0.12	-0.20
	1.3	0.21	-0.12	0.07	0.38
DSC	1.0	1.11	1.12	1.74	1.92
	1.3	1.78	1.58	1.13	0.60
DSE	1.0	1.52	2.35	2.46	2.62
	1.3	0.98	1.26	1.59	0.98
DSW	1.0	1.23	1.51	1.68	1.81
	1.3	1.39	1.77	2.27	1.61
DWA	1.0	0.60	0.60	0.48	0.51
	1.3	0.37	-0.06	0.27	0.29
DWM	1.0	2.48	2.22	1.74	0.85
	1.3	2.37	1.97	1.26	0.05
DYH	1.0	1.57	0.62	0.33	0.21
	1.3	1.69	0.88	0.47	1.79

Note: m = embedding dimension; e is a measure of variability of the series, and
e = $\left(\frac{\hat{\sigma}}{r}\right)^{\beta} r$, where $\hat{\sigma}$ is the estimated standard deviation and r is the spread

The DNT series appears to belong to Category 1. It has symmetric output, a linear transmission mechanism and i.i.d. residuals. However, there is evidence of ARCH effects in the residuals from the linear model according to the McLeod-Li (M-L) statistics. We also concluded that DWM belonged to Category 1 on the basis of its linear transmission mechanism and i.i.d. residuals. We do not have a suitable explanation as to why the transformed unemployment rate was asymmetrical according to the DeLong and Summers test. The economy of the West Midlands would appear to be similar to that of the Northern region with a steadily declining heavy industrial base. Why apparent asymmetry, and therefore nonlinearities in the regional unemployment rate, should be present is puzzling.

In the case of the DEA series, category 2 seems the most likely candidate to explain the asymmetry in the series. A linear transmission mechanism is the best-fitting model and, according to the BDS statistics, the residuals appear to be non i.i.d. As pointed out by Mittnik and Nui (1993) models from category 2 are being more widely considered in recent times in business cycle research.

5.2 Dynamics of the Aggregate U.K. Unemployment Rate

For the aggregate data, based on the criteria of minimising the AIC, the exponential autoregressive (EAR) model provided a better goodness-of-fit than either the logistic smooth transition (LSTAR), or exponential smooth transition (ESTAR) varieties of the STAR class of models. This result is consistent with the findings of Rothman (1992) and Peat and Stevenson (1996) who found that an EAR model was the best fit for aggregate U.S. and Australian unemployment rate series respectively.

Given that the first-difference of the seasonally-adjusted aggregate U.K. unemployment rate series can be represented by an EAR model, we are in a position to determine whether this model gives rise to cyclical behaviour. Priestley (1988) outlines a sufficient condition for the existence of a limit cycle in an EAR model which is a function of the parameters of the model.[17] For our 3rd order EAR model,

[17] Given the pth-order EAR model given by:

$$Y_t = \sum_{j=1}^{p}[\alpha_j + \beta_j \exp(-\gamma Y_{t-1}^2 / \sigma_y^2)]Y_{t-j} + \varepsilon_t$$

then a sufficient condition for the existence of a limit cycle is

$$\hat{Y}_t = 0.427Y_{t-1} + 0.246Y_{t-2} + [0.256 - 0.382\exp(-\gamma Y_{t-1}^2)]Y_{t-3}$$

where $\gamma = 876.332 > 0$,

we have

$$\frac{1-\sum_i \alpha_i}{\sum_i \beta_i} = \frac{0.071}{-0.384} = -0.185 < 0$$

and the sufficient condition for limit cycle behaviour holds.

As suggested in the introduction, the recovery from asymmetrical data of an empirical time series model which exhibits limit cycle behaviour has important implications for the validation of theoretical business cycle models.

5.3 Spatial Aggregation

Of the ten transformed regional U.K. unemployment rate series that we modelled, a nonlinear structure was the best indicated model for seven cases. Of the three remaining cases, while a linear autoregressive model appeared the best transmission mechanism, in one case (DNT) there remained significant ARCH effects in the residuals which could indicate bilinear nonlinearity, while in another (DEA) the residuals were non i.i.d. In the remaining case (DWM) a linear function and i.i.d. residuals were inconsistent with asymmetrical output. A straightforward linear autoregressive model dies not appear to be a satisfactory structure on which to model the disaggregated U.K. unemployment rate series.

It appears to us that nonlinearities in the disaggregated data are driving nonlinearity in the aggregate. This is a far stronger result than that of Peat and Stevenson (1996). While they concluded that nonlinearities in the aggregate were being masked by the disaggregation process, evidence for that proposition was not conclusive. In their study one large region was modelled by a nonlinear process and the two remaining large populated regions were a borderline call as to whether they were best modelled by a linear or nonlinear process.

6. CONCLUSIONS

$(1-\sum_i \alpha_i)/\sum_i \beta_i > 1$ or < 0

In this study we tested for asymmetry in the business cycle by testing time series of the aggregate and regional United Kingdom unemployment rates. As well as applying a direct test for asymmetry we embraced the notion that if a business cycle is to be explained, then its behaviour should be related to a nonlinear process. Asymmetrical patterns in a time series are not unlike the patterns generated by nonlinear structures. All series were tested for nonlinear autoregressive structure which were drawn from the EAR, STAR and SETAR classes. These classes are all capable of models which can produce cyclical behaviour - a feature of empirical time series models which is part of the validation process of nonlinear theoretical business cycle models. Nonlinear models were fitted to the data where appropriate. Table 12 summarised our general findings.

Table 12 Summary of General Findings

Series	Asymmetry Test	Nonlinearity Test	Best-Fitting Model
Great Britain	Asymmetric	Nonlinear	Nonlinear
East Anglia	Asymmetric	Linear	Linear
East Midlands	Asymmetric	Nonlinear	Nonlinear
North	Symmetric	Linear	Linear
North West	Asymmetric	Nonlinear	Nonlinear
Scotland	Symmetric	Nonlinear	Nonlinear
South East	Asymmetric	Nonlinear	Nonlinear
South West	Asymmetric	Nonlinear	Nonlinear
Wales	Asymmetric	Nonlinear	Nonlinear
West Midlands	Asymmetric	Linear	Linear
Yorkshire and Humberside	Asymmetric	Nonlinear	Nonlinear

From the last column of Table 12 we observe that nonlinearities are present in the aggregate series and, further, that the process of aggregation across spatial market series serves to induce rather than remove evidence of nonlinearity in the aggregate series.

REFERENCES

Acemoglu, D. and Scott, A. (1994), “Asymmetries in the Cyclical Behaviour of UK Labour Markets”, The Economic Journal, 104, November, 1303-1323.

Akaike, H. (1974), “A New Look at Statistical Model Identification,” IEEE Transactions on Automatic Control, AC-19(6), December, 716-723.

Brock, W.A., Hsieh, D. and LeBaron, B. (1991), Nonlinear Dynamics, Chaos and Instability, MIT Press, Cambridge, MA.

Brock, W.A., Dechert, W.D., and Scheinkman, J.A. (1987), “A Test For Independence Based on the Correlation Dimension,” Working Paper, Department of Economics, University of Chicago.

Cao, D.Q. and Tsay, R.S. (1992), “Nonlinear Time Series Analysis of Stock Volatilities,” Journal of Applied Econometrics, 7, S165-S185.

Chan, K.S. (1990), “Percentage Points of Likelihood Ratio Tests for Threshold Autoregression,” Technical Report No 272, Department of Statistics, University of Chicago.

Day, R.H. and Lin, T.Y. (1992), “A Keynesian Business Cycle”, in Nicholas Kaldor and Mainstream Economics, Nell, E.J. and Semmler, W. (eds), St. Martin’s Press, New York.

DeLong, J.B. and Summers, L.H. (1986), “Are Business Cycles Symmetrical?”, in Gordon, R.J. (ed.), The American Business Cycle, Chicago: NBER and University of Chicago Press.

Dickey, D.A. and Fuller, W.A. (1981), “Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root,” Econometrica, 49, 1057-1072.

Enders, W. (1995), Applied Econometric Time Series, Wiley.

Frank, M. and Stengos, T. (1988), “Some Evidence Concerning Macroeconomic Chaos,” Journal of Monetary Economics, 22, 423-428.

Frank, M., Sayers, C.L. and Stengos, T. (1989), “Evidence Concerning Nonlinear Structure in Provincial Unemployment Rates,” Universities of British Columbia, Houston and Guelph.

Ham, M.R. and Sayers, C.L. (1990), “Testing for Nonlinearities in United States Unemployment by Sector,” Department of Economics, University of Virginia, Charlottesville, and University of Houston, Houston.

Jones, D., Manning, N., and Stevenson, M. (1994), "The Unemployment Rate and the Business Cycle in Britain: An Aggregate and Regional Analysis," Regional Studies, Vol. 28, No. 6, 591-604.

Luukkonen, R., Saikkonen, P. and Teräsvirta, T. (1988a), "Testing Linearity Against Smooth Transition Autoregressive Models," Biometrika, 75, 491-499.

Luukkonen, R., Saikkonen, P. and Teräsvirta, T. (1988b), "Testing Linearity in Univariate Time Series Models," Scandinavian Journal of Statistics, 15, 161-175.

Luukkonen, R., and Teräsvirta, T. (1991), "Testing Linearity of Economic Time Series Against Cyclical Asymmetry," Annales d'Economie et de Statistique, 10/21, 125-142.

McLeod, A. and Li, W. (1983), "Diagnostic Checking ARMA Models Using Squared-Residual Autocorrelations," Journal of Time Series Analysis, 4, 269-273.

Mittnik, S. and Niu, Z. (1993), "Asymmetries in Business Cycles: Econometric Techniques and Empirical Evidence," Department of Economics, State University of New York at Stony Brook, May.

Neftci, S. and McNevin, B. (1986), "Some Evidence on the Nonlinearity of Economic Time Series: 1890-1981," Research Report No. 86-26, C.V. Starr Centre, New York University.

Ozaki, T. (1980), "Nonlinear Time Series Models for Nonlinear Random Vibrations," Journal of Applied Probability, 17, 84-93.

Peat, M. and Stevenson, M. (1995), "Testing for Nonlinearities in Economic and Financial Time Series," Working Paper No. 48, School of Finance and Economics, University of Technology, Sydney.

Peat, M. and Stevenson, M. (1996), "Asymmetry in the Business Cycle: Evidence From the Australian Labour Market," forthcoming Journal of Economic Behavior and Organization, 30:3, August, 1996.

Pfann, G.A. (1992), "Employment and Business Cycle Asymmetries: A Data Based Study," unpublished paper, University of Limbury, The Netherlands.

Priestley, M.B. (1988), Nonlinear and Nonstationary Time Series Analysis, Academic Press.

Rothman, P. (1991), "Further Evidence on the Asymmetric Behaviour of Unemployment Rates Over the Business Cycle," Journal of Macroeconomics, 13, 291-298.

Rothman, P. (1992), "Forecasting Asymmetric Unemployment Rates," Working Paper No. 92-03, Department of Economics, University of Delaware.

Sims, C.A. (1988), "Baynesian Skepticism on Unit Root Econometrics," Journal of Economic Dynamics and Control, 12, 463-474.

Teräsvirta, T. and Anderson, H. (1992), "Characterizing Nonlinearities in Business Cycles Using Smooth Transition Autoregressive Models," Journal of Applied Econometrics, 7, S119-S136.

Teräsvirta, T. (1994), "Specification Estimation, and Evaluation of Smooth Transition Autoregressive Models," Journal of the American Statistical Association, Vol. 89, No. 425, March, 1994, 208-218.

Tong, H. (1990), Nonlinear Time Series: A Dynamical System Approach, Clarendon Press, Oxford.

Tsay, R.S. (1989), "Testing and Modelling Threshold Autoregressive Processes," Journal of the American Statistical Association, 84, 231-240.

Tsay, R.S. (1991), "Detecting and Modelling Nonlinearity in Univariate Time Series Analysis," Statistical Sinica, 1, 431-451.

Weiss, A.A. (1986), "ARCH and Bilinear Time Series Models: Comparison and Combination," Journal of Business and Economic Statistics, January, 4(1), 59-70.