Confidence Intervals for the Underground Economy in Australia

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Abstract

Economists have proposed several different techniques for measuring the underground economy but the literature gives only point estimates not standard errors. This makes it difficult to see if different estimates are contained in one confidence band. This paper provides an approach to estimating confidence intervals for estimates of illicit economic behaviour. The approach is adaptable to existing econometric techniques and is likely to offer a better way to assess the reliability of point estimates of the underground economy than subjective opinion.

Keywords: underground economy, bootstrap, currency demand, taxes, welfare benefits.

* I would like to thank Ross Milbourne, Glenn Otto and Graham Voss for their comments and suggestions. Naturally, any errors and omissions are my own.
I. Introduction

Recently there have been a number of attempts to estimate the size of the underground economy in many countries. The underground economy is defined as unmeasured economic activity that has contributed to value added according to the national accounting convention, but which is not captured in official statistics because of the failure to report income in whole or in part. This means that activities that are not usually measured by the national accounting convention will not form part of the measure of the underground economy\(^1\). This is important because the existence of a non-negligible underground economy may distort the size of important economic variables, such as the tax base and the growth of GDP, which are most often used to gauge the functioning of the economy. Further still, a non-negligible and volatile underground economy may have significant implications for the business cycle [Bajada (1999)].

Estimates of the underground economy can vary widely even within one country and have often given cause for concern about their reliability. For example in the United States, estimates of the 'true' extent of illicit behaviour have varied between 8 and 33\% of GNP. Similarly for Sweden the estimates ranged between 7\% and 17\% of GNP, while for Germany between 16\% and 24\% of GNP [Frey and Pommerehne (1984)].

As opposed to the most standard econometric analysis, the underground economy literature gives only point estimates not standard errors. This makes it difficult to see if different estimates are contained within one confidence band. Part of the reason for this is that many of the methodologies used to estimate the size of the underground economy are not based in statistical or distribution theory. For example many are often based on voluntary surveys and samples [see Pestiau (1983), Dilnot and Morris (1981) and Censis (1976)], non-voluntary tax auditing [see Kinsey (1987), Simon and Witte (1982) and

\(^1\) For a complete and comprehensive taxonomic framework which distinguishes between various concepts of underground activity see Feige (1985).
OECD (1980), and labour market statistics [see Langfelt (1989), Myrsten (1989) and Frey and Pommerehne (1984)].

This paper provides the first known approach to estimating confidence intervals for the estimates of the underground economy. It applies a bootstrap methodology to Bajada's (1999) model of the underground economy. The paper is organized as follows. Section II describes the time series model of Bajada which is used as the basis for this paper. Section III then discusses the method of estimation of the confidence intervals and in Section IV a modified approach is used to construct the appropriate confidence intervals for the underground economy in Australia. Section V presents our results and Section VI our major conclusions.

II. The Underground Economy in Australia

Among the many estimates of the underground economy abroad, Bajada (1999) offered a time series estimate of the underground economy in Australia for the period 1967 to 1996. The estimates are based on an excess sensitivity approach which suggest that transactions in the underground economy are undertaken in cash and are motivated principally by individuals’ attempts to either avoid the payment of taxes or to attempt to prevent the loss of any government assistance. Using a variant of the monetary approaches of Cagan (1958), Gutmann (1977), Feige (1979) and Tanzi (1980, 1983), Bajada estimates the size of the Australian underground economy to be much larger than was previously expected - 10% of GDP [CBA (1980)]. In fact for the period 1967 to 1996 the underground economy in Australia on average measured approximately 15% of GDP, clearly a significant part of overall economic activity.

Motivated by the desire to avoid the payment of taxes or prevent the loss of any government assistance, participants in the underground economy expressly request cash
when receiving payment to minimize their chances of being detected. Appropriately the demand for currency is modelled in order to estimate the size of these subterranean activities. By convention the demand for real money balances depends on the level of income and interest rates. Since taxation and welfare benefits affect currency through disposable income, the demand for real money balances is modified to depend on disposable income. Taxes and welfare benefits are introduced as additional variables to capture any excess sensitivity on currency. By measuring these excess sensitive components it is possible to quantify the size of this illicit economic behaviour.

The demand for currency was also found to depend on a number of additional variables namely, inflation, consumption expenditure as a percentage of GDP and technological development. For example, rising rates of inflation erode the value of money and induce individuals to hold less while a growing consumption/income ratio, a proxy for consumption behaviour, is expected to add to the stock of currency. Improvements in technology, which have changed the means by which we pay for many goods and services, is expected to reduce the use of currency.

Modelling the demand for currency using an error correction model, and calculating the stock of currency in the absence of the excess sensitivity of taxes and welfare benefits, produced an estimate of legal currency. The discrepancy between this measure and the total stock of currency defined the amount of illegal currency which when multiplied by the velocity of currency produced estimates of the size of the underground economy. These point estimates using quarterly data for the period 1966.4 to 1996.2 \((n = 119)\) were calculated using the following expression. (A full description of the data is given in Appendix A).

\[
U\hat{G}_t = V_t \times \left[ \exp(X_t \beta + \ln(C)_{t-1} + \Delta \ln(P)_t + \Delta \ln(N)_t) \times (\exp(T, \alpha + W_t \delta) - 1) \right]
\] (1)
where

\[ C = \text{currency}; \quad P = \text{GDP price deflator}; \quad N = \text{population}; \quad V = \text{Velocity of circulation of currency in the official economy}; \]

\[ X = \text{a 119 \times 14 matrix of explanatory variables namely intercept, } \Delta \ln(YD), \Delta \ln(E), \Delta \ln(R), \Delta \ln(\pi), \ln(YD_{t-1}), \ln(E_{t-1}), \ln(\pi_{t-1}), \ln(R_{t-1}), \text{ D2, D3, D4, Tr, and } \ln(C_{t-1}); \]

\[ \beta = \text{a 14 \times 1 vector of coefficients corresponding to the explanatory variables in } X; \]

\[ T = \text{a 119 \times 2 matrix of two tax variables, namely } \Delta \ln(T) \text{ and } \ln(T_{t-1}); \]

\[ \alpha = \text{a 2 \times 1 vector of coefficients corresponding to the tax variables in } T; \]

\[ W = \text{a 119 \times 1 vector of } \Delta \ln(W); \text{ and } \]

\[ \delta = \text{the coefficient on } \Delta \ln(W); \]

However an interval estimate, in contradistinction with a point estimate, takes into account the possibility that a sample estimate may differ from its true value because of sampling fluctuations. Since each point estimate generated by (1) has variability around its mean and subject to sampling fluctuations, we cannot be certain that the unknown mean size of the underground economy is adequately represented by these point estimates. An assessment on the robustness of these estimates can be determined by constructing intervals for the underground economy at each data point. We construct a 95\% confidence interval from which we can be 95\% confident that the unknown mean size of the underground economy lies between the constructed intervals. However to do this we require the sampling distribution of (1) and this is not known because of its unique calculation. It is possible however to approximate this distribution using a bootstrap procedure.
III. Method of Bootstrapping

The idea behind the bootstrap technique is as follows [see Veall (1988) and Horowitz (1997)]. Suppose a currency demand equation is given by

\[ \ln C = \Psi \rho + \epsilon \]  

where \( \Psi \) is a \((n \times m)\) matrix of all the explanatory variables and intercept, \( \rho \) is a \((m \times 1)\) vector of all coefficients and \( \epsilon \), an \((n \times 1)\) vector of white noise residuals.

The OLS estimated equation is given by

\[ \ln(\hat{C}) = \Psi \hat{\rho} + e \]  

where \( \hat{\rho} = (\Psi'\Psi)^{-1}\Psi'\ln(C) \)

The random error terms are assumed to have come from an unknown distribution with a mean of zero and a variance of \( \sigma^2 \). The result yields a vector of estimated residuals \((e_1, e_2, ..., e_n)\). Suppose that \( F(\epsilon) \) takes any distribution. These values of the disturbance terms generate a vector of values for \( \ln C(\ln C_1, \ln C_2, ..., \ln C_n) \) where

\[ \ln C_t = f(\Psi_t; \rho) + \epsilon_t \]

The distribution will depend on the distribution function of \( F(\epsilon) \) but this in unknown since we do not know the distribution of the \( \epsilon \) terms. We are therefore prevented from getting a small sampling distribution for both \( \hat{\rho} \) and \( \ln(\hat{C}) \). We can overcome this problem by using the bootstrap method because it allows the replacement of \( F(\epsilon) \) by \( F(\epsilon) \). The estimated residuals then become,
\begin{equation}
e_i = \ln C_i - f(Y_i;\hat{\rho})
\end{equation}

From this vector of estimated residuals a sample of size \( n \) is drawn with replacement, so that from this vector of estimated residuals, \( n \) residuals are randomly selected and a second vector of residuals is created. However each random selection is followed by replacement so that if \( e_1 \) is selected, it is replaced and has a probability of being selected again of \( 1/n \). In the extreme case, it is possible to have constructed a vector of estimated residuals from the original vector made up of a single estimate repeated \( n \) times, however the probability of this is very small. This second vector, \( e^* \) of re-sampled residuals, are substituted into equation (2) and a new vector of estimated \( \ln C^i \) is generated holding of course the original estimate of \( \rho \) constant.

\begin{equation}
\ln C^i = \Psi \hat{\rho} + e^*
\end{equation}

The vector \( \Psi \) remains unaltered throughout this procedure. Given the newly generated \( \ln(C) \) we regress \( \ln C^i = \Psi \hat{\rho} + \epsilon \) in order to re-estimate \( \rho \). The number of estimates of \( \rho \) will equal the number of replications that one performs. The process is repeated a large number of times and an empirical distribution of \( \hat{\rho} \) is obtained which will serve as a proxy for the true distribution of \( \hat{\rho} \). This distribution can be used to calculate a confidence interval for \( \rho \).
IV. A Modified Bootstrap

Because of the nature of our estimation we are concerned not with the empirical distribution of the estimated coefficients in the currency demand equation but rather the distribution of (1).

Consequently a slight modification of the standard bootstrap routine was undertaken and the general process in estimating the confidence intervals for the underground economy in Australia for each data point may be summarized in the following seven steps.

1. Using the error correction model defined by Bajada (1999) to estimate the size of the underground economy, as

\[ \Delta \ln(C) = X\beta + T\alpha + W\delta + \varepsilon \]  

(7)

where \( \varepsilon \) is white noise, we estimate equation (7) to obtain the estimates of \( \hat{\beta}, \hat{\alpha}, \hat{\delta} \) and residuals \( \hat{\varepsilon} \).

2. Sample randomly 119 white noise errors from 119 \( \hat{\varepsilon} \) with replacement. Denote these re-sampled errors by \( \hat{\hat{\varepsilon}} \).

3. Generate a new set of observations on the dependent variable by

\[ \Delta \ln(C_t') = X_t\hat{\beta} + T_t\hat{\alpha} + W_t\hat{\delta} + \hat{\hat{\varepsilon}}_t \]

(8)

However since \( \ln(C_{t-1}) \) is an independent variable in the regression, for each new set of observations on the dependent variable in (8), we solve for \( \ln(C_t') \) (using \( \ln(C_t) \) as the
first observation) which we use in the next period as \( \ln(C_{t-1}) \). This dynamic bootstrap ensures consistent estimates of \( \Delta \ln(\hat{C}_t) \).

4. Regress \( \Delta \ln(\hat{C}_t) \) on \( X_t, T_t, \) and \( W_t \) to obtain a new set of coefficients \( \hat{\beta}, \hat{\alpha}, \hat{\delta} \) and residuals \( \hat{\varepsilon} \).

5. Obtain the predictor of \( \Delta \ln(\hat{C}_t) \) based on these estimates as

\[
\Delta \ln(\hat{C}_t) = X_t \hat{\beta} + T_t \hat{\alpha} + W_t \hat{\delta} + \hat{\varepsilon},
\]

(9)

6. Estimate the volume of illegal currency \(^2\) \( (\bar{H}_t) \) defined as the difference between the existing levels of currency holdings \( (\bar{C}_t) \) and a level estimated to be held when there exists no underground economy \( (\bar{C}_{wt}) \) - calculated as the holdings of currency in the absence of the excess sensitivity of taxes and welfare benefits.

\[
\bar{H}_t = \bar{C}_t - \bar{C}_{wt} = \exp(\Omega_t) - \exp(\Omega_t - T_t \hat{\alpha} - W_t \hat{\delta})
\]

(10)

where \( \Omega_t = \ln(\bar{C}_t) + \ln(C_{t-1}) + \Delta \ln(P_t) + \Delta \ln(N_t) \)

\(^2\) Bajada (1999) defines illegal currency as the sum of currency in the hands of agents engaging in underground activity.
7. Finally estimate the size of the underground economy by

\[ UG_i = \hat{H}_i \times \hat{V}_i \]

\[ = \hat{V} \times \left( \exp \left( X_i \hat{\beta} + \ln(C)_{t-1} + \Delta \ln(P)_i + \Delta \ln(N)_i \right) \times \left( \exp \left( T_i \hat{\alpha} + W_i \hat{\delta} \right) - 1 \right) \right) \] (11)

We repeat steps (1) to (7), 10,000 times for each quarter in order to generate an empirical distribution of the predictor \( UG_i \). By sorting out the 10,000 replications of \( UG_i \) in ascending order, we obtain 2.5 and 97.5 percentiles which we will use to construct the 95% confidence interval for our point estimates of the underground economy in Australia.

V. Results

In Table 1 we report the lower (2.5 percentile) and upper (97.5 percentile) interval estimates constructed for the underground economy using the bootstrap procedure outlined in the previous section. The intervals are expressed both as a percentage of GDP [columns 4 and 6] and in millions of dollars, expressed in 1989/90 prices [columns 1 and 3]. The point estimates are given in columns 2 and 5 of the same table. The empirical frequency distribution of the average bootstrapped estimates of the underground economy are given in Figure 1\(^3\). This distribution appears normal so it is acceptable to take the lower 2.5 percentile and upper 97.5 percentile to construct the intervals. In Figure 2 we plot the interval estimates as a percentage of GDP for the entire sample period.

The band width of the 95% confidence intervals is only 2.8% of GDP. The narrow band width appears to reaffirm the claim by Bajada (1999) that the point estimates of the size of the underground economy in Australia are relatively robust. These confidence intervals

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\(^3\) The single period distributions have very similar skewness and kurtosis as that represented by the average which is plotted in Figure 1.
can be interpreted as follows - we can say that we are 95% confident that the unknown mean size of the underground economy lies between the constructed intervals. For example, in 1967 we can say that we are 95% confident that the unknown population mean size of the underground economy to be between 12.68% and 15.27% of GDP. Equivalently the size of the underground economy is expected to measure between $20.2 billion and $24.3 billion. Similarly for 1985, we can say that we are 95% confident that the unknown population mean size of the underground economy lies between 13.77% and 16.63% of GDP. As recently as 1995, the interval is 13.97% to 16.86% of GDP or equivalently between $58.6 billion and $70.7 billion.

The Cash Economy Task Force (CETF), established in November 1996 by the Australian Commissioner of Taxation following a growing community perception that the underground economy was manifesting itself among a growing number of participants [ATO (1998)], was required to undertake an extensive investigation of the underground (or cash) economy. The CETF did not provide numerical estimates of its size but speculated that it represented approximately 10% of GDP. However this estimate falls well outside the confidence bands and is unlikely to represent the true unknown mean size of the illicit activities in Australia. In fact it may be acceptable to denote the lower confidence interval as a lower bound estimate and the upper confidence interval as an upper bound estimate of the size of subterranean activities in Australia.

\[4\] This guesstimate is based on an earlier estimate of the size of the underground economy [CBA (1980)].
VI. Conclusion

The foregoing analysis has sought to demonstrate that the excess sensitivity approach to estimating the size of the underground economy in Australia is quite reliable in light of the fact that the constructed intervals are quite close to the point estimates previously estimated by Bajada (1999). The band width was found to be on average less than 2.8% of GDP. However constructing interval estimates not only substantiate the robustness of the point estimates, they also allow us to say that we are confident with some degree of certainty that the unknown mean size of the underground economy lies between these constructed intervals. Our findings also suggest that earlier estimates of the underground economy lie well outside the confidence bands. It is unlikely that these earlier estimates adequately represent the unknown mean size of the underground economy in Australia.

This confidence interval approach is directly applicable to existing methodologies used to estimate the size of the underground economy elsewhere and is likely to offer a better way to assess the reliability of point estimates. It is strongly recommended that interval estimates be used along side existing point estimates to judge their reliability. It is quite possible that the various estimates of the underground economy in the United States are unreliable because their confidence intervals are very wide.
Table 1 - *Interval Estimates of the Underground Economy*  

<table>
<thead>
<tr>
<th>Date</th>
<th>Lower Interval ($m)</th>
<th>Point Estimate ($m)</th>
<th>Upper Interval ($m)</th>
<th>Lower Interval (% of GDP)</th>
<th>Point Estimate (% of GDP)</th>
<th>Upper Estimate (% of GDP)</th>
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Notes:  
(a) Based on Dec-31 each year.  
(b) March and June quarters 1996 are quarterly estimates only.
Figure 1 - Empirical Frequency Distribution of Bootstrapped Estimates of the Underground Economy (% of GDP).

Figure 2 - Upper and Lower Bound Estimates of the Underground Economy (% of GDP)
Appendix A

Australian Data (Seasonally unadjusted values in millions of Australian dollars)


T Average Taxes, calculated as Direct Taxes on Incomes, current prices, (Source: ABS National Accounts: Income and Outlay Account - Commonwealth General Government - Table 5204-55) expressed as a percentage of GDP(I) (defined below).

W Personal Benefit Payments, current prices. (Source: ABS Time Series Statistics: General Government Income and Outlay Account - Table 5206-33) as percentage of YD.

YD Disposable Income calculated as GDP(I) (income based and at current prices; Source: ABS Time Series Statistics: Domestic Production Account - Table 5206-23) less Direct Taxes on Incomes (as defined in T above) plus Personal Benefit Payments.

E Private Final Consumption Expenditure (current prices, Source: ABS Time Series Statistics: Household Income and Outlay Account - Table 5206-29) expressed as a percentage of GDP measured at current prices.


π Inflation rate calculated from the Implicit GDP Price Deflator. (see below)

L Population ('000). Source ABS NIF-10s Model Database: Labour Market -Table 1342-21.

P Price Deflator calculated as the ratio of GDP (expenditure based, current prices; Source: ABS Time Series Statistics: Domestics and Production Account - Table 5206-23) and GDP (expenditure based, 1989/90 prices; Source: ABS Time Series Statistics: Measures of GDP - Table 5206-1).

Tr Linear Trend

D(x) Seasonal Dummies (x = 2, 3, 4);

$$D_2 = \begin{cases} 1 & \text{for second quarter} \\ 0 & \text{otherwise} \end{cases}; \quad D_3 = \begin{cases} 1 & \text{for third quarter} \\ 0 & \text{otherwise} \end{cases}; \quad D_4 = \begin{cases} 1 & \text{for fourth quarter} \\ 0 & \text{otherwise} \end{cases}$$
References


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