Towards Applied Disequilibrium Growth Theory: I

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Towards Applied Disequilibrium Growth Theory:
I. The starting model.*

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Abstract

In this paper we build a hierarchically structured continuous-time model of Keynesian monetary growth. The model is sufficiently rich with respect to markets, sectors and agents and consistent with respect to budget constraints to capture sufficient broad details of actual macroeconomies and so serve as a macrotheoretic basis for many large scale macroeconometric models. We describe the model at the level of national accounts and then derive its extensive form dynamics. We also give detailed discussion as to how our model is related to the large scale Murphy model of the Australian economy. Our model provides a basis for understanding the various economic feedback chains contained in such models and their dynamic interaction.

*We have to thank G. Groh and C. Köper for helpful discussions on the building blocks of the model of this paper, see also our joint paper Chiarella, Flaschel, Groh Köper and Semmler (1999a). There, the approach of the present paper is extended to allow in particular for smooth input and output substitution. There is also a book manuscript in preparation, see Chiarella, Flaschel and Semmler (1999), which will place our theoretical approach to macroeconometric model building into a broader perspective.
1 Introduction

In this paper we extend the hierarchically structured continuous-time models of Keynesian monetary growth, introduced and generalized in already many respects in Chiarella and Flaschel (1999a, Chs 4-7, 1999b,c), Chiarella, Flaschel, Groh and Semmler (1999, Chs 4-6), Chiarella, Flaschel, Franke and Lux (1999, part III) for closed as well as open economies, in the direction of the macroeconometric Murphy model of the Australian economy. The resulting modeling framework leads towards an empirically motivated modeling of a small open economy with a Keynesian short- and medium-run and with classical or monetarist features in the medium- as well as the long-run behavior of the economy. The Murphy model (see Powell and Murphy (1997) for its detailed description) and our theoretical reformulation of it in this paper, therefore blend demand and supply side approaches into an integrated and coherent whole with a (from a theoretical point of view) very detailed description of the structure of a small open economy, like Australia (in the case of the Murphy model).

The present paper, however, approaches this task still from the macrotheoretic perspective developed in Chiarella and Flaschel (1999a) and Chiarella, Flaschel, Groh and Semmler (1999, Chs 4-6) and thus mirrors the approach chosen in Powell and Murphy (1997) only to a certain degree. In an outlook we will sketch what may be the consequences of the present approach for macroeconometric model building. In subsequent papers, see the references section, we shall provide more details on the dynamic structure and analysis of the present model (its intensive form, steady state analysis, more or less complex attractors, transients, etc.). We shall also attempt to move closer to the structure of the Murphy model, by revising the equations of the model of this paper towards the inclusion of smooth factor substitution and other flexibilities, a more standard view of the money market and money supply (as contained in the Murphy model). We will continue to use a continuous time approach (without any discrete lags) so as to allow for a compact representation and analysis of the dynamics implied by the model.

In this paper we added to the structural form of the Murphy model a complete set of fully specified budget equations for all sectors and thereby take account of all feedback structures implied by such budget restrictions. Furthermore all equations are specified in a consistent way from the perspective of dimensional analysis and are nevertheless at first chosen as linear as possible. This allows for a discussion of intrinsic (natural) nonlinearities before attempts are made to design nonlinearities that may keep the economy a viable one should it depart too much from the steady state in its naturally nonlinear design. This in particular means that we shall start our theoretical reconsideration of the Murphy model on the basis of a fixed proportions technology which has the additional advantage that the rate of capacity utilization of the capital stock is easy to define and to analyze with regard to the economic consequences implied by under- or over-utilized capital.

This choice of starting point for the analysis of the dynamic properties of structural macroeconometric models of open economies thus allows us, on the one hand, to study the implications of its intrinsic or ‘natural’ nonlinearities first, as they derive from unavoidable growth rate formulations, products or quotients of state variables and the like. We shall see in this setup that viability or boundedness of the dynamics (and in particular convergence to the steady state) will often depend on the assumption of sufficiently low adjustment speeds for quantities, prices and expectations, while local and even global stability will normally get
lost if these adjustment parameters are chosen sufficiently high.

On the other hand, our approach allows us to introduce extrinsic nonlinearities into the assumed technological or behavioral relationships in a systematic way at a later stage, as a (theoretically reflected) response of the economy to the specific local instabilities observed in the working of the basic form of the model, often already known from partial dynamic macro-models. For example, a kink in the money wage Phillips curve (which reflects that money wages may rise quickly in the boom but will only fall slowly – if at all – in the depression) is often already sufficient to avoid the inflationary instability that derives in such models from the existence of so-called Mundell effects (an institutional nonlinearity much neglected in the theoretical and applied debate on Phillips-curves, inflation and stagflation).

We therefore will attempt her and in future work to proceed step by step to a detailed and systematic theoretical and numerical analysis of the dynamic features (steady states, attractors, transients, etc.) of complete and coherently formulated structural macroeconomic models, including those applied to actual economies, a theoretical discussion which so far has been basically lacking in the literature.\(^1\)

Similarities of the Murphy model with the theoretical work on hierarchically structured Keynesian monetary growth models of Chiarella and Flaschel (1999a), a project that was started in 1993 during a three month stay of one of the authors at the University of Technology, Sydney, became more and more apparent as this work progressed. Extending the working model of Chiarella and Flaschel (1999a, Ch.6) to the open economy, as in Chiarella, Flaschel, Franke and Lux (1999, part III), then provided the impetus for not only continuing with the hierarchical structure of Keynesian monetary growth models established in Chiarella and Flaschel (1999a) towards more and more elaborate versions of them. This impetus also became the starting point for a further project of developing fully integrated and coherent Keynesian models from the other side of the tunnel, namely from the structural forms of small (or large) open economies as they are used in macroeconomic model building. Here the Murphy model was of particular usefulness, due to its many similarities with the theoretical work of Chiarella and Flaschel (1999a), Chiarella, Flaschel, Groh and Semmler (1999, Ch.s 4-6), Chiarella, Flaschel, Franke and Lux (1999, part III), already observed above, but also due to its very detailed and thorough presentation and discussion in Powell and Murphy (1997).

We use the structural model of their book in a simplified as well as in a more complex way. We suppress many of the lags included in the Murphy model and also some secondary structural components (and also primary ones, like smooth factor substitution, for the time being). We write down for the model all sectoral identities or budget equations of agents and include all the feedbacks they imply (in particular for asset accumulation). Finally, we modify in the present paper (and in subsequent work) more or less the equations of the Murphy model in the light of the dynamic equations used in the main parts of Chiarella and Flaschel (1999a), Chiarella, Flaschel, Groh and Semmler (1999), Chiarella, Flaschel, Franke and Lux (1999). In this way we arrive at theoretical presentations of such structural macroeconometric models which at one and the same time intend to be descriptive (to a

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\(^1\)See however Barnett and He (1998) for an interesting approach to the analysis of applied macroeconometric models from the theoretical and the numerical perspective. We believe that this type of investigation (and more) is urgently needed with respect to applied macroeconometric model building.
certain extent) and theoretically consistent in the sense of dimensional analysis and of the budget equations the various agents of the model are facing.

In the basic version of the model, we have endogenous natural rates of growth, of Harrod neutral technical change and of NAIRU-employment, but exogenous output growth in the rest of the world. There is a detailed set of direct and indirect taxation schemes, including various types of wage taxation and payroll-taxes. We have two types of households, pure asset holders and workers, with differentiated saving habits, where the latter group only saves in the form of savings deposits (or fixed price bonds). Wage income (of three types of workers' households) is taxed at a different rate than interest income of asset holders (and of workers). Sluggish price dynamics, accompanied by Metzlerian quantity dynamics and varying degrees of capacity utilization of the capital stock characterizes the market for the non-traded domestic commodity, and we have of course also sluggish adjustment of money wages and varying employment rates on the labor market, augmented by insider and outsider considerations combined with a sluggish adjustment of the outside employment rate in view of the over- or underemployment of the employed workforce (the insiders). We have as a module of the model the market for housing services and consider investment in housing besides the investment of firms in fixed business capital, both depending on profitability and rates of capacity utilization. There is a detailed description of the government sector with respect to tax receipts and expenditures and – based on that and the pure equity financing of firms (still assumed in this paper) – also a full set of asset accumulation equations accompanied by asset price or interest rate dynamics (in the place of a full portfolio approach to asset markets). Pensions and unemployment benefits and their financing via payroll taxes are treated explicitly. There is no role for money in the model at present, but only an interest rate policy rule of the monetary authority which fixes the short-term rate of interest of domestically traded bonds in the light of certain measures of economic activity adopted by the central bank. There are finally exports of finished goods and imports of raw materials or semi-finished goods and there is (though still somewhat limited) international trade in financial assets, and a foreign exchange market that is always cleared by the actions of the private sector and the government despite the assumption of only a finite speed of adjustment of the rate of exchange.

This brief list of some structural components of the model of monetary growth of this paper indicates that it will include various elements that are of importance in the current discussion of the macroeconomic problems that governments have to face. It goes without saying, however, that there are still important components of a macroeconomy that are missing in the present theoretical reformulation of the Murphy model. The model of the present paper therefore only represents a first step into the direction of formulating an integrated model of monetary growth for an open economy, which, on the one hand, is related to empirical work and which, on the other hand, allows for a complete computation of the steady state values of the model as well as numerical and sometimes also theoretical analyses of the behavior of the dynamics around the steady state of such a high dimensional dynamical system.

In the next section we provide an overview on the structure of the real and the financial part of the model, characterizing the sectors, markets and activities that will be included into our model. Section 3 then presents this structure from the viewpoint of the system of national accounts and provides thereby a detailed introduction to the notation that is used in this paper. In section 4 we present the structural equations of the model (in their extensive
form) by way of an appropriate subdivision into important modules that build up the model. Section 5 finally gives an outlook on what to do with this model type when it is reduced from extensive form to per unit of capital expressions or intensive form, to the laws of motion for the state variables of the model. We will find that there is a (basically) uniquely determined interior steady state for this dynamical system which appears to be locally attracting for low adjustment speeds, but which generally is surrounded by centrifugal forces leading to limit cycles or more complex attractors or to pure explosiveness for large displacements from the steady state or for adjustment speeds that are chosen sufficiently large. Further (extrinsic) nonlinearities are therefore needed in general to get bounded dynamics for a larger set of parameter constellations.

2 The Real and the Financial Structure of the Economy

The following two tables provide a survey of the structure of the economy to be modeled\(^2\) that is related – but not identical – to the description of the Australian economy given in Powell and Murphy (1997). Note in this respect that the aim of the present paper is to establish an integrated continuous time model - leading to an autonomous system of differential equations – where all sectors are fully specified with respect to their behavior and their budget constraints from the viewpoint of complete theoretical models of monetary growth thereby providing a bridge between the Keynes-Metzler type monetary growth models of Chiarella and Flaschel (1998a,d) and the Powell and Murphy (1997) approach. Whenever needed from this perspective we therefore deviate from the framework given in Powell and Murphy (1997). We will comment on the differences between their discrete time macroeconometric dynamical model and the continuous time model of this paper briefly in section 4.

2.1 The structure of the real sector of the economy

Let us start with a presentation of the variables that comprise the real part of the economy to be considered. The following table 1 provides data on the temporary equilibrium position of the economy, based on given prices and expectations and also shows real stocks and their rates of growth.

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\(^2\)See the appendix to this paper for the employed notation.
<table>
<thead>
<tr>
<th></th>
<th>Labor</th>
<th>Non traded Goods</th>
<th>Exports</th>
<th>Imports</th>
<th>Dwellings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workers</td>
<td>$L = \alpha_t L_1$</td>
<td></td>
<td>$C_w$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>Asset holders</td>
<td>$-$</td>
<td></td>
<td>$C_e$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>Firms</td>
<td>$L^d_1, L^w_1$</td>
<td>$Y^p, Y, I, \mathcal{I}$</td>
<td>$X$</td>
<td>$J^d$</td>
<td>$-$</td>
</tr>
<tr>
<td>Government</td>
<td>$L^d_9$</td>
<td></td>
<td>$G$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>Prices</td>
<td>$w^b, w$</td>
<td>$[w^r, w^u]$</td>
<td>$p_v = (1 + \tau_u)p_y$</td>
<td>$p_x = e p_x^*$</td>
<td>$p_m = (1 + \tau_m)e p_m^*$</td>
</tr>
<tr>
<td>Expectations</td>
<td>$\pi = \bar{\rho}_o^s$</td>
<td>$\pi = \bar{\rho}_o^e$</td>
<td>$-$</td>
<td>$-$</td>
<td>$\pi = \bar{\rho}_o^e$</td>
</tr>
<tr>
<td>Stocks</td>
<td>$L_1$</td>
<td>$K, N$</td>
<td>$-$</td>
<td>$-$</td>
<td>$K_h$</td>
</tr>
<tr>
<td>Growth</td>
<td>$\tilde{L}_1 = n$</td>
<td>$\dot{K} = I/K - \delta$</td>
<td>$-$</td>
<td>$-$</td>
<td>$\dot{K}_h = \frac{I}{K_h} - \delta_h$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\dot{N} = Y - Y^d$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table 1: The real part of the economy.
(Foreign country data: $\gamma_t, p^*_z, p^*_m, \tau^*_c = \tau_c$)

Table 1 describes the real sector of the considered economy. We have a labor market, three commodity markets and the housing market. Domestic production $Y$ concerns one good that is only domestically used (for all private consumption $C_w + C_e$, all investment $I, I_h, \mathcal{I}$, also into housing, and all government consumption $G$ and which uses up all the imports $J^d$ as intermediate goods) and one that is only used for exports $X$. There is thus only a single commodity used in domestic absorption – up to the housing services $C_h^d$ demanded by workers.

Our model exhibits three domestic sectors: Households, firms and the government, but with heterogeneous agents in the household sector, workers and (pure) asset holders, the former supplying their labor $L$ at the wage level $w^b$ (which includes payroll taxes) and the latter the housing services $C_h^e$ for the workers as far as real flows are concerned. Firms produce a non-traded domestic and an exported commodity and employ labor $L^w_1$ (with varying rates of utilization $L^w_1$) and imports $J^d$ besides their capital stock $K$ for these purposes, and invest into fixed business capital $I$ and inventories $\mathcal{I}$. Government finally provides public consumption goods $G$, pays rents $w^r$ and unemployment benefits $w^u$ and also employs part of the workforce $L^d_9$. There are a number of variables needed to describe (the laws of motion of) the quantities, prices $p_v$ (including value added taxes), and expectations about their rates of change, which will be explained in detail when we turn to the description of the various equations of the model in section 4. There is endogenous growth $n$ of the potential labor force $L_1$, of the capital stock $K$, and of the stock of housing $K_h$ (supplied at price $p_h$ for

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3Powell and Murphy (1997) do not have an explicit description of heterogeneous agents in their household sector, but basically use a uniform life cycle hypothesis for the modeling of the consumption demand of this sector, see however their p. 117 for a brief remark on income distribution.
rental services) and also actual change of inventories \( N \) that is different from their desired rate of change \( \tau \).

2.2 The structure of the financial sector of the economy

Let us next consider the financial part of the economy. The following table 2 provides data on the changes in financial stocks, corresponding prices, and the growth of stocks in the financial part of the economy.

<table>
<thead>
<tr>
<th></th>
<th>Money</th>
<th>Short-term Bonds</th>
<th>Long-term Bonds</th>
<th>Equities</th>
<th>Foreign Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workers</td>
<td>–</td>
<td>( \dot{B}_w )</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Asset holders</td>
<td>–</td>
<td>( \dot{B}_e )</td>
<td>( \dot{B}_1 )</td>
<td>( \dot{E} )</td>
<td>( \dot{B}_2 )</td>
</tr>
<tr>
<td>Firms</td>
<td>–</td>
<td>–</td>
<td>( \dot{B}_2 )</td>
<td>( \dot{E} )</td>
<td>–</td>
</tr>
<tr>
<td>Government</td>
<td>–</td>
<td>( \dot{B} )</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

| Prices               | 1     | \( 1 \ [r] \)   | \( p_b = 1/r^i \) | \( p_e \) | \( e \cdot p_b = e \cdot 1/r^i \) |
| Expectations         | –     | –                | \( \pi_b = \hat{\pi}_b \) | \( \pi_e = \hat{\pi}_e \) | \( \epsilon = \hat{\epsilon} \) |
| Stocks               | –     | \( B = B_w + B_e \) | \( B^i = B^i_1 + B^i_2 \) | \( E \) | \( B_2 \) |
| Growth               | –     | \( \dot{B}, \dot{B}_w, \dot{B}_e \) | \( \dot{B}^i = \dot{B}_1^i + \dot{B}_2^i \) | \( \dot{E} \) | \( \dot{B}_2 \) |

Table 2: The financial part of the economy. (Foreign country data: \( r^* \))

The first column in the above table shows that we do not consider money holdings here. Cash management and transactions money will be introduced in another paper, see Chiarella, Flaschel, Groh, Köper and Semmler (1999a,b), in the usual form of an aggregate Cagan money demand function and also in a more disaggregated form than in the Murphy type model. We exclude money holdings in our basic modeling framework, by assuming that money is a costless medium of exchange for firms, household and the government that returns at the ‘end’ of each point in time \( t \) to the ‘local branches’ of the central bank by the balancing of the budget restrictions of these sectors. At present there are only (four) interest-bearing financial assets in our model that can be held by the (pure) asset owners and by the workers of our economy (as shown in the above table). As in the Keynes-Metzler model of monetary growth of Chiarella and Flaschel (1999a) we here assume, in order to start with a simple representation of financial flows, that bonds are only issued by the government, that firms use only equity financing and pay out expected earnings as dividends, and that there exist also long-term bonds issued by the ‘foreign government’. Financial flows between the sectors

\[^4\text{there are no commercial banks in the model of this paper.}\]
of our economy are therefore very narrowly defined (in order to simplify the flow budget restrictions to a sufficient degree). The laws of motion of the real part of our economy do not yet depend too severely on this financial structure of the economy since – as in Powell and Murphy (1997) – we do not use a full portfolio approach towards the description of the stock equilibria of the economy. Rather we determine asset prices and asset returns through certain simple laws of motion, while the inflow of financial assets is basically determined from the supply side.\(^5\) This is done in a way that implies equilibrium on the market for foreign exchange with respect to the flows appearing in the current as well as in the capital account so that there is no change in reserves held by the central bank and thus no need to consider this item explicitly in the balance of payments to be discussed in the next section.

There is work in progress where loans to firms, inside debt of the household sector, supply side rationing, market imperfections and more may be introduced into the Keynes-Metzler framework of Chiarella and Flaschel (1999a, Ch.6), see Chiarella and Flaschel (1999g-i), Chiarella, Flaschel, Groh and Semmler (1999).\(^6\)

Note that we allow for savings out of wages (in a Kaldorian way) and that workers save only in the form of short-term debt (interest-bearing saving deposits\(^7\) held at the local branches of the central bank). All other assets (plus the remainder of short-term debt) are exclusively held by the (pure) asset holders of our model. We stress that this approach serves the purpose of simplifying the budget constraints of the agents, but needs refinement in future reformulations of the model. Note that the government sector include the activities of the central bank (and its branches), which in the following model type boil down to setting the interest rate of (only domestically held) short-term government bonds according to some type of Taylor rule.\(^8\)

### 3 The Structure of the Economy from the Viewpoint of National Accounting

We shall consider in this section the production accounts, income accounts, accumulation accounts and financial accounts of the four internal agents in our economy: firms, workers, asset holders and the government (including the monetary authority). These accounts – plus the balance of payments – provide basic information on what is assumed for these four sectors and which of their activities are excluded from the present theoretical framework. These accounts furthermore serve the purpose of checking that all ex post results of the considered economy are consistent with each other and showing how the usual basic identities of national accounting hold.

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\(^5\)Powell and Murphy (1997) use perfect substitute assumptions, as for example the interest rate parity condition, and rational expectations to describe the behavior of the asset markets, while we use certain delayed adjustment processes towards such an outcome and thus avoid use of the jump variable technique for the description of the financial part of the economy.

\(^6\)Compare also Franke and Semmler (1997) for a full portfolio approach to asset market behavior.

\(^7\)or fixprice bonds, which are thus perfectly liquid.

\(^8\)in the place of the indirect steering of this rate of interest through a monetary supply rule and money market equilibrium as in Powell and Murphy (1997).

\(^9\)The fifth agent, the foreign economy, is represented by the balance of payments at the end of this section and will be confined to steady state behavior later on. All demands of this foreign sector are indexed by \(*\), while its supply of long-term bonds \(B_2\) to domestic residents is indexed by \(2\).
accounting (concerning output and income, savings and investment) can be derived from them.

### 3.1 The four sectors of the economy

We start with the accounts of the sector of firms — shown below — which organize production $Y$, employment $L_f^d$ of their workforce $L_f^w$ and gross business fixed investment $I$ and which use (in the present formulation of the model) only equities $E$ as financing instrument (no debt in the form of bank loans or bonds issued by firms). There are value added taxes $\tau_v$ on consumption goods, import taxes $\tau_m$ and payroll taxes $\tau_p$ with respect to hours worked $L_f^d$, but no further taxation in the sector of firms and there are no subsidies.

All accounts are expressed in terms of the domestic currency. Firms build dwellings — which are of the same type as all other domestic production — and sell them to the asset holders (as investors) and thus have no own investment occurring in the housing sector. They sell consumption goods to workers, asset holders and the government, export goods to the world economy, organize fixed gross investments with respect to their capital stock (as well as voluntary inventory changes $I$ with respect to finished goods) and experience involuntary inventory changes $Y - Y^d$ due to the deviation of aggregate demand $Y^d$ from output $Y$ (which is based on expected sales $Y^e$ and planned inventories $I$).\(^\text{10}\)

Firms use up all imports as intermediate goods which thereby become part of the unique homogeneous good that is produced for domestic purposes. They have replacement costs with respect to their capital stock, pay indirect taxes and wages including payroll taxes. Their accounting profit is therefore equal to expected profits (based on sales expectations and paid out as dividend to equity owners) and retained profits (equal to planned inventories). As is obvious from the narrow income account of firms, firms thus only save an amount equal to their intended inventory changes. The accumulation account is self-explanatory as is the financial account which repeats our earlier statement that the financial deficit of firms is financed by the issuing of new equities solely.

Note that all investment is valued (and performed) net of value added tax and thus at producer prices $p_y$ in the place of the consumer prices $p_y = (1 + \tau_v)p_y$. Indirect taxes (value added taxes)\(^\text{11}\) thus only fall on consumption activities and not on gross investment, i.e., here also not on housing investments and the inventory investment of firms). There are furthermore no direct (capital) taxes in the sector of firms, neither on property nor on profits, since all expected profits are distributed to asset holders and since there are no taxes on windfall profits (unexpected retained earnings — or losses — of firms that help to finance investment). Note however that the wages $w^b$ paid by firms include payroll taxes $\tau_p w$ (for unemployment insurance, health and other social insurance, and retirement pensions) and that wage income $w$ of workers is taxed at the rate $\tau_w$ in addition. Note finally that the accumulation account of firms is based on realized magnitudes and thus does not refer to their intended inventory changes explicitly.

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\(^{10}\)No other type of inventory holding is considered in the model of this paper.

\(^{11}\)There is however a tax on the imports made by the firms.
Production Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imports $e p_m J^d$</td>
<td>Consumption $p_r C_w$</td>
</tr>
<tr>
<td>Depreciation $p_y \delta K$</td>
<td>Consumption $p_r C_c$</td>
</tr>
<tr>
<td>Indirect Taxes $\tau_y p_y (C_w + C_c + G) + \tau_m e p_m J^d$</td>
<td>Consumption $p_r G$</td>
</tr>
<tr>
<td>Wages (including payroll taxes) $w^b L^d_f$</td>
<td>Exports $p_x X$</td>
</tr>
<tr>
<td>Profits $\Pi = \rho^c p_y K + p_y I = \rho^c p_y K + p_y \dot{N}$</td>
<td>Gross Investment $p_y I$</td>
</tr>
<tr>
<td></td>
<td>Durables (Dwellings) $p_y I_h$</td>
</tr>
<tr>
<td></td>
<td>Inventory Investment $p_y \dot{N}$</td>
</tr>
</tbody>
</table>

Income Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividends $\rho^c p_y K$</td>
<td>Profits $\Pi$</td>
</tr>
<tr>
<td>Savings $S^m_f = p_y I$</td>
<td></td>
</tr>
</tbody>
</table>

Accumulation Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Investment $p_y I$</td>
<td>Depreciation $p_y \delta K$</td>
</tr>
<tr>
<td>Inventory Investment $p_y \dot{N}$</td>
<td>Savings $S^m_f$</td>
</tr>
<tr>
<td></td>
<td>Financial Deficit $FD$</td>
</tr>
</tbody>
</table>

Financial Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Deficit $FD$</td>
<td>Equity Financing $p_e \dot{E}$</td>
</tr>
</tbody>
</table>
Consider next the sector of asset-holders. Investment in housing as well as the supply of housing services has been exclusively allocated to this sector. The production account thus shows the actual sale (not the potential sale) of housing services (= demand for housing services by assumption) which is divided into replacement costs and actual earnings or profits on the uses side of the production account.

Income of asset holders comes from various sources: interest payments on short- and long-term domestic bonds and on long-term foreign bonds (net of tax payments which must be paid abroad), dividend payments of firms (based on their expected profit) and profits from housing rents. All domestic profit income is subject to tax payments at the rate $\tau_c$ and after tax income by definition is divided into the consumption of domestic commodities (including houses, but not housing services) and the nominal savings of asset owners.

The accumulation account shows the sources for gross investment of asset-holders in the housing sector, namely depreciation and savings, the excess of which (over housing investment) is then invested into financial assets as shown in the financial account. Note here that short-term bonds are fixed price bonds $p_b = 1$ (which are perfectly liquid), while long-term bonds have the variable price $p_b = 1/\tau^t$ (and fixed nominal interest payments of one unit of money per period) which shows that they are of the type of consols or perpetuities (the same holds true for imported foreign bonds, which are of long-term type solely).13

There is no taxation of financial wealth (held or transferred) in the household sector. Furthermore, though asset holders will consider expected gross rates of return on financial markets in their investment decision, there is no taxation of capital gains on these markets, which is descriptively seen realistic.

---

12 and are thus not perfectly liquid, since there is no ‘money back’ guarantee here for the sector of asset owners as a whole.

13 Due to the assumption of a given nominal rate of interest on foreign bonds, these bonds can be liquidated if this is desired by domestic residents, but they are of course subject to exchange rate risk. Foreign bond purchases by domestic residents will be treated as a residual in the wealth accumulation decisions of the asset holders.
Production Account of Households (Asset Owners including Housing Investment):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation $p_y \delta_h K_h$</td>
<td>Rent $p_h C_h^d$</td>
</tr>
<tr>
<td>Earnings $\Pi_h$</td>
<td></td>
</tr>
</tbody>
</table>

Income Account of Households (Asset Owners):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax payment $\tau_c r B_c$</td>
<td>Interest payment $r B_c$</td>
</tr>
<tr>
<td>Tax payment $\tau_c B_1^l$</td>
<td>Interest payment $B_1^l$</td>
</tr>
<tr>
<td>Taxes $\tau_c (p_h C_h^d - p_y \delta_h K_h)$</td>
<td>Interest payment $e (1 - \tau_c^* ) B_2^l$</td>
</tr>
<tr>
<td>Tax payment $\tau_o^p p_y K$</td>
<td>Dividend payment $\rho^p p_y K$</td>
</tr>
<tr>
<td>Consumption $p_c C_c$</td>
<td>Earnings $\Pi_h$</td>
</tr>
<tr>
<td>Savings $S_c^m$</td>
<td></td>
</tr>
</tbody>
</table>

Accumulation Account of Households (Asset Owners):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Investment $p_y I_h$</td>
<td>Depreciation $p_y \delta_h K_h$</td>
</tr>
<tr>
<td>Financial Surplus $FS$</td>
<td>Savings $S_c^m$</td>
</tr>
</tbody>
</table>

Financial Account of Households (Asset Owners):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term bonds $B_c$</td>
<td>Financial Surplus $FS$</td>
</tr>
<tr>
<td>Long-term bonds $p_h B_1^l$</td>
<td></td>
</tr>
<tr>
<td>Foreign Bonds $e p_h B_2^l$</td>
<td></td>
</tr>
<tr>
<td>Equities $p_\epsilon E$</td>
<td></td>
</tr>
</tbody>
</table>
The next set of accounts, the ones of worker households, are fairly simple and easy to explain. First, there is no production account in this sector. Income of the members of the workforce, which may be employed, unemployed or retired, thus derives from wages, unemployment benefits or pension payments where \( L = \alpha_1 L_1 \) denotes the total number of persons in the current workforce (\( L^w \) the part that is employed) and \( \alpha_1 L_2 \) the number of retired people which have access to pension funds (\( \alpha_1 = \text{const.} \) the participation rate of the potential workforce \( L_1 \)). To this we have to add the interest income on saving deposits (short-term bonds) which is taxed at the general rate used for financial asset income. All wage type incomes are subject to taxation at the rate \( \tau_w \) and are again by definition divided into nominal consumption (consumption goods and housing services) and savings. Note here that the employment \( L^d \) of the employed \( L^w \) can differ from their normal employment which is measured by \( L^w \), the number of persons that are employed. Note also that wages \( w \) are net of payroll taxes (used to finance unemployment benefits, social insurance and pensions in particular).

We assume in the following that workers have a positive savings rate and that they hold their savings in the form of short-term bonds solely which is mirrored here in the accumulation and finance account in a straightforward way.

**Production Account of Households (Workers):**

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
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</table>

**Income Account of Households (Workers):**

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxes ( \tau_w ) ( wL^d + w^u(L - L^w) + w^r \alpha_1 L_2 ) + ( \tau_w r B_w )</td>
<td>Wages ( wL^d )</td>
</tr>
<tr>
<td>Consumption ( p_v C_w + p_h C^d_h )</td>
<td>Unemployment benefits ( w^u(L - L^w) )</td>
</tr>
<tr>
<td>-</td>
<td>Pensions ( w^r \alpha_1 L_2 )</td>
</tr>
<tr>
<td>Savings ( S^a_w )</td>
<td>( r B_w ) Interest payments</td>
</tr>
</tbody>
</table>

**Accumulation Account of Households (Workers):**

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Surplus ( FS )</td>
<td>Savings ( S^a_w )</td>
</tr>
</tbody>
</table>
Financial Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term bond accumulation $\hat{B}_w$</td>
<td>Financial Surplus $FS$</td>
</tr>
</tbody>
</table>

There are finally the accounts of the fiscal and monetary authority which due to the many taxation schemes and transfer payments that are assumed are more voluminous than the preceding accounts - at least with respect to the income account. There is first however a fictitious production account where the supply of public goods is valued at production costs so that there will result a negative amount of nominal savings $S^n_g$ which balances the income account of the government.

The resources of government income consist of taxes on the various forms of workers’ income (taxed at a uniform rate), of taxes on the various forms of profit, interest and rent income (again taxed at a uniform rate), payroll taxes, value added taxes and import taxes. Uses of this tax income of the government are interest payments, transfers to the unemployed and the retired persons and the costs of the mentioned government ‘production’. In general all these uses of the tax income of the government will exceed its income so that there will be a financial surplus $FS$ which balances the income account of the government.

There is no accumulation of real assets in the government sector which means that we only have to look into the financial account of the government to see how the excess of government outlays over government revenue is financed through short- or long-term debt. Note that there is some type of accounting money in the economy that however only fuels the economy during the transactions period, but does not appear as flow in the financial accounts of asset owners, and workers, and the government, but instead returns to the banking sector at the end of each transaction period $t$ by the settlement of all budget restrictions in the economy. In striking contrast to a cash in advance constraint we thus assume in this paper that agents can obtain all money they need for transaction purposes during the transaction period $t$ (as intra-day credit in one form or another), but that they have to satisfy their budget constraint at the end of each such period $t$ where money holdings are not needed and thus not present in them. Instead all liquid asset holdings concern the short term bonds of the government as some form of interest bearing saving deposit.
Production Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government expenditure for goods $p_G G$</td>
<td>Costless Provision of public goods</td>
</tr>
<tr>
<td>Government expenditure for services $u^b L_g^d$</td>
<td></td>
</tr>
</tbody>
</table>

Income Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest payment $r_B$</td>
<td>Wage income taxation $\tau_w [w L^d + w^u (L - L^u) + w^r \alpha_1 L_2]$</td>
</tr>
<tr>
<td>Interest payment $B_i$</td>
<td>Profit and interest taxation $\tau_c [\rho^s p_y K + r_B + B_i]$</td>
</tr>
<tr>
<td>Pensions $w^r \alpha_1 L_2$</td>
<td>Rent income taxation $\tau_c (p_n C_h^d - p_n \delta K_h)$</td>
</tr>
<tr>
<td>Unemployment benefits $w^u (L - L^u)$</td>
<td>Payroll taxes $\tau_p w L^d$</td>
</tr>
<tr>
<td>Government consumption $p_G G$</td>
<td>Value added tax $\tau_v p_y (C_w + C_c + G)$</td>
</tr>
<tr>
<td>Salaries $u^b L_g^d$</td>
<td>Import taxes $\tau_m e p_m^d J^d$</td>
</tr>
<tr>
<td>Savings $S_g^n$</td>
<td></td>
</tr>
</tbody>
</table>

Accumulation Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings $S_g^n$</td>
<td>Financial Deficit $FD$</td>
</tr>
</tbody>
</table>

Financial Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial deficit $FD$</td>
<td>Short-term debt $\dot{B}$</td>
</tr>
<tr>
<td>Long-term debt $p_k \dot{B}_i$</td>
<td></td>
</tr>
</tbody>
</table>

There are a variety of further types of taxes that could have been included into the structure of the model as we have discussed it so far. The most important types from a macroeconomic
point of view are probably the following ones: Corporate profit taxation; Investment taxes (or subsidies) – fixed business investment and housing investment – including a treatment of depreciation allowances; Financial wealth taxation and inheritance taxes; Capital gains taxation; Real property taxation; Taxes on the rents imputed in the case of asset holders.

In the model of this paper corporate income taxation would reduce to a taxation of the windfall profits of firms, since all expected earnings of firms are distributed to workers and as dividend payments to asset holders. We leave (gross) investment as untaxed in order to stress that this type of activity is to be supported for the future development of the economy by the government. Financial wealth taxation will be considered in future work where a more advanced and much more interdependent structure for the small open economy introduced in section 4 is considered. Similarly capital gains are not taxed in order to stimulate financial investment into risky assets (and since it is also difficult to treat in its exact amount and dating and - from an institutional perspective – with respect to capital losses). Real property taxation is probably a major item in many countries, but is left here aside for simplicity as are the imputed rents of the housing services consumed by the asset owners of our economy.

There are further taxes on the state level and on the local level of government administration (which are here completely ignored), taxes on private insurance and pensions (which do not exist in our model), product specific taxes, subsidies for employment and investment on the margin, turnover taxes, which may be important from a partial microeconomic analysis of public economics, but which have to be left aside here in our broad picture of a macroeconomy. Furthermore, we also do not impose internal constraints on the uses of the taxes that are actually received by the government of our macroeconomy and which might restrict the use of certain taxes to certain expenditures or transfers made by the government.

Let us finally describe the balance of payments of the economy under consideration. This will be done from the viewpoint of the foreign sector which is thus a fifth agent of the economic structure considered in this paper. The description of the behavior of this agent will however be confined to steady state behavior in the later presentation of the structural equations of the model.
The balance of payments:\textsuperscript{14}

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ep^*X$</td>
<td>$ep^*J^d$</td>
</tr>
<tr>
<td>$e(1 - \tau^<em>_c)B^</em>_2$</td>
<td>$(1 - \tau^<em>_c)B^</em>_1$</td>
</tr>
<tr>
<td>$\dot{B}_1^*/r^t$</td>
<td>$e\dot{B}_2^<em>/r^{1</em>}$</td>
</tr>
</tbody>
</table>

This balance of payments shows the trade account (exports $X$ and imports $J^d$), the international component of interest payments (to foreigners and from abroad) that are all assumed to cross borders and the outflow and the inflow of new capital (long-term bonds). Note that this account does not show any reserve changes of the central bank due to foreign exchange market operations. This is possible in the approach chosen in this paper, despite a temporarily given exchange rate $e$, since on the one hand the supply of bonds of the government domestically and abroad of the equities of firms are channeled here into the savings decisions of households without readjustments. On the other hand the excess of domestic private savings is going into foreign bonds, which in turn implies that the balance of payments must be balanced without any intervention from the central bank. This will be checked in the next subsection from the viewpoint of the ex post equality of aggregate savings with aggregate investment plus the current account balance on the one hand and with aggregate investment plus the capital account balance on the other hand, where both equalities can be established without any interference from the central bank. The obtained result is basically due to the fact that all foreign exchange market operations can be settled without any help from the central bank, and without any rationing processes, since the residually determined item $e\dot{B}_2^*/r^{1*}$ just provides the balancing item for this account.

This concludes our description of the four accounts of the three typical sectors of a small open economy (with heterogeneous agents in the household sector) plus a foreign sector that is here represented via the balance of payments solely.

3.2 Domestic product, savings, investment and further aggregates

In this subsection we derive some basic concepts of national accounting in the specific form they receive in our model economy and also the relationships between nominal aggregate savings and nominal total investment. Considering nominal gross domestic product first, we see that this concept aggregates (with respect to uses) total private and government consumption, net exports, total investment (including housing and all inventory investment $\dot{N}$) and finally the services of housing actually demanded and supplied. The sources of these expenditures are the depreciation of the capital stock of firms and of the housing stock, indirect taxes, wage payments of firms and profits in production and in housing supply.

\textsuperscript{14}viewed from the foreign economy and in terms of the domestic currency.
Gross Domestic Product (GDP):\textsuperscript{15}

\[ p_y \delta K + p_y \delta_h K_h + \tau_v p_y (C_w + C_c + G) + \tau_m e \rho_m J^d + w^h L_j^d + \Pi + \Pi_h \]
\[ = p_v (C_w + C_c + G) + p_x X - e \rho_m J^d + p_y I + p_y I_h + p_y \dot{N} + p_h \rho_h^d \]

Net domestic product is then obtained (also in nominal terms and at market prices) by moving the depreciation items from the left hand side of the preceding equation to its right hand side and thus by deducting them from the corresponding gross investment, giving rise to net investment descriptions for firms as well as for the housing sector of the economy.

Net Domestic Product at market prices (NDP):\textsuperscript{16}

\[ \tau_v p_y (C_w + C_c + G) + \tau_m e \rho_m J^d + w^h L_j^d + \Pi + \Pi_h \]
\[ = p_v (C_w + C_c + G) + (p_x X - e \rho_m J^d) + p_y (I - \delta K) + p_y (I_h - \delta_h K_h) + p_y \dot{N} + p_h \rho_h^d \]

Net domestic product at factor costs, finally, follows from net domestic product at market prices by deducting indirect taxes from both sides of the preceding equation which simply leads to a revaluation of consumption goods and imports, both measured now at prices without value added taxes and without import taxes.

Net Domestic Product at factor costs (NDP-F):\textsuperscript{17}

\[ w^h L_j^d + \Pi + \Pi_h \]
\[ = p_y (C_w + C_c + G) + (p_x X - p_m J^d) + p_y (I - \delta K) + p_y (I_h - \delta_h K_h) + p_y \dot{N} + p_h \rho_h^d \]

On the basis of the uses of nominal savings of the four sectors considered, see their accumulation and financial accounts, one furthermore gets via their aggregation:

\[ S^n = S^n_w + S^n_c + S^n_f + S^n_g = I^{na} + [e \dot{B}_1^t / \tau - (\dot{B}_1^t - \dot{B}_1^f) / \tau] \quad \text{with} \]
\[ I^{na} = p_x \dot{E} + p_y I + p_y (I_h - \delta_h K_h) = p_y (I - \delta K) + p_y \dot{N} + p_y (I_h - \delta_h K_h) \]

We here see that total nominal savings are ex post always equal to total nominal net investment plus net capital exports. Note that the home country’s capital exports are equal (in value) to the import \( \dot{B}_1^t \) of foreign bonds and that its capital imports are given by the value of the export of the home country’s long-term bonds \( \dot{B}_1^t - \dot{B}_1^f \). Note also that actual net investment consists of business fixed investment, of actual inventory changes and of net investment in the supply of dwellings. This important identity of national accounting is based on the four identities that relate the nominal savings of the various sectors to the uses

\textsuperscript{15}GNP=GDP + e(1 - \tau_c^*)B_3^t - (1 - \tau_c)(B^t - B_1^f).

\textsuperscript{16}NNP=NDP + e(1 - \tau_c^*)B_3^t - (1 - \tau_c)(B^t - B_1^f).

\textsuperscript{17}National Income is defined on this basis by NDP-F + e(1 - \tau_c^*)B_3^t - (1 - \tau_c)(B^t - B_1^f).
made of these savings. Approaching aggregate nominal savings from the definitions of the various savings items, i.e., from the income side, by contrast gives rise to:

\[
S^n = S^n_0 + S^n_c + S^n_f + S^n_g = I^{na} + [p_yX - \nu Y^d] + [e(1 - \tau^*_c)B^1_x - (1 - \tau_c)(B^d - B^d_1)],
\]

\[
I^{na} = p_y(I - \delta K) + p_y\dot{N} + p_y(I_h - \delta_h K_h)
\]

i.e., aggregate nominal savings equals aggregate nominal actual net investment plus nominal net exports plus international nominal net transfers. The identities just discussed thus in sum show that basic concepts of the system of national accounts can already be quite complicated in the model economy we are considering in this paper.

Appendix: The following adds a detailed calculation to what has just been asserted and in particular shows that there indeed is no intervention needed from the monetary authority on the market for foreign exchange, due to the assumed budget restrictions for households, firms and the government.

Starting from the definitions of the nominal savings of the four sectors the identity on nominal savings and investment just stated can also be shown as follows:

\[
S^n = Y^n_w - p_v C_w - p_h C_h^d + Y^n_c - p_v C_C + p_y I + T^n - w^u(L - L^u) - w^r \alpha_L L^2 - (rB + B^d) - (p_v G + w^b L^d_g)
\]

which gives rise to

\[
S^n = wL^d + w^u(L - L^u) + w^r \alpha_L L^2 + rB_w - p_v C_w - p_h C_h^d + \rho^v p_y K + rB_c + B^d_1 + p_h C_h^d + p_y \delta_h K_h + e(1 - \tau^*_c)B^1_x - p_v C_c + p_y I + \tau_m \nu Y^d - \tau_c(B^d_1 - B^d_1) + \tau_m \nu Y^d + \nu Y^d - I - I_h - w^u(L - L^u) - w^r \alpha_L L^2 - (rB + B^d) - (p_v G + w^b L^d_g).
\]

These expressions can be rearranged in the following way:

\[
w^b L^d - p_v C_w + \rho^v p_y K + rB + B^d_1 - p_y \delta_h K_h + e(1 - \tau^*_c)B^1_x - p_v C_c + p_y I + \tau_m \nu Y^d - \tau_c(B^d - B^d_1) + \tau_m \nu Y^d - \tau_c(B^d - B^d_1) + \nu Y^d - I - I_h - w^u(L - L^u) - w^r \alpha_L L^2 - (rB + B^d) - (p_v G + w^b L^d_g).
\]
\begin{align*}
  &= p_y \dot{N} + p_y (I - \delta K) + p_y (I_h - \delta_h K_h) \\
  &+ p_x X - e p_m^* \dot{J}^d + e (1 - \tau^*_c) B_2^l - (1 - \tau_c) (B^l - B^*_l) \\
  &= I^{na} + e (1 - \tau^*_c) B_2^l - (1 - \tau_c) (B^l - B^*_l)
\end{align*}

This proves the asserted identity from the viewpoint of the definitions of nominal savings. Note here that \(B^l - B^*_l = B^*_l\) holds by definition for the international allocation of domestic long-term bonds and that aggregate goods demand is defined by the expression: \(Y^d = C_w + C_c + I + I_h + G\).

Having presented the model from the ex post point of view by means of structured tables and the system of national accounts we now turn to the structural form of the model and present in the following section its technological foundations, its behavioral relationships, various definitions and the budget equations of the four agents of the domestic economy, and finally also its laws of motion for quantities, prices and expectations.

4 The model

In this section we develop the extensive form equations of our model based on the structure laid out in section 3. We reformulate the Murphy model for the Australian economy, as presented in Powell and Murphy (1997), from a macrotheoretic perspective, by making it a continuous time dynamic model of monetary growth, suppressing all discrete lag structures of their quarterly period model in particular. The present reformulation of their model is furthermore based on the experience gained in Chiarella and Flaschel (1999a,d) in the modeling of integrated Keynes-Metzler models of monetary growth for closed as well as open economies. Certain features of this dynamical system approach to growth and fluctuations are therefore retained in the formulation of our continuous time version of the Murphy model which of course means that its dynamical structure will differ from that of the Murphy model to a certain degree.\(^{18}\)

Our interest in this section is not such much to fully mirror the dynamical structure and implications of the Murphy model, but to formulate and to investigate, in a first approximation of this 100 equations approach to macroeconometric model building, the set of more or less prominent feedback structures of macrodynamic theory it basically contains and the role they play for stability analysis, and this, as stated, in an integrated macrotheoretic monetary growth framework that in its generality is comparable to this type of macroeconometric model building. This section therefore attempts to build a bridge between empirically motivated work on structural model building (where there generally is no analysis of the mechanisms that are hidden in the formulated structure) and theoretical investigations of reasonably large representation of concrete economies, where the interest is to see what the steady state of such economies will look like in all of its details and what stabilizing (or destabilizing) effects are present around it (or have to be added far off the steady state in order to ensure the boundedness of the considered dynamics).

\(^{18}\)This remark in particular applies to our treatment of financial markets where we attempt to avoid the so-called jump variable technique of models with only rational expectations by allowing for heterogeneous expectations formation and somewhat delayed adjustments towards interest rate parity conditions.
4.1 Introduction

Let us start with some notation to be used in the structural equations we shall employ in our approach to Keynesian monetary growth. Module 1. of the model provides definitions of important rates of return $\rho^e$, $\rho^a$, $\rho^n$, $\rho_h$, of nominal wealth $W^n$ and of hourly wages including payroll taxes, $w^b$, prices $p_o$ including value added tax, of pension payments per retired worker (in the workforce) per time unit, $w^r$, and unemployment benefits per unemployed worker (of the workforce) per time unit, $w^u$, with $w$ denoting the money wage exclusive of payroll taxes but still including wage income taxes. We here in particular define the currently expected rate of profit based on the sales expectations $Y^e$ of firms (net of depreciation $\delta K$) and on actual exports $X = x_y Y$, imports $J^d = j_y Y$ and the actual employment $L^d = l_y Y$ of the workforce of the firms (and in a similar fashion also the actual and the normal rate of return on business fixed investment, $\rho^a$, $\rho^n$, based on actual sales and normal rates of capacity utilization of the capital stock. Our choice of notation of production coefficients already indicates that we are assuming a technology with fixed input/output coefficients where export supply is in fixed proportion to actual output $Y$, as is import demand and labor demand. Furthermore, potential output is defined on the basis of a given capital stock as $Y^p = y^K K$, $y^p = \text{const}$, and is used in the definition of normal profits in a specific way that has still to be explained. We use fixed coefficients technology for the same reasons as in Chiarella and Flaschel (1999a), since it allows clearer insight into the dynamic feedback structure of the model.

1. Definitions (Rates of Return, Nominal Wealth, Wages and Prices):

\[
\rho^e = \frac{p_y Y^e + p_x x_y Y - w^b l_y Y - p_m j_y Y - p_y \delta K}{p_y K} \quad (1)
\]

\[
\rho^a = \frac{p_y Y^d + p_x x_y Y - w^b l_y Y - p_m j_y Y - p_y \delta K}{p_y K} \quad (2)
\]

\[
Y^{dp} = \frac{\bar{U} Y^p}{1 + \gamma \beta n^t}, \quad Y^n = \bar{U} Y^p \quad (3)
\]

\[
\rho^n = \frac{p_y Y^{dp} + p_x x_y Y^n - w^b l_y Y^n - p_m j_y Y^n - p_y \delta K}{p_y K} \quad (4)
\]

\[
\rho_h = \frac{p_h C^d_h - p_y \delta_h K_h}{p_y K_h} \quad (5)
\]

\[
\rho^r = (1 - \tau_e) r^l - \pi^l \quad (6)
\]

\[
W^n = B + B^l_1 r^l + \varepsilon B^l_2 r^l + p_e E + p_y K_h \quad (7)
\]

\[
w^b = (1 + \tau_y) w \quad (8)
\]

\[
w^r = \alpha^r w \quad (9)
\]

\[
w^u = \alpha^u w \quad (10)
\]

\[
p_v = (1 + \tau_v) p_y \quad (11)
\]

Note that the various rates of profits are defined on the basis of output prices $p_y$ net of value

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19Wealth effects will however only be studied in future extensions of the model of this paper, but should of course be kept in mind when interpreting the behavior of the present model.
added tax, since they measure what can actually be distributed to equity owning households (with the rate $\rho^e$ measuring actual dividend payments at each moment in time, while $\rho^n$ measures the actual rate of profit of firms based on their actual sales).

Firms have a desired rate of capacity utilization $\bar{U} < 1$ (which is not endogenized in the present model) and thus plan a normal output $Y^n = \bar{U}Y^p$ less than potential output in order to have capacity reserves in the case of unforeseen demand shocks. Furthermore they have to hold inventories $N^d = \beta_{rd}Y^n$ which have to grow at the given world growth rate $\gamma$ in the steady state which means that the demand $Y^d_p$ they consider as adequate in the light of their potential output (or as satisfying in the steady state) must be less than normal production since part of the latter is going into inventories). The normal rate of profit, $\rho^n$, is then defined on the basis of this concept of normal output as the other rates of profits just discussed. The rate of return $\rho_h$, finally, refers to the housing sector and its actual sale of (=demand for) housing services $C^d_h$ at price $p_h$. It is diminished through the depreciation of dwellings at rate $\delta_h$ and set into relation to the net value of the capital stock $K_h$ in the housing sector. All capital goods are – or have been – bought at price $p_i$ in the market for non-traded (domestic) goods, since value added taxes only concern consumption expenditures in the present model.

We often compare profitability as measured by the above rates of return with required profitability given by the expected long-term real rate of interest $(1 - \tau_c)r^l - \pi^l$ which is related to the price of long-term bonds (consols or perpetuities) in the following well known way $p_b = 1/r^l$, $\pi^l$ a weighted (long-term) expected rate of inflation to be defined in module 5b of the model. Note that we calculate this required rate of return net of interest taxation, while all other rates in this block are gross rates of return.

Aggregate nominal wealth of asset holders and workers (the latter only hold short-term debt of the government as saving deposits) is composed of short-term fix-price (= perfectly liquid) bonds of the domestic government, $B$, held only domestically, long term bonds issued by the domestic and the foreign government in the amounts held by domestic residents, $B'_1$, $B'_2$, equities $E$ and the value of the housing capital stock $K_h$, again measured at producers' prices. We assume that there is no resale market for houses (and goods in general) and thus do not have a secondary market in this segment of the economy in order keep things simple in our one domestic good economy (once these commodities are sold they do not reappear on the market). Financial assets by contrast are traded in secondary markets and give rise to certain price adjustment equations that are implicitly based on stock reallocations in the financial markets considered. Asset markets are therefore still treated in a preliminary way, since portfolio decisions are not yet modeled explicitly (see module 6 of the model).

Note that government bonds are treated as net wealth in our model. However, since wealth effects are still excluded from the behavioral equations to be introduced later on, this concept of wealth is here presented solely for pointing to the necessity of treating such wealth effects in future extensions of the model. Note furthermore that central bank money is not treated as component of financial wealth in the present paper. Such money is here assumed to be used for intra-day transaction purposes solely and is supplied by branches of the central

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20 A Ricardian equivalence argument might lead to the exclusion of workers' saving deposits $B_{wc}$ from aggregate wealth, since the wage taxation rate is endogenous in our model and is varied by the government in order to establish a desired 'Government Debt / GDP ratio' in the economy.
bank without user costs (cash ATM and credit cards) for the public during the day who however transfer this type of money back to these branches at the end of each 'trading period', by fulfilling their budget equations and due to the loss of interest rate payments that would otherwise arise. Households, firms and the government thus do not need to hold money balances for intra-day trading (due to flexibilities in the management of their intra-day income accounts) and are thus not forced to devote part of their asset holdings to pure cash holdings. Paper money fuels the economy within each period, but is simply stored in the banking sector at the end of it, while all savings decisions go into interest-bearing asset solely.\footnote{21} A Keynesian liquidity preference function – if it were explicitly present in our model – would thus concern the allocation of wealth (of wealth owners) between liquid short-term and illiquid long-term bonds and is thus not related to the specific treatment of cash or means of exchange management chosen in the present paper.

The remaining equations in the above module define (on the basis of before tax money wages \( w \)) gross wages \( w^b \) that include payroll taxes (as the intended basis for government transfers to the unemployed, the retired, etc.), pensions \( w^r \) and unemployment benefits \( w^u \), which are all in constant proportion to money wages \( w \). Finally, \( p_y \) is the consumer price of the domestic good that is assumed to be in fixed proportion to the producer price \( p_y \) on the basis of a given value added tax rate \( \tau_v \).

*Powell and Murphy's (1997, p.291ff.) list of consolidated equations exhibits with respect to this module as eq. 32 the definition of gross operating surplus, see also their p.233, which is identical to the numerator in our definition of the expected profit rate \( \rho^e \) of firms. As eq. 29 it provides the definition of wages including payroll taxes, \( w^b \) as defined above. In the equation for fixed business investment they use a rate \( r_B^{ALR} \), see eq. 44 in their consolidated list and their pages 171, 248, in the place of our rate \( \rho^l \), which is following the rate \( \rho^e \) with some time delay, where they define the rate \( r_B^{ALR} \) to be the gross rate of return that would be realized on business fixed investment before depreciation is deducted and inclusive of the risk premium required for equity investment) if all prices (including the wage rate) were frozen indefinitely at their current values and they relate this rate to the real rental price of capital \( r_B^{LR} / p_y \). It appears that this definition of gross return is closer to our definitions of the rates \( \rho^e, \rho^p, \rho^b \) than to the rate \( \rho^l \) that we will use in our investment function.*

*There is no equation for the actual gross rate of return of housing investment in the consolidated list of equations, which is however provided on p. 145 of their book. This rate differs from our rate \( \rho_h \) (which is net of depreciation) in that we neglect taxes on rental services and maintenance costs as measured by their term \( \beta_5^{GB} \). Furthermore we have set the parameter \( \beta_1^{IS} \) equal to one and have thus identified housing supply with the stock of housing that supplied for rental services. On this basis we however distinguish occupancy rates \( C_h^d \) from corresponding supply and will enter this discrepancy as a demand pull term into the housing investment equation as well as into the dynamics of rental prices. Note here also that the rate of return on housing services must then be based on rented units \( C_h^r \) not on the amount of housing services \( C_h^p \) that is offered on the market.*

*The required real rate of return \( \rho^r \) of the above module is basically of the same type as the one defined in eq. 52 of the consolidated list of Powell and Murphy (1997), see also their page 52. These authors however use an estimated average inflation rate \( \xi_{10} \) based on model-consistent expectations over a time horizon of ten years (coupled with a ten year nominal interest rate on government bonds) in the place of our medium-run concept of expected inflation \( \pi^l \). We do not make use of the average interest rates on outstanding foreign and domestic debt, the eq.s 54 and 55 in the consolidated list...*
of Powell and Murphy (1997), see also their page 229, in the calculation of private sector property income, but simply make use here of the two actual rates \( r^1, r^2 \) in their place when calculating the interest rate income received by asset holders at each moment of time. Eq. 88 of their list simply provides a symbol for interest rate differential used to capture the influence of tight monetary policy on the real investment of firms and asset holders which is used in explicit form in our following model. Finally the measure of nominal wealth used by Powell and Murphy (1997) is not contained in their consolidated list, but can be found on p.120 of their book. It differs from the measure defined in our module 1 insofar as total money holdings are included, private sector debt to foreigners contracted in two ways is allowed for as well as real stock owned by foreigners. In the place of equities Powell and Murphy (1997, p.120) immediately use the current value of the capital stock owned by the private sector and they also include inventories into the definition such measures of the value of the stock of firms.

Their concept of private wealth (coupled with a life cycle hypothesis for a single representative consumer) is thus fairly different from the one we employ in the above module 1. We will add money holdings in Chiarella, Flaschel, Groh, Köper and Semmler (1999a,b), Chiarella, Flaschel and Zhu (1999c), but will not reflect here and there measures of the capital stock of firms that include their stock of inventories (which in our model are only present for the domestically marketed good), but will use the issued equities instead in our concept of private wealth. We will also not consider indebtedness of the private sector to foreigners, but use public indebtedness of this sort instead (and will consider the purchase of foreign bonds by domestic residents in addition).

Module 1 finally provides the definitions of unemployment benefits, rents paid to retired worker households and consumer prices (producer prices plus value added tax) which are not explicitly represented in the consolidated list of equations that Powell and Murphy (1997) supply, but which are not different from their use of these concepts. Crucial differences in the equations considered so far therefore basically concern their use of model consistent inflations in the calculation of the required rate of return used by investors and their definition of private wealth which they use in the single optimal consumption function of their model.

Module 2 concerns the household sector where two types of households are distinguished, pure workers and pure asset holders or wealth owners. Of course, these two types of households are only polar cases in the actual distribution of households types. Nevertheless we believe that it is useful to start from such polar household types before intermediate cases are introduced and formalized. Powell and Murphy (1997) consider only one type of household explicitly (though they briefly refer to effects of income distribution implicitly contained in their formulation of a consumption function) whose consumption behavior is based on the life cycle hypothesis with respect to wage income and wealth. We shall use differentiated saving habits for the two types of households instead (as they can be derived from Cobb-Douglas utility functions) and will ignore wealth as a direct argument in our consumption functions (leaving this for later reformulations of the model).

### 4.2 Households

We consider the behavioral equations of worker households first:

\[ \text{Note in this respect also that the relative price } p_v/p_h \text{ does not yet play a role in the consumption decisions of workers.} \]
2a. Households (Workforce):

\[ Y_{w}^{dn} = (1 - \tau_{w})[wL^{d} + w^{u}(L - L^{w}) + w^{r}\alpha_{1}L_{2}] + (1 - \tau_{c})rB_{w} \]

\[ = Y_{w1}^{dn} + (1 - \tau_{c})rB_{w} \] (12)

\[ L^{w} = L^{w}_{f} + L^{w}_{g} \] (13)

\[ L^{d} = L^{d}_{f} + L^{d}_{g} = L^{d}_{f} + L^{w}_{g} \] (14)

\[ C_{w}^{on} = c_{1}(1 - \tau_{w})[wL^{d} + w^{u}(L - L^{w}) + w^{r}\alpha_{1}L_{2}], \quad C_{w}^{o} = C_{w}^{on}/p_{v} \] (15)

\[ C_{w}^{d} = \alpha_{1}^{w}(C_{w}^{o}/C_{w} - 1) + \alpha_{2}^{w}(V - \bar{V}) + \gamma \] (16)

\[ p_{h}C_{h}^{d0} = c_{2}(1 - \tau_{w})[wL^{d} + w^{u}(L - L^{w}) + w^{r}\alpha_{1}L_{2}] \] (17)

\[ C_{h}^{d} = \alpha_{1}^{h}(C_{h}^{d0}/C_{h}^{d} - 1) + \alpha_{2}^{h}(V - \bar{V}) + \gamma \] (18)

\[ S_{w}^{n} = Y_{w}^{dn} - p_{v}C_{w} - p_{h}C_{h}^{d} = \hat{B}_{w} \] (19)

\[ \tilde{L}_{1} = \tilde{L}_{2} = \tilde{L}_{0} = n \quad (L_{0}(0), L_{1}(0), L_{2}(0) \text{ given}) \] (20)

\[ \tilde{n} = \beta_{nw}(\tilde{n} - n), \quad \bar{n} = \bar{n}(V, \bar{V}) \] (21)

\[ L = \alpha_{1}L_{1} \] (22)

The first equation in this module defines the aggregate disposable income of workforce households by the sum of the wage incomes of the employed, unemployment benefits for the unemployed, unemployment being measured by \( \alpha_{1}L_{1} - L^{w} \), and the pensions of retired people, after taxes (with the tax rate \( \tau_{w} \) uniformly applied to these three types of workers' incomes). Furthermore workers as a group also have interest income from their holding of saving deposits which is taxed as all other interest payments (which goes to pure asset owners) by means of the rate \( \tau_{c} \). Note that retired persons \( L_{2} \) receive pension payments in an amount that is scaled down by the given participation rate \( \alpha_{1} \) (of the persons \( L_{1} \) between 16 and 65) which is constant in the present model. Pensions are thus paid to both employed and unemployed workers in the workforce once they retire.\(^{24}\)

Next we consider the number of employed workers \( L^{w} \) which are working in the sector of firms, \( L^{f}_{w} \), or for the government, \( L^{w}_{g} \), there providing public services. In contrast to \( L^{w} \) we denote by \( L^{d} \) the actual employment of the employed which can be larger or smaller than the normal hours of work \( L^{w} \) of the employed workforce due to over- or undertime work (such situations by assumption only occur in the firm sector, but not in the government sector, see equation (14)).

Desired consumption (in nominal and in real terms) \( C_{w}^{on}, C_{w}^{o} \) of workers is proportional to their nominal and real wage income, respectively, with \( c_{1} \) denoting the uniform marginal propensity to consume of both employed and unemployed workers as well as for retired persons. Note that we always use consumer prices \( p_{v} \) when going from nominal to real magnitudes (we thus ignore the influence of the price \( p_{h} \) of housing services here and later on). Note also that the interest income of worker households does not influence their consumption

\(^{23}\)\( \alpha_{1} \) the participation rate of the workforce which is endogenously determined in the Murphy model for the Australian economy.

\(^{24}\)The participation rate is also applied – for reasons of simplicity – to the growth rate in pension-receivers that is caused by the assumed migration of whole families.
plans here, since we assume that all of their interest income is saved in order to simplify the feedback from asset accumulation into the real part of the model. Following Powell and Murphy (1997) we assume that actual consumption plans \( C_w \) of all types of workers adjust towards desired consumption with a time delay that depends on their deviation from desired consumption and on the state of the labor market which is measured by the rate of employment \( V \) (plus a trend term \( \gamma \) that ensures the existence of steady growth paths later on). The demand for desired housing services is treated in the same way as the demand of workers for consumption goods which means that we assume for this type of consumption:

\[
p_h C_h^{*} = c_2(1 - \tau_w)[wL^d + w^u(L - L^u) + w^r\alpha_i L^i],
\]

with \( c_2 \) as marginal propensity to consume these services. Note that we have to use the price for these services on the left hand side of this consumption function. Note also that adjustment towards desired levels is of the same type as the one for the consumption of the domestic goods produced by firms.

It is of course questionable whether the considered marginal propensities to consume are really uniform with respect to the three types of situations adult workers may be in and whether they all use the actual rate of employment as an expression on the prospects of their future incomes. Introducing different behavior in this place is easily possible, but should be left to investigations with a more pronounced empirical orientation.

The next equation defines nominal savings \( S_w^n \) of workers and states that these savings are held in the form of short-term bonds solely. As already explained in the preceding section money is assumed to fuel real transactions via its circulation, since there is no cash or income in advance constraint for obtaining such means of payments for intra-day trading.\(^{25}\) It is thus considered to be stored in the banking sector (here only the branches of the central bank) after each round of transactions. Note that workers do not accumulate wealth in the form of real estate which is of course not true for example for the Australian economy. Including this into the present module and adding a resale market for houses is thus left here for future extensions of the model.\(^{26}\)

When we say ‘workers’ we have, as already noted above, three groups of persons in mind: \( L_1 \), the potential workforce, \( L_0 \) the young people (below 16 years) and \( L_0 \) the retired people (above 65 years). All three components of the workforce households grow – via migration into the considered country (and possibly also by reasons internal to the economy) – at the same rate \( n \), for reasons of simplicity and for the purpose of later steady state analysis. Note that these new members of the workforce are immediately treated as the residents of the country under consideration. This rate \( n \) follows with a delay the growth rate \( \bar{n} \) which represents the desire to migrate (with constant population shares) into the labor market of our economy and which is here endogenously determined through the state of the labor market \( V \) and its rate of change \( \dot{V} \).

Actual labor supply (in terms of persons), finally, is given by \( \alpha_i L_1 \) with \( \alpha_i \) the participation rate, and is divided according to the state of the economy into employed and unemployed people, \( L^w, L - L^u \). Note again that we have assumed that the participation rate is constant in time and thus do not make any use of the encouraged / discouraged worker effect as in

\(^{25}\)Note again that the temporal budget equations of all agents in the economy must be fulfilled at each end of the trading period \( t \).

\(^{26}\)It is however possible to assume that the consumption of the domestic good through workforce households is partly going into the purchase of houses if it is assumed that goods purchased cannot be sold anymore at a later point in time to another sector of the economy.
Powell and Murphy (1997, 6.7).

Summing up the module 2a thus basically describes the two consumption decisions of workers households based uniformly on their various sources of wage income. It is easy to derive such consumption functions by assuming Cobb-Douglas utility functions. Powell and Murphy (1997) make use of a life cycle approach in the place of our description of the consumption behavior of workers and thus immediately include wealth effects into the consumption decisions of their single type of household. We shall apply their approach to consumption behavior in our two agents framework in another paper, see Chiarella, Flaschel, Groh, Köper and Semmler (1999a,b), and will then study the role of wealth effects in such an extended framework.

Powell and Murphy’s (1997, p.291ff.) list of consolidated equations contain this module of our model in form of a representative consumer only (and thus aggregated with the next module of our model), but occasionally refer to the role of income distribution which they represent as a time trend in the equation they provide for optimal consumption on their page 117. We shall make use of their life cycle approach in our disaggregated two household setup in Chiarella, Flaschel, Groh, Köper and Semmler (1999a,b) and shall distinguish there and here between equilibrium consumption and the dynamic adjustment of consumption towards equilibrium consumption, see the eq.s 5,6 in their consolidated list and their chapter 7. Not here, but also in Chiarella, Flaschel, Groh, Köper and Semmler (1999a,b) we will include a consumer price index as in their eq.s 19, 21 (see also their pages 132/3) in the calculation of the real consumption of workers, so that their consumption of housing service then forms an exponentially weighted average with the consumer price of the goods marketed domestically, also to be applied in the money wage Phillips curve as the cost-push expression (in a purely adaptive way in the Murphy model). with respect to the dynamic adjustment of consumption plans we use somewhat different functional forms than Powell and Murphy (1997, p.121), a difference that should not be too important in a comparative evaluation of these two approaches. Their inclusion of (real) interest rate effects may however strengthen the nominal instability that we will investigate in another paper of this series of papers, i.e., the so-called Mundell effect of inflationary expectations. Note here also that the rate of employment enters this adjustment equation in derivative form in the Murphy model. Eq. 3 (see also their p.112) of their consolidated list provides a long expression for labor force participation which is constant in our formulation of the model, in order to avoid the complicated calculation of total unemployment benefits and also of total rents that go to retired workers. With respect to the treatment of labor supply the Murphy model is thus much more advanced than our approach, but also much more complicated as far as a proper (still lacking) treatment of the budget equations of all worker households is concerned. We neglect their logistic trend term in the propensity to consume dwelling services, i.e., their eq. 20 (see also p.126) and apply a different approach to the endogeneity of the natural rate of growth of the working population, their eq. 94 (discussed on their page 140). Furthermore, the number of unemployed people is not adjusted by a weight ω that accounts for the difference of two unemployment rate measures.

Wage income \( Y_L \) is defined on page 117 in Powell and Murphy (1997) and it consists of the wages of the employed (after taxes) plus unemployment benefits plus rent payment and also plus further transfers that are not considered in our model (in particular to families with children). It is further modified by taking account of certain lump sum taxes and of transfers from overseas to this sector of the economy. This extends our above definition of worker households’ income to some extent, but is not basically different from it. Note also that the Murphy model does not relate unemployment benefits and rent payments to the time profile of the actual participation of potential workers in this social insurance system. As in our model, the Murphy model distinguishes between private and
public employment, but it does not explicitly consider over- and undertime work of the employed, though the effects that we will describe in this matter are basically present in the employment equation and the money wage dynamics, to be considered later on.

We observe again that the demand $C^d_h$ for housing services is identified with the supply for these services in Powell and Murphy (1997, p.128), so that the above module contains to more equations in this respect as compared to the Murphy model (but reserves on the other hand a treatment of a consumer price index of the type $p^\alpha_c p^h_h(1-\alpha)$ for workers expenditure composition to another paper, see Chiarella, Flaschel, Groh, Köper and Semmler (1999a,b). There is no separate savings decision $S^w_w$ of workers in the Murphy model and the issue and absorption of new assets is determined in this model basically from the supply side and is thus similar to the treatment in the model of this paper. However all adjustment speeds that we will discuss in module 6 of our model are set equal to infinity in the Murphy model which is less problematic than the case of a finite adjustment speed for asset prices and expectations about them in such a setup.

Finally, we have only a very simple demographic module in the sector of worker households, where we assume that all age groups of workers grow with the same rate (made endogenous by assuming an appropriate type of migration) and also that the participation rate of the people in the potential workforce stays constant in time (see chapter 6 in Powell and Murphy (1997) for details on this matter).

Next, we consider the other type of household of our model, the (pure) asset owners who desire to consume $C_c$ (goods and houses as supplied by firms through domestic production $Y$) at an amount that is growing exogenously at the rate $\gamma$ and which is thus in particular independent of their current nominal disposable income $Y^D_c$. The consumption decision is thus not an important decision for pure asset holders. Their nominal income diminished by the nominal value of their consumption $p_c C_c$ is then spent on the purchase of financial assets (three types of bonds and equities) as well as on investment in housing supply (for worker households). Note here that the one good view of the production of the domestic good entails consumption goods proper and houses (both at consumer prices $p_c$) so that asset holders buy houses for their consumption as well as investment purposes.

\[
2b. \quad \text{Households (Asset-Holders)}: \\
Y^D_c = (1-\tau_c)\left[\rho^c p^h K + rB_c + B^d_1 + p_h C^d_h - p_y \delta_h K_h\right] \\
\dot{C}_c = \gamma \\
S^n_c = Y^D_c - p_c C_c \\
= \dot{B}_c + \dot{B}^d_1/r^t + e\dot{B}^d_2/r^t + p_Y d + p_y (I_h - \delta_h K_h), \quad \dot{B}_c = \dot{B} - \dot{B}_w \\
C^a_h = K_h \quad [C^d_h = ... \text{see module 2a.}] \\
g^d_h = (I_h/K_h)^d \\
= \alpha_h^1((1-\tau_c)\rho_h^d - \rho^c) + \alpha_h^2(r^t - (r + \xi)) + \alpha_h^3(C^d_h/C^d_h - \dot{U}_h) + \gamma + \delta_h \\
\dot{\rho}_h = \beta_h(\rho_h - \rho^d_h) \\
\dot{g}_h = \beta_g (g^d_h - g_h), \quad g_h = I_h/K_h
\]
\[
\begin{align*}
\dot{p}_h &= \beta_p \frac{C_n^h}{C_h^d} - \bar{U}_h + \kappa_h \dot{p}_u + (1 - \kappa_h)\pi_t \\
\dot{K}_h &= I_h/K_h - \delta_h
\end{align*}
\]

Equation (23) defines the disposable income of asset holders which consists of dividend payments of firms (which distribute their whole expected profit to equity holders), interest on government bonds, \( rB_c + B_1^1 \), insofar they are held by domestic residents, rents for housing services net of depreciation, and interest payments on foreign bonds held by domestic households (after foreign taxation and expressed in domestic currency by means of the exchange rate \( e \)). Private savings of asset holders \( S^e_c \) concerns short-term and long-term bonds (domestic and foreign ones with respect to the latter), equities and net housing investment.

We assume in the following that the amount of savings of asset holders that goes into short term bonds, \( \dot{B}_c \) is given by \( \dot{B} - \dot{B}_v \), which means that asset holders passively accept the inflow (or even the outflow) of short-term bonds that is implied for them by independent decision of the government on its short-term debt policy and by the savings decision of workers (that only concerns short-term bonds). This is clearly a very restrictive assumption which – together with the treatment of the other flows of financial asset accumulation must be improved in further elaborations of the asset market dynamics of the model.

Note that there is no inside debt of the household sector (lending of asset owners to worker households). Note also that we have supplied – and not only here – a full treatment of budget equations including all feedbacks on asset accumulation that are implied by them, a degree of completeness which is missing in the Powell and Murphy (1997) model.

The supply of housing services \( C_h^e \) is assumed to be proportional to the existing stock of houses that is devoted to the supply of such services (there are no maintenance costs in the housing sector as in Powell and Murphy (1997)). We assume for simplicity that there is no resale market for dwellings. Note again that the production of dwellings is part of the production activities of firms and thus part of the homogeneous supply of the domestic (non-traded) output.

The demand for housing services has already been defined in module 2a. We assume that housing demand is always served and we can guarantee this in general – up to certain extreme fluctuations in the demand for housing services – by assuming that house owners voluntarily hold excess capacities as measured by the exogenously given desired rate of capacity utilization \( \bar{U}_h \) of the housing service supply. We have assumed in the workforce sector that their demand for housing services grows beside short term influences with trend rate \( \gamma \) (underlying the steady state of the model). This implies that housing services per household grow with trend rate \( \gamma - n \), where \( n \) is the natural rate of growth of the workforce. Therefore, over the growth horizon of the economy, we have that worker households consume more and more housing services (measured by square meters per housing unit for example).\(^{27}\)

Equation (27) of module 2b. describes the desired rate of gross investment of asset holders, which depends on the (expected!) long-run profit rate \( \rho_h^p \) in the housing sector compared with the required rate of return, measured in reference to government consols by \( \rho^* = r^l - \pi_t \) (via Tobin’s \( q \) as relative profitability measure), on the interest spread \( r^l - (r + \xi) \) as a

\(^{27}\)such a construction is needed for the discussion of steady states of the considered economy.
measure for the tightness of monetary policy (here based on an interest rate policy rule) and its perceived (or only believed) effects on the level of economic activity and employment, on the actual rate of capacity utilization with respect to housing services (representing the demand pressure in this investment behavior), \( \frac{C_h}{C_h^*} - \bar{U}_h \), on the trend rate of growth \( \gamma \) and on the rate of depreciation \( \delta_h \) in the housing sector. We assume that the actual rate of investment \( g_h \) in houses follows the desired one, \( g_h^d \), with some delay. Furthermore, the long-term rate of profit \( \rho_h \) in the housing sector follows the actual profit rate in this sector, \( \rho_h \), with a delay, an approach towards long run views in investment behavior that will also be used in the description of the dynamics of the capital stock of firms.

The rate of inflation of the rental price in housing, \( \bar{p}_h \), depends as investment on the rate of capacity utilization in the housing sector (the demand pull component) and on a weighted average formed by the actual rate of inflation of consumer of producer prices in the production of the domestic good and on the level of this inflation that is expected as a long-term average, the rate \( \pi_l \), whose the law of motion will be provided later on (the cost-push components). Finally actual gross investment plans are always realized and thus determine the rate of growth of the housing stock by deducting depreciation from them.

Summing up we can state that consumption decisions of asset owners are basically driven by exogenous habits that are independent of their income and wealth position and that their investment decision into the housing sector is preceding the other asset accumulation decisions as they derive from their choice of nominal savings. These latter decisions are in the present framework governed by supply side forces based on the new issuing of bonds by the domestic government and of equities by firms. Note here that asset holders accumulate or decumulate short-term bonds depending on the difference between their flow supply by the government and the flow demand of workers. Asset holders are thus simply adapting themselves to the decisions of these two other agents. Furthermore, their choice of accumulating or decumulating foreign long-term bonds is here determined as the residual to all these flows in or out of short-term and long-term domestic debt of the government and the flow of new equities issued by firms and is thus determined as a last step in the savings decision of asset holders. The essential decisions in this block of the model are therefore the housing investment decision and the pricing rule for housing services which is based on demand pressure as well as cost-push elements.

Powell and Murphy's (1997, p.291ff.) list of consolidated equations exhibits with respect to this module as eq. 53 the definition of private property income, see also their p.233. With respect to this equation and the whole approach chosen by Powell and Murphy (1997) we neglect in this section all investment of foreigners in the domestic capital stock (and also all public investment as a separate item in this regard). We have also no private debt to foreigners in the model. We distinguish however explicitly between short-term bonds and long-term bonds in the budget equations of asset owners and the government, a distinction that is not so obvious in the equations, such as the one under consideration, that are used in the Murphy model. Furthermore we have domestic

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28 Liquidity and risk premium with respect to long-term bond holdings.
29 Powell and Murphy (1997) use the rate of employment on the labor market in the place of this rate which is a more indirect way of expressing the demand conditions on the market for housing services.
30 Related to but also different from the approach chosen in Powell and Murphy (1997), which introduces some inertia into the housing investment decisions of asset owners.
31 This adjustment equation for rental prices in housing differs considerably from the one chosen in Powell and Murphy (1997).
private holdings of foreign long-term bonds, and thus a positive entry in the income account of asset holders in the place of their indebtedness to the foreign sector which appears as a negative entry in Powell and Murphy (1997, p. 233).\textsuperscript{32} In sum Powell and Murphy's (1997) treatment of property income may be the more appropriate one for the consideration of actual economies like Australia\textsuperscript{33} and thus needs to be considered in future elaborations of our theoretical model of a small open economy.

We have already commented on a uniform treatment of consumption behavior in the preceding subsection of this section and only note here that the above module chooses a particular simple approach in this respect as far as asset holders are concerned. Private savings $S_C^o$ (or $S_C^o + S_B^o$) is not treated explicitly in Powell and Murphy (1997), but only appears in the form of asset accumulation equations. At present it is not clear to us whether these asset accumulation equations can be consistently checked by means of the private savings that are implied by the income and the consumption equations of the Murphy model. Note also again that saving decisions of our pure asset holders concern the issue of new equities and residually the purchase of foreign bonds (if their savings exceeds their domestic wealth accumulation).

Equations 76 and 77 of the consolidated list of equations of the Murphy model, see also their pages 260 and 261, describe the accumulation identity for private debt to foreigners (in two types of currencies) as they derive from actions of the government which we provide indirectly via the saving decisions of asset holders (and which is denoted only in the foreign currency). In the Murphy model there is instead a rigid allocation rule assumed with respect to the denomination of such private debt (so that only part of it is subject to exchange rate revaluations). Eq. 73, see Powell and Murphy (1997, p.257), describes foreign owned fixed business investment as a fixed proportion of overall private business fixed investment, the latter of which is treated in the next module of our model while the former is simply lacking in our model.

Housing is treated separately in Murppy and Powell (1997, Ch.8,9), but is integrated here into the above module of the behavior of asset holders. As in the Murphy model we have strict proportionality between the stock of houses and the supply of rental services, the eq. 22 of their consolidated list (see also their p.138). The accumulation equation 79 is a direct definitional consequence of the description of housing investment (to be considered next) and is of course the same in their and our model. The dwelling investment equation 7 of their list (discussed on their p.144) exhibits a strong influence of past rate of housing investment on current housing investment which we represent in our formulation of this equation as a lagged response of actual rates of investment to the desired one. All further discussion of this investment equation therefore now refers to this desired accumulation rate for housing investment. As in the Murphy model we have the influence of the rate of return of housing services in its deviation from the required rate on the rate of housing investment, where we make use of an average concept $\rho^d$ in the place of their actual rate of return and use a rate of inflation that is expected to hold over the medium run in the place of their use of model consistent expectations.

In the place of the impact of the rate of employment on the housing investment decision we use the rate of utilization of the housing stock that is offered for rent. But as they do we use the differential between the short- and the long-term rate of interest in order to give expression to the tightness of

\textsuperscript{32}Our model will give rise to such an item if $E_B^o$ turns out to be negative in which case domestic wealth allocation of asset owners is financed to some extent by reductions in their foreign bond holdings or by foreign debt if there are no such bond holdings. Our model therefore in principle allows for foreign indebtedness of the private sector (in which case the capital account will exhibit only negative entries).

\textsuperscript{33}though their use of average rates of return in the calculation of actual properties returns is somewhat problematic from the purely theoretical point of view and its use of consistent budget equations.
monetary policy and its impact on housing investment. There is furthermore trend growth in our model as in the Murphy model, but not the constant coefficients $\alpha_0^*$, $\alpha_*^*$ that they discuss in detail in chapter 9. We bypass their equation 2 of the consolidated list, see also their p.129, since this equation is difficult to justify as an equilibrium price of rental services, which is used in the equation 1 of their list (see also p.130) describing the price dynamics of housing services. This equation is basically based on past values for actual and the just mentioned equilibrium price of housing services and is replaced in our model by a simple demand pull cost-push approach to the rate of inflation concerning rental prices.

4.3 Firms

In the following module 3. of the model we describe the sector of firms, whose planned investment demand is also assumed to be always served, just as all other consumption and investment plans. We thus assume for the short-run of the model that it is always of a Keynesian nature since aggregate demand is never rationed, due to the existence of excess capacities, inventories, overtime work and other buffers that exist in real market economies. There is thus only one regime possible, the Keynesian one, for the short-run of the model, while supply side forces come to the surface only in the medium and the long run of the model. Up to certain extreme episodes in history this may be the appropriate modeling strategy for the macro-level of a market economy. This is shown in more detail in Chiarella, Flaschel, Groh and Semmler (1999, Ch.5) for an integrated Keynes-Metzler model of monetary growth of a closed economy.

3. Firms (Technology, Production, Employment and Investment):

$$Y^p = y^K, \quad y^p = \text{const}$$

(32)

$$J^d = j^p Y, \quad j^p = \text{const}$$

(33)

$$X = x^p Y, \quad x^p = \text{const}$$

(34)

$$L^d_f = l^p \exp(-n_t)Y, \quad l^p = \text{const}$$

(35)

$$\bar{n}_t = \beta_n(\bar{n}_t - n), \quad \bar{n}_t = \bar{n}_t(g_k)$$

(36)

$$U = Y/Y^p$$

(37)

$$\tilde{L}^u_f = \beta(L^d_f - \tilde{V}^u_f L^u_f) + (\gamma - n_t)\tilde{L}^u_f, \quad \tilde{V}^u_f \in (0, 1)$$

(38)

$$g^d_k = (I/K)^d$$

(39)

$$g^d_k = \alpha_1^{(1 - \tau_e)}(r^d - \rho^d) + \alpha_2^{k_r}(r^d - (r + \xi)) + \alpha_3^k(U - \bar{U}) + \gamma + \delta$$

$$\rho^d = \beta_\rho^d(\rho^e - \rho^d)$$

(40)

$$\dot{g}_k = \beta_g(g_k - g_k), \quad g_k = I/K$$

(41)

$$Y_f = Y - Y^e = \bar{I}$$

(42)

$$S^p_f = p^b Y_f$$

(43)

$$p_e \ddot{E} = p^b(I - \delta K) + p^b(\dot{N} - \bar{I})$$

(44)

$$\dot{I} = I + \bar{N}$$

(45)

$$\dot{K} = I/K - \delta = g_k - \delta$$

(46)
As already stated we assume in the sector of firms a fixed proportions technology, with respect to the three inputs, labor $L_f^d$, imports (raw materials) $J^d$, and capital $K$, and its two outputs, (internationally) non-traded and traded goods, $Y, X$ (which are not constrained on the world markets for these two goods). Imports and exports are thus inelastically demanded and supplied by domestic firms. In addition we have endogenous Harrod neutral technological progress at the rate $n_t$ with respect to labor productivity $z = Y/L^d$ – which follows the rate of innovations $n_t(g_t)$ with some delay. We stress that the capital stock is used to measure potential output $Y^p = y^pK$ in the following, while all other magnitudes are provided by the Keynesian regime and its demand determined output rate $Y$. The rate of capacity utilization $U$ is defined on the basis of this concept of potential output and will receive importance when describing the investment behavior and the pricing policy of firms. Firms employ a labor force of amount $L_f^u$ which supplies labor effort of amount $L_f^d$ as determined by the present state of sales expectations (plus voluntary inventory production). This labor force of firms is adjusted in a direction that reduces the excess or deficit in the utilization of the employed labor force, $L_f^d - L_f^dL_f^u$, which means that firms intend to return to the normal usage of their labor force thereby. An additional growth term for the employed labor force takes account of the trend growth $\gamma$ of domestic output, but is diminished by the effect of Harrod neutral technical change which when working in isolation would allow to reduce the workforce of the firms.

Next there is the formulation of the desired gross rate of capital stock accumulation of firms which depends on four factors. Firstly, relative profitability, measured by the deviation of the long-term rate of profit $\rho$ from the required rate of interest $\rho^* = r^f - \pi^t$ via the type of calculations underlying Tobin’s $q$. Secondly, on the interest rate spread $r^f - (r + \xi)$, again representing the tightness of monetary policy and its believed effects on economic activity and employment. Thirdly, on the rate of capacity utilization $U$ of the capital stock of firms in its deviation from the desired rate of capacity utilization $\bar{U}$, which is given exogenously. Fourthly, on trend growth $\gamma$ and the rate of depreciation $\delta$ of business fixed investment. As in the case of housing investment, we assume that the actual rate of accumulation $g_k$ is following the desired one with some time delay. Furthermore, also the expected long-term rate of profit $\rho_t$ is adjusted towards the currently expected rate of profit $\rho^e$ with some time delay.

Firms produce output to cover expected demand for it and intended additions to inventories. Expected sales are also the basis of the dividend payments of firms and thus do not allow for retained earnings of firms, whose income $Y_f$ is by definition equal to their output $Y$ minus the expected sales $Y^e$, which in turn must be equal to desired inventory changes, to be defined below. Valued at producer prices $p_y$ these inventory changes thus also represent the nominal savings of firms.

Due to these assumptions, on the dividend policy of the firm in particular, and due to our assumption that firms only use equities for financing their expenditures, we get as budget

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34Note that this normal usage includes a certain amount of absentism and is thus less than the full normal usage of this labor force ($V_f^w < 1$).

35We could include here a dependence of the gross rate of investment on the rate of change of the rate of capacity utilization which would add Harrod’s accelerator to the present framework (which is in fact done in a similar fashion in Powell and Murphy (1997)).
firms finance net investment and unintended inventory changes by issuing new equities (no bonds and no bank loans are allowed at present). Note here that unintended inventory disinvestment gives rise to windfall profits to firms which are retained, not subject to taxation and used to finance part of the fixed business investment as shown in the above equation. We stress once again that this particular financing rule is not crucial for the dynamic evolution implied by the model, but should of course give way to more realistic financing conditions in later reformulations of the model. The last two equations of module 3. then define the total actual investment of firms (for national accounting purposes) and the growth rate of the capital stock which is determined by the net rate of capital accumulation planned by firms.

Powell and Murphy (1997) allow for substitution in production by using a nested input technology of CES type and CES - transformation curves with respect to the two outputs that are produced by firms. With respect to such smooth transformation functions they then define medium-run marginal cost and revenue pricing procedures which act as attractors for the development of short run prices in a particular way. We shall reconsider their approach to substitution and competitive pricing in Chiarella, Flaschel, Groh, Köper and Semmler (1999a). Furthermore, Powell and Murphy (1997) do not distinguish between actual working time and the normal working time of the employed, but use a single employment equation in their place which differs from our definitions of (efficient!) employment \( L^d \) of the employed \( L^w \) and which is not backed up by budget equations. Finally there some minor differences in the description of the investment behavior of firms which, however, do not matter very much (it can be shown that the medium run target prices of Powell and Murphy (1997) can be reformulated in terms of rates of capacity utilization; see Chiarella, Flaschel, Groh, Köper and Semmler (1999a) in this regard).

Note finally that there is here no value added tax on depreciation, investment, and planned or unplanned inventories as in the housing sector considered beforehand and that there is no direct taxation of firms. Summing up the above module of the model basically provides descriptions of the output, the employment and the investment decisions undertaken in the sector of firms and this on the basis of various delays concerning employment, investment and underlying profitability measures (the delayed output adjustment decision is described under the heading ‘quantity adjustment’ in module 5a and the price adjustments undertaken by firms are considered in block 5b of the model).

*Powell and Murphy's (1997, p.291ff.) list of consolidated equations is in this sector very different from the equations that we employ above for the description of the behavior of firms and this basically due to their use of nested CES technology descriptions with smooth factor (labor, imports, and capital) and smooth output substitution (domestically marketed commodities and exports, to be subdivided later on in addition). This is the 'neoclassical' heart of the Murphy model, see their page 149, and it will be introduced into our modeling framework (overcoming the limitations of fixed proportions technologies) in a separate paper, see Chiarella, Flaschel, Groh, Köper and Semmler (1999a). In its place we have used above a fixed proportions technology of the same generality as in the smooth substitution approach, including Harrod neutral technological change at an endogenously determined rate and also an obvious and unambiguous definition of the rate of capacity utilization of firms, to be generalized to neoclassical production technologies in Chiarella, Flaschel, Groh, Köper and Semmler (1999a). Eq.s 33 – 39 of the consolidated list of equations of the Murphy model thus*
play no role in our current representation of the enterprise production block. The same applies to the eq.s 40, 41 and 100 which have no counterpart in our present formulation of the model of this paper.

Eq. 12 of the consolidated list of the Murphy model is the business sector employment equation, see also p. 186 in Powell and Murphy (1997). In our reading of this equation, see also Chiarella, Flaschel, Groh, Köper and Semmler (1999a) in this regard, this equation is based on various measures of the capacity utilization of firms and the development of medium run employment, but not on an efficient use of labor as it is determined by means of the assumed production technology and the state of effective demand on the domestically marketed goods as in the module of the firm of the present model. In Chiarella, Flaschel, Groh, Köper and Semmler (1999a) we will therefore propose that firms employ labor in an efficient way and thus determine employment on the basis of the data that exist at each moment in time for the capital stock, for imports and exports and for domestic output, while the rate of capacity utilization will only appear in the investment and pricing behavior of firms.

The eq. 15 of the consolidated list in Powell and Murphy (1997), see also page 182 in their book, is basically the same as ours if we interpret their measure $\frac{\bar{Y}}{\bar{P}}$ as measure for the rate of capacity utilization of firms as we shall do it in Chiarella, Flaschel, Groh, Köper and Semmler (1999a), since the profitability differential with respect to the required rate of returns is the same as in the housing investment equation and thus subject to the same comments there made and since the tightness of monetary policy is also expressed in their impact on investment in the way we explained already for housing investment. As in the Murphy model there is endogenous trend growth in our fixed business investment equation. There is however an additional term $b$ in Murphy’s investment equation which refers to changes in the business cycle and which could be built into our investment equation by assuming that investment not only depends on the level of capacity utilization but also on its time rate of change, that is on a proportional and a derivative control in this respect. This is a meaningful extension of the determinants of investment behavior which however will not be integrated into our present model. As in the case of housing we distinguish the desired rate of investment (whose determinants have just been discussed) from the actual one, and use a feedback loop from desired to actual investment in the place of the influence of past investment decision on the current ones in the equation that is used in the Murphy model.

We do not allow for a capital stock outside the enterprise sector and thus do not need eq. 31 of the consolidated list of the Murphy model, see also their p.226, as well as the eq.s 27, 62 and 84 from this list which also concern public enterprise fixed business investment and which therefore are not present in our model. There remains from the list of 100 eq.s of the Murphy model the accumulation identity for the capital stock of firms which as in our model is demand determined (eq. 86 and p.262 for the Murphy model, and eq. 95 (see p.140 in Powell and Murphy (1997)) which provides the description of endogenous Harrod neutral technical change which differs formally from our approach to an endogenous rate of technical progress, but which is nevertheless similar in spirit to it.

Next, import and export prices are treated in the simplest way possible by assuming that they are fixed in terms of the foreign currency and thus need only to be multiplied with the exchange rate in order to arrive at domestic producer prices. There is no subsidy or tax on exports, but there is a tax rate on domestic producer prices of size $\tau_m$. This module of the model basically impacts the profitability of firms as measured by the expected rate of profit $\rho^e$ in the first block of our model.
3a. Export Prices and Import Prices in Domestic Currency

\[ p_m = (1 + \tau_m)ep_m^* \]
\[ p_x = ep_x^* \]  

(47)  
(48)

In contrast to Powell and Murphy (1997) there are here no inventories held with respect to imports or exports. Since imports only serve as intermediate inputs of firms there is also no need to represent them in the consumer price index of the domestic country. Note furthermore that imports are demanded and exports supplied independently of their price changes, since they are in fixed proportions to the output \( Y \) of the domestic commodity. Note finally that the PPP theory cannot be valid here, since there is no common basket of goods that is produced and used internationally.

Powell and Murphy (1997) allow for certain price responses on the world market – due to varying imports and exports of the Australian economy – which we will briefly consider in Chiarella, Flaschel, Groh, Köper and Semmler (1999a). Furthermore, they divide exports (in fixed proportions) into agricultural and non-agricultural exports which from a theoretical perspective does not contribute much to the generality of the model, but which may provide extra descriptive relevance.

Powell and Murphy’s (1997, p. 291ff.) list of consolidated equations contain a detailed presentation of import and export relationships, including two types of exports (in fixed proportions to each other however), inventories in exported commodities, lagged adjustment of imports and exports in view of desired medium run values, price reactions on the world markets with respect to price taking Australian export and import decisions, the use of price indices and an equation concerning import duties, see the eq.s 8, 9, 11, 13, 16, 17, 23, 25, 28, 30, 42, 43 and 82 of the consolidated list of equations in Powell and Murphy (1997). It is obvious that our above module on the exports and imports of firms is only a special case of this approach to internationally traded goods for the Australian economy. We will extend this approach in Chiarella, Flaschel, Groh, Köper and Semmler (1999a) and will then be much closer to the framework chosen in this respect in Powell and Murphy (1997), though we would like to stress that also the present simple form of our export and import equations captures the spirit of the treatment of trade in the Murphy model.

4.4 The government

In the next module 4. we describe the public sector of the economy in a way that allows for government debt in the steady state and for a monetary policy that fixes the rate of interest on short-term debt in view of the level of the long-term world rate of interest, the domestic rate of inflation and the domestic level of activity of firms.

\[ T^m = \tau_w[wL^d + w^n(L - L^w)] + \tau'_p wL^d + \tau_v p_y(C_w + C_o + G) \]
\[ p_N G = g p_N Y^e, \quad g = \text{const.} \]  
\[ L^d_g = \frac{L^w_g}{\alpha_g G/ \exp(n \delta)} \]  
\[ \dot{\tau} = -\beta_1 (r + \xi - \tau^*(\tau^*)) + \beta_2 (\tilde{p}_w - \tilde{\pi}) + \beta_3 (U - \tilde{U}) \]  
\[ \dot{\tau}_w = \alpha_{\tau w 1} (d/\bar{d} - 1) + \alpha_{\tau w 2} \bar{d}, \quad d = \frac{B + B^I/r^I}{p_N Y^e} \]  
\[ \dot{\tau}_m = \alpha_{\tau m} \frac{p_{2z} X - p_m J^d}{p_{2z} X} \]  
\[ S^m_g = T^m - w^u (L - L^w) - w^r \alpha_1 L_2 - (r B + B^I) - (p_N G + w^b L^d_g) \]  
\[ \dot{B} = \alpha_6^g (p_N G + r B + B^I) \]  
\[ \dot{B}^I/r^I = (1 - \alpha^g_6) (p_N G + r B + B^I) \]  
\[ \dot{B}^I = \alpha^g_6 \dot{B}^I \]  
\[ \dot{B}^I* = (1 - \alpha^g_6) \dot{B}^I \]  
\[ \tau_c = \rho p y K + r B + B^I \]  
\[ e^d_h - \delta_h K_h] \]  
\[ J^d \]

The first equation in the government module describes the tax collection by the government which consists of taxes on wages, unemployment benefits and pensions, payroll taxes as the basis of state transfers to worker households, value added taxes on consumption goods, capital taxes on profit, interest and rent (net of depreciation), and import taxes. Note that symmetric to the treatment of interest payments received and taxed abroad – we have here that all interest payments of the government (to domestic residents or foreigners) are taxed domestically and thus contribute to a reduction in domestic government debt. Note also that there are no taxes on wealth, investment, depreciation and inventories and of course none on firm income which is equal to intended inventories plus windfall profits or losses, \( Y^d - Y^e \), solely.

Government expenditures are assumed to be a fixed proportion of expected sales\(^{36}\) (at consumer prices) and employment in the government sector is a fixed proportion of real government expenditure. In view of later steady state calculations, see Chiarella and Flaschel (1999e), we assume that this employment relationship is also subject to Harrod neutral technical change (of the same type as in the sector of firms), but that government employees have fixed normal working hours and thus are never over- or underemployed as is the case for the workers in the sector of firms. Note that this implies that there is no lag in the employment policy of the government in view of its employment function shown above.\(^{37}\)

With respect to monetary policy we assume that monetary authorities determine by legislature (the change in) the nominal rate of interest on short-term bonds (which are not traded

\(^{36}\)An easy extension of this rule for government expenditures would be to assume the ratio \( g \) depends negatively on short-term and long-term interest rates \( r, r^I \), which could also be extended to the consumption decision of workforce households and which thereby give extra power to the interest policy rule of the central bank to be considered below. Note also that we do not consider delays in the adjustment of government expenditure (and the employment decisions that accompany it).

\(^{37}\)Note here that the employment level in this part of the economy is much larger than any other employment level within each firm and may therefore allow for such a direct employment policy simply due to retirement effects and the like.
internationally) by means of a Taylor type policy rule\textsuperscript{38}. There are no money holdings in the private sector of the economy and there is therefore no need to specify the new supply of money, used for open market operations as well as foreign exchange market operations. With respect to the first type of operation we observe that there is no need for it in an economy where the short-term rate of interest is directly set by the monetary authority and where transactions are performed by costless temporary credit by the branches of the central bank that must be settled in accordance with the budget constraints at the end of each day. With respect to foreign market operations we have shown in the preceding section that they are not needed as long as the private sector just absorbs the inflows of domestic bonds and equities and invests the remaining savings into long-term foreign bonds. This considerably simplifies the feedback structure between the real and the financial sector of the economy, but of course should give way to more realistic descriptions of the financial markets in future extensions of the model.

With respect to the interest rate policy of the central bank we assume that it attempts to move the actual rate of interest, \( r \), toward the steady state short-term rate of interest, \( r_o \),\textsuperscript{39} as it is determined by the world rate of interest on long-term bonds minus the liquidity premium that applies to them, but that it at the same time aims at moving the actual rate of inflation, \( \dot{p} \), toward some target rate \( \bar{\pi} \), for example from above by raising the rate of interest in order to reduce economic activity (as measured by the rate of capacity utilization \( U \)) and thus the demand pressure on the rate of inflation. Of course, high levels of economic activity \( U \) will make this decision for a tight monetary policy more pronounced than low levels of activity which explains the third term in our interest rate policy rule. In view of the fact that we will not consider inflationary processes in the world economy, but shall assume given world market prices for imports and exports in module 8 of the model, we set the target rate of inflation \( \bar{\pi} \) of the central bank equal to zero throughout this paper (in order to simplify the presentation of the interior steady state of the model).\textsuperscript{40}

Note again that money is not held as a store of liquidity and wealth by the private sector of the economy, but is only used as means of transactions that flows from banks (here only branches of the central bank) to households, firms and the government and then back to the banking sector on each trading day, without any income-in-advance restrictions.

We use \( d \) to denote the ratio between government debt and expected sales (Debt-GDP-ratio) at consumer prices and assume as policy rule for the tax rate on wages that this rate is adjusted such that government moves debt into the direction of a desired Debt-GDP-ratio \( d \) augmented by a term that describes reactions to the rate of change of the Debt-GDP-ratio \( d \) as in a derivative control feedback loop. The burden of too high debt thus falls entirely on wage income which supports our view that government bonds are net wealth. In addition, the import tax rate \( \tau_m \) is adjusted in order to reduce any possible surplus or deficit in the trade balance in terms of the domestic prices for exports and imports (which include import taxation).

\textsuperscript{38}See Flaschel, Gong and Semmler (1998) with respect to this particular choice of an interest rate policy rule.

\textsuperscript{39}or at least attempts to not let it go too far away from it.

\textsuperscript{40}We shall show in future work however that this rate should be chosen positive in order to avoid certain problems caused by the actual behavior of money wages (and that central banks actually generally have a target that is greater than zero).
The next two equations, on the debt financing of the government, are based on their left hand sides on the actual government deficit or surplus. Government revenue is based on nominal taxes $T^m$ and is used to finance nominal government expenditures $p,G$, interest payments on short- and long-term debt $(rB + B^l)$, unemployment benefits, pensions and the wage sum of state employees. The deficit that generally will come about in this way is then financed by issuing new short-term or long-term debt. We here assume that the portion $\alpha^g_t$ of the new government debt is financed short-term, while the remainder is financed long-term.

For accounting purposes we have added the definition of nominal government savings $S^m_g$ which – if negative – is financed in the just stated way through short- and long-term debt. Finally we have to state how the new long term government debt is distributed in the world. As before we here too assume that this is done in constant proportions with respect to the domestic and the foreign market for domestic long-term debt.

New assets are therefore distributed to asset owners in fairly rigid proportions (on primary asset markets) supplemented by a procedure whereby we will only introduce laws of motion for the various asset prices in the following, but will not develop a full portfolio approach to the determination of asset prices (or their rates of change) and the implied portfolio adjustments on secondary as well as on primary asset markets. Hence, asset markets are represented here solely by way of certain interest rate adjustment processes (and their impact on the investment decisions of firms and in dwellings). Asset markets are in this paper thus surely modelled less complicated than in a full portfolio approach (and the liquidity preference schedule this approach would imply for the holding of short-term debt). More or less assets therefore just flow into the private sector of the economy in proportions that are determined by the government and the firms, which represents a very tranquil way of asset absorption. It is therefore quite obvious that asset markets are the substructure of the model that need improvement most urgently, see also the module on asset price dynamics.41

For the moment we justify this approach to asset accumulation and their price dynamics by the fact that we at least provide by it a complete – though not yet really convincing – description of the dynamics of asset markets, which must be improved later on by a static or dynamic portfolio approach to the behavior of these markets that gives more role to the demand side. At present however supplies of new assets just flow into the economy – up to the foreign investment of asset holders – and lead through some type of not explicitly formulated process to interest rate differentials and the dynamics of asset prices and expectations about them as they are described in this section of the paper.

Note that this description of the government sector excludes Open Market Operations as well as Foreign Exchange Operations of the central bank. The first type of policy is not needed in an economy where the interest rate on short-term bonds is set by the central bank and where accounting money only serves the purpose of intraday trading until all budget equations are settled again. The second type of policy is not needed since the supply side description of asset markets and the accommodating behavior of asset holders with respect to foreign bonds always clears the market for foreign exchange as we have seen in the preceding

41Köper and Flaschel (1999) integrate a portfolio approach into the real dynamics of the 6D Keynes-Metzler model of Chiarella and Flaschel (1999a) and find that the implications of this portfolio approach to the real-financial interaction share many similarities with model types where the present approach to asset market dynamics is used instead.
section.

In sum we have a target rate of inflation \( \pi \) of the central bank which is here zero by assumption, a debt target per unit of expected nominal GDP which is given by \( d \), government’s expenditures which are a given share in expected GDP, and the attempts of the government to establish external trade equilibrium via import taxation. In a sense the behavior of the government is therefore still fairly neutral, though we allow for steady state debt and deficit according to certain rules. There are certain similarities between our description of the government sector and the one of Powell and Murphy (1997), in particular with respect to the wage tax rate adjustment rule. It is not difficult to add further policy rules to this module of the model, as e.g. an anticyclical government expenditure and employment policy rule, an anticyclical behavior of payroll taxes or other formulations of the Taylor interest policy rule. Later extensions and modifications of the model should concern the introduction of a banking industry (which transforms short-term debt into long-term debt, issues loans to firms and the like) and a less rigid diversification and distribution scheme for the allocation of government debt to the various other agents of the model (where we in principle have followed Powell and Murphy (1997) for the time being).

Powell and Murphy’s (1997, p.291ff.) list of consolidated equations does not contain the equation that describes government revenue, which however is presented on p.233 in Powell and Murphy (1997). This revenue is composed of wage income taxation, the taxes (at a different rate) that stem from property income, some sort of lump-sum taxation that is proportional to nominal GDP, value added taxes paid on the consumption of households, import duties, payroll taxes on the private sector’s wage bill, taxes on housing rental services and gross operating surplus from public enterprises. In our model we do not have public investment and therefore such profit incomes of the government, and no lump-sum taxes and taxes on housing services (apart from taxes on the profit that derives from them). Instead we also tax unemployment benefits, rents of retired persons (to the extent they are actually paid), payroll taxes on all employment and value added taxes on all consumption. It is of course obvious that all the difference to the Murphy approach reappears under expenditures of the government and will thus cancel with each other and thus lead to the expressions used in the Murphy model. Recall in this respect also that our definition of property income is different from the one given in Powell and Murphy (1997) as was explained in the description of the module of asset holders.

Government expenditure, eq. 66 of the consolidated list (see also p.233 in Powell and Murphy (1997)), consequently only concern wage payments net of payroll taxes, corrected by a term that corresponds to enterprise activities of the government (not present in our model), public consumption (and investment, not present in our model) at producer’s prices, inventory investment (not present in our model), nominal transfers to the old, the unemployed and for children (not present in our model), and further transfers to the whole population (not present in our model), all of course calculated net of wage taxation. Remaining terms then concern foreign aid (not present in our model), interest rate payments on government debt (only denominated in domestic currency in our model and at actual rates in the place of the average one of the Murphy model). All these expressions and concepts concern the equations 66, 63 and 64 of the consolidated list of their model and they are also built on the eq.s 58 – 61 of this list which define the aggregate value of interest-bearing debt of the public sector and its allocation to the domestic and the foreign economy, similar to the way we have done in the above module of the government sector of our model. The same holds true for the accumulation identities of government debt which are definitional in essence, see eq.s 78 and 83 of their consolidated list.
There is an equation in this list, eq. 65 (see also p.237), which represents an adjusted ratio of transfer payments to after-tax wage rate, which is not present and needed in our approach as is eq. 69 which represents some average tax rate concept as far as wage taxation is concerned (integrating an exogenous component of wage taxation that is not present in our model).

The final equation of the government block is eq. 68 of the consolidated list (see also their page 236), which concerns the wage taxation rate. As in our approach this tax reaction is based on a proportional and a derivative term as far as a particular measurement of total government debt is concerned and also on a historical value of the endogenous part of wage taxation. This extra component as well as an exogenous component of wage taxation are neglected in our present reformulation of the Murphy model.

Our treatment of government expenditures on domestic goods makes them dependent on expected sales just as the choice of government employment while Powell and Murphy (1997) seem to treat them as exogenously determined. Further differences are the interest rate policy rule we employ in the place of their conventional LM treatment of the money market, see the comments on the asset market module below and the import taxation rule which is in our model intended to give rise to balanced trade in the steady state. Finally, government savings and the allocation of government debt have already been discussed above.

4.5 Quantity and price adjustment processes

We now come to the description of the dynamics of quantities (module 5a) and prices (module 5b). Module 5a of the model basically describes a Metzlerian inventory adjustment process for the non traded good produced by firms. Module 5b describes the nominal adjustments in the goods and in the labor market, as well as the adjustment of long-term inflationary expectations $\pi'$.

\[
\begin{align*}
5a. & \quad \text{Quantity Adjustments in the Production of the Domestic (Non-traded) Good:} \\
Y^e & \neq Y^d = C_w + C_c + I + I_h + G \quad (60) \\
S^n & = S_p^n + S_i^n + S_g^n = I^{na} + NCGX^n = I^{na} + NX^n + NFLX^n \quad (61) \\
I^{na} & = p_y(I - \delta K) + p_y(I_h - \delta_h K) + p_yN \quad (62) \\
N^d & = \beta_n Y^e \quad (63) \\
I & = \beta_n(N^d - N) + \gamma N^d \quad (64) \\
Y & = Y^e + I \quad (65) \\
\hat{Y}^e & = \beta_y(Y^d - Y^e) + \gamma Y^e \quad (66) \\
\hat{N} & = Y - Y^d \quad (67)
\end{align*}
\]

The first equation in 5a contrasts expected sales $Y^e$ with aggregate demand and actual sales $Y^d$ of the non traded good for our Keynesian description of the short-run of the model. Actual sales = aggregate demand consists of five different items here (two types of consumers' goods produced...)

\[42\text{There are no sales and delivery constraints for traded goods and there is thus no direct need to consider inventory adjustment processes in their case.}\]
demand, two types of investors' demand and the government's demand for domestic goods. Next we consider once again (for consistency reasons) the accounting identity for actual total savings, actual total investments and the balance in the current or the capital account, where nominal actual total investment $I^a$ is defined by net fixed business investment and net investment in houses and by total inventory changes—everything valued at producers' prices. This equation provides an important consistency check for our analysis of goods market disequilibrium in the context of a small open economy. It also implies, see subsection 2 of section 3 of this paper, that the flows of new assets supply are equal to the absorption of these supplies by the household sector (workers and asset holders).

The remaining five equations describe the inventory adjustment process. Desired inventories $N^d$ are a constant fraction of expected sales $Y^e$. Intended inventory changes $I$ are proportional to the gap between desired inventories and actual ones, $N$, plus a term that accounts for the fact that inventory formation takes place in a growing economy with trend growth $\gamma$. Output decisions $Y$ are based on the sum of expected sales and intended inventory changes, while sales expectations $Y^e$ are changed in an adaptive way through the observation of the discrepancy between actual sales $Y^a$ and the expected ones $Y^e$, again augmented by a term $\gamma Y^e$ that accounts for the trend growth underlying the evolution of this economy. Finally, actual inventory changes $\dot{N}$ are just given by the difference between actual output and actual sales, which once again gives expression to our general assumption that the short-run of our economy is always of a Keynesian nature and not perfectly foreseen by the agents of our economy. This inventory adjustment process is the same as the one in Powell and Murphy (1997) with the exception that sales expectations are always correct in the Murphy model.

Powell and Murphy's (1997, p.291ff.) list of consolidated equations is in this case basically identical to our treatment of inventories of the domestically marketed commodity, up to the fact that we allow for disappointed sales expectations and thus for a truly Metzlerian inventory adjustment process and neglect inventory activities of the government. There are three accounting identities used for sales, GDP and GNE in the Murphy model, see the eqs 26, 96 and 97 of the consolidated list of this model. The sales composition equation for the domestic good, see their page 224, contains as additional items with respect to our formulation of it, purchases for housing maintenance and public investment, which are both missing in our model. We stress again that these sales are correctly perceived by firms in the Murphy model so that there is no adjustment mechanism needed for sales expectations. GNE is defined as eq. 97 of the consolidated list and consists of sales of the domestic good and inventories of this good, valued at producer prices, the nominal value of the supplied rental services net of maintenance costs, inventories in export commodities, gross wages in the government sector (net of payroll taxes) and value added tax on the consumption goods supplied to the household sector. Gross domestic product is then defined, in eq. 96 (see also page 260), by adding net exports to GNE. The GNE concept is used in defining money demand in the Murphy model and the GDP concept appears in the description of the balance of payments, both still to be discussed in the modules that follow below.

The remaining equations describe domestic production, see eq. 81 in the consolidated list, as in our model as the sum of (expected) sales plus voluntary inventory changes, determine planned and actual domestic inventory investment somewhat differently from our formulation, see equation 10 in the consolidated list, by referring to trend growth (as we do) but also by including a time trend component and import considerations. The similarity to our approach becomes more obvious on the pages 216/7 in Powell and Murphy (1997) where the derivation of this inventory adjustment equation is explained in some detail. In our model we neglect the term which formulates a partial proportionality of planned inventories with excess import supply. The equation 81 of the consolidated
list of the Murphy model for the actual change in inventories is of course only a definitional matter. Note here again that there is no difference between desired and actual stocks of inventories in the Murphy model.

Next we consider the wage price dynamics of the model. This type of dynamics is receiving more and more attention in recent studies of primarily empirical orientation\(^{43}\) and thus represents an important module of the present stage of modeling the details of a small open economy with an integrated treatment of its short-, medium- and long-run behavior. We stress however that we do not yet treat consumer price indices and the role of import prices in the formation of the money wage and the price level Phillips curves, respectively, see Chiarella, Flaschel, Groh, Köper and Semmler (1999a) in this regard.

\[5b. \quad \text{Wage-Price Adjustment Equations, Expectations:}\]

\[\dot{w}^h = \beta_{wh}(V - \bar{V}) + \beta_{wh}(V_f^w - \bar{V}_f^w) + \kappa_w(\bar{p}_v + \bar{n}_t) + (1 - \kappa_w)(\pi^t + \bar{n}_t) \quad (68)\]

\[\dot{p}_v = \bar{p}_y = \beta_{p}(U - \bar{U}) + \kappa_p(\dot{w}^h - \bar{p}_v) + (1 - \kappa_p)\pi^t \quad (69)\]

\[\pi^t = \beta_{\pi}(\alpha_{\pi}(\bar{p}_v - \pi^t) + (1 - \alpha_{\pi})(\bar{n}_t - \pi^t)) \quad (70)\]

\[L^w = L_f^w + L_g^w = L_f^w + L_g^w \quad (71)\]

\[V = L^w/L = V_f + V_g = L_f^w/L + L_g^w/L \quad (72)\]

\[V_f^w = L_f^d/L_f^w, \quad [V_g^w = L_g^d/L_g^w = 1] \quad (73)\]

\[\dot{\bar{V}} = \beta_{\ell}(V - \bar{V}) \quad (74)\]

Wage inflation \(\dot{w}^h = \dot{w}\) is nearly of the same type as in Powell and Murphy (1997). Wage inflation responds in the traditional Phillips curve manner; to the state of the demand pressure in the labor market as measured by the deviations of the rate of employment \(V\) from its NAIRU-level \(\bar{V}\); to the deviation of the employment rate \(V_f^w\) of the employees of firms from their norm (including absentism) which is measured by \(\bar{V}_f^w\) (and which corresponds to the derivative term for the rate of employment, \(V\), that Powell and Murphy (1997) employ in this place); and to the usual accelerator term of price inflation which is here measured as a weighted average of actual price inflation based on short-term perfect foresight (plus the actual rate of productivity growth) and expected long-term price inflation (plus the long-run rate of productivity growth) in the place of the simple adaptive scheme used by Powell and Murphy (1997). Wage inflation is therefore governed by demand pull terms augmented by a weighted average of cost-push expressions.

The law of motion for consumer prices \(p_v\) of the non-traded commodity is formulated in a similar way, as a second type of Phillips curve. In the place of the concept of medium run prices used by Powell and Murphy (1997) we use the demand pressure measure \(U - \bar{U}\), the deviation of actual capacity utilization from its norm, as one cause of price inflation. In Chiarella, Flaschel, Groh, Köper and Semmler (1999a) we shall show that this measure is closely related to the medium run price concept of Powell and Murphy (1997) in the case of smooth factor and output substitution. The cost push term in the price inflation equation is given as a weighted average of current wage inflation and the one expected for the long-run

\(^{43}\)See Fair (1997a,b), and Stock and Watson (1997).
(both made less severe in their influence on price inflation by the existence of a positive growth rate of labor productivity, now and in the longer run).

Expected long-term inflation $\pi^f$ in turn is based on a weighted average of two expectations mechanisms, an adaptive one with weight $\alpha_\pi$ and a forward looking one with weight $1 - \alpha_\pi$. Forward looking expectations are here simply based on the inflation target of the central bank $\pi$, in the usual way of a regressive scheme of expectations revision. Inflationary expectations are thus following a weighted average of actual inflation and the target rate of the monetary authority.

This description of the wage-price spiral is based on formulations used extensively in Chiarella and Flaschel (1998a) and is therefore not explained in more detail here.

The following equations describe some definitions concerning total employment (through firms and the government) and the outside rate of employment $V$ as well as the inside rate of employment $V^w$ of the employed. The last equation, finally, assumes that the NAIRU rate of employment follows the actual rate of employment with some delay. We here deviate again from Powell and Murphy (1997) who consider this rate of employment as being determined exogenously. Note that firms follow the rate $V^w$ when deciding on the change in the workforce they employ.

We can see from the above description that only the inflation rate of non-traded domestic goods matters in the wage price module of our economy. Housing, i.e., the rental price of dwellings (and its rate of change $\dot{p}_h$) is thus completely ignored in this description of the wage price interaction. This simplifies the feedback structure of the model, but should give way to a domestic price index of the form $p_c = p^a_n p_h^{1-a}$ and its rate of change in the wage equation in future reformulations of the model.

Powell and Murphy's (1997, p.291ff.) list of consolidated equations contains with respect to the module of the present section only two equations, eqs 4 and 14, that are discussed in Powell and Murphy (1997) on pages 107 and 219 respectively. Eq. 4 is the money wage Phillips curve where adaptively formed inflationary expectations are already inserted in the usual way of an augmented Phillips curve of the monetarist type (with unity coefficient). This equation is nonlinear (strictly convex) in the rate of unemployment and it exhibits the rate of change of the rate of employment as a derivative control term (as in the original Phillips approach) which is implied in our model by the assumptions made on insiders and outsiders and the employment policy of the firm with respect to the over- or underemployment of their insiders. The other equation, the price dynamics for the domestically marketed good, is primarily based on the concept of medium run target prices $p^M_N$ of firms as they derive from the nested CES technology used in Powell and Murphy (1997). These prices are determined such that the current output becomes an optimal output of firms and they will be related to capacity utilization calculations in such a setup in Chiarella, Flaschel, Groh, Körper and Semmler (1999a). Murphy's domestic inflation equation thereby basically reduces to an equation where inflation is driven by capacity utilization considerations of firms augmented by a cost-push term that is based on the rate of change of $p^M_N$. There is a constant term in this equation which in our reformulation of it will imply that the rate of inflation will rise before full capacity utilization is achieved and which therefore need not create the problems that are observed in Powell and Murphy (1997, p.220) in this respect (since it simply defines the desired rate of capacity utilization which is less than one, see Chiarella, Flaschel, Groh, Körper and Semmler (1999a) in this regard).

In the place of our concept of capacity utilization Powell and Murphy (1997) use an output capital ratio $b$, see the eq. 99 in the consolidated list, to introduce accelerator relationships into the in-
vestment behavior of firms, but not into the pricing behavior of them as it has just been discussed. The eq. 49 of their list finally introduces the symbol $\xi$ to denote the actual rate of inflation which in the form of model consistent expectations is used in investment equations but not in the price wage dynamics. With regard to such expectations and the adaptive ones employed in the money wage equation we therefore have formulated in our wage price dynamics an intermediate situation which is based on forward and backward looking expectations, but not either only 'rational' or purely adaptive ones. Note also that our wage and price dynamics is based on two Phillips curves of the demand pull - cost push variety.

Note finally that our model assumes that the NAIRU level of the rate of employment is determined endogenously in the place of the exogenous determination of the this rate in Powell and Murphy (1997, p. 106/7).

4.6 The dynamics of asset market prices and expectations

The sixth module lists the dynamic adjustment equations we assume to hold for the asset prices of our model: long-term domestic bonds, $p_b$, equities, $p_e$, and for the exchange rate (in view of the given US $-rate of return on foreign bonds). We stress that reallocations of the stock of wealth are not considered explicitly in the present version of the model (which also does not yet allow for wealth effects in the behavioral assumptions that are employed).

As already discussed in the preceding descriptions of the modules of the model, asset flows and asset accumulation are determined by supply side conditions in the main in the present form of the model and are thus just absorbed by asset holders, at least as far as short-term bonds (leaving aside those already purchased by worker households), long-term domestic bonds and equities are concerned. In contrast to this, asset holders are supplied with an investment demand function as far as their housing investment is concerned (which is never rationed) and they balance their savings account thereafter by purchasing or selling foreign bonds on the world market. This is surely only a preliminary approach to the accumulation of financial wealth and its distribution to the two household sectors we consider. We shall return to this question in Chiarella, Flaschel, Groh, Köper and Semmler (1999a) where money holdings are added to the model and are adjusted in time via changes in the short-term rate of interest.

The adopted approach to asset accumulation is acceptable in a continuous time framework, if there is subsequent stock reallocation according to a specified money demand function and if all other asset can be considered as perfect substitutes for each other, since asset holders are in such a case indifferent with respect to their holding of interest bearing bonds, see Sargent (1987) for example. In the present approach there is however no stock demand for transaction balances and thus no explicit reallocation of stock positions that have been changed by flows of new assets into the asset markets. Furthermore, due to somewhat delayed responses of asset prices to expected interest rate differentials we depart in the following from the perfect substitutability assumption. Thus we have to acknowledge that asset market dynamics is not yet well-founded as far as conceivable behavior of individual holders of financial assets is concerned. Franke and Semmler (1997) provide a portfolio approach with imperfect substitutabilities to the determination of the temporary structure of interest rates of the economy which will be adapted to the present framework in either
stock or flow form in future extensions of the model.

6. Asset Prices, Expectations and Interest Rate Adjustments:

\[ \dot{p}_b = \beta_{p_b}[(1 - \tau_c)r^l + \pi_b - ((1 - \tau_c)r + \xi)] \quad \dot{p}_b = -\dot{r} \]  
\[ \dot{p}_e = \beta_{p_e}[(1 - \tau_c)\rho^e p_b K \overline{p}_e E + \pi_e] - ((1 - \tau_c)r^l + \pi_b) \]  
\[ \dot{\epsilon} = \beta_{\epsilon}[(1 - \tau_c^*)r^* + \epsilon - ((1 - \tau_c)r^l + \pi_b)] \]  
\[ \dot{\pi}_{bs} = \beta_{\pi_{bs}}(\dot{p}_b - \pi_{bs}), \quad p_b = 1/r^l \]  
\[ \pi_{bc} = \hat{\pi}_b \]  
\[ \pi_b = \alpha_s \pi_{bs} + (1 - \alpha_s)\pi_{bc} \]  
\[ \dot{\pi}_{es} = \beta_{\pi_{es}}(\dot{p}_e - \pi_{es}) \]  
\[ \pi_{ec} = \hat{\pi}_e \]  
\[ \pi_e = \alpha_s \pi_{es} + (1 - \alpha_s)\pi_{ec} \]  
\[ \dot{\epsilon}_s = \beta_{\epsilon_s}(\dot{\epsilon} - \epsilon_s) \]  
\[ \epsilon_c = \hat{\epsilon} \]  
\[ \epsilon = \alpha_s \epsilon_s + (1 - \alpha_s)\epsilon_c \]  

Note first of all with respect to the three laws of motion for the bond price, the share price and the nominal rate of exchange, that they have to be based on interest rate differentials after taxes, but that there is no taxation of actual capital gains in the model, and thus no tax term applied to expected capital gains in the formulae shown above.

Instead of a full portfolio approach to asset market equilibria it is assumed in the above adjustment equations for asset prices \( p_b, p_e \) and the exchange rate \( \epsilon \) that stocks in asset markets give rise to forces that imply certain laws of motion for their prices. The law of motion for the price of long-term domestic bonds, \( p_b = 1/r^l \), for example, states that the rate of change of \( p_b \) is determined by the differential between the net rate of return \( (1 - \tau_c)r^l + \pi_b \) on long-term bonds (including expected capital gains \( \pi_b \)) and the short-term rate of interest \( (1 - \tau_c)r \) (of fixed-price bonds and after taxes) augmented by a liquidity and risk premium \( \xi \) that is exogenously given. In the limit, \( \beta_{p_b} = \infty \), we interpret this dynamic law as an equilibrium relationship: \( (1 - \tau_c)r^l + \pi_b = ((1 - \tau_c)r + \xi) \) which could then be used as in Blanchard (1981) to study the conventional type of saddlepoint adjustment processes based on the jump variable technique. Yet it is not at all clear under which circumstances fast, but finite adjustments of consols prices will lead to a dynamics that mirrors such saddlepoint dynamics obtained in the limit. On the contrary, considerations of small size models as in Flaschel, Franke and Semmler (1997) have shown that nothing of this type can be expected in general. Therefore, we stick to the assumption that the rate of interest \( r^d = 1/p_b \) of long-term bonds follows the movement of the short-term rate of interest \( r \) (in the above assumed way) with some delay which may be very short, but which is larger than zero.

The next equation describes the evolution of equity prices \( p_e \) in a similar way. Their rate of change, \( \dot{p}_e \), is driven by the discrepancy between the rate of return on equities (dividend payments of firms after taxes and expected capital gains \( \pi_e \) per equity value) and the rate
of return on long-term bonds after taxes (including expected capital gains). Note here that 
\[(1 - \tau_c)e^{r_d E} \beta_p K\] describes the actual dividend payments per unit of equity value and represents 
a nominal rate of return which – leaving capital gains aside – has to be compared with the 
nominal rate of interest on bonds, \((1 - \tau_c)r^l\) in order to make the correct rate of return 
comparison. Assuming \(\beta_p = \infty\) would again imply the often used assumption that 
long-term bonds and equities are considered as perfect substitutes and would thus lead to asset 
market representations of a more conventional type.

Finally, we assume that the dynamic of the exchange rate, \(\hat{e}\), is also based on an expected 
interest rate differential, namely between domestic and foreign long-term bonds (both after 
taxes), the latter augmented by the expected capital gains from possible devaluations, \(\epsilon\), of 
the domestic currency and the former by expected capital gains on domestic bonds. Note 
here that the foreign rate of interest is exogenously given so that there is no comparable 
capital gain on long-term foreign bonds. An increase in the above differential makes foreign 
bonds more attractive which leads to a capital outflow and thus an increase of the demand 
for the foreign currency which is the (here implicit) cause for the increase of the exchange 
rate \(\hat{e}\) implied by equation (77). Assuming \(\beta_p = \infty\) would lead us to the limit assumption of 
uncovered interest rate parity (UIP) – often employed in the literature – which in this paper 
is however subject to some time delay.

With respect to expected capital gains on long-term bonds and equities we assume heteroge-
neous expectation formation. On the one hand, there is technical or time series analysis (for 
a certain group of asset owners), which here boils down to an exponentially weighted formula 
based on past observations or simply an adaptive formation of expectations with speeds of 
adjustment \(\beta_{\pi_\alpha}, \beta_{\pi_e}\) (lazy ‘retired’ or less well informed asset owning agents which we here 
identify with the fraction of elderly people among the asset holders). On the other hand, 
there exist a portion of asset owners with correct expectations \(\hat{p}_\alpha, \hat{p}_e\) (ambitious ‘younger’ or 
well informed agents who achieve myopic perfect foresight by sacrificing leisure time).

We thus assume that the establishment of myopic perfect foresight is very time consuming 
(reducing the leisure time of the ambitious wealth owners significantly). There is thus only 
a certain fraction, the ‘younger’ ones of the population of all pure asset owners, who devote 
themselves and their leisure time to this formidable task (represented by \(1 - \alpha_s\)), while 
the other (the lazy ones among the wealth owners) rely on less time consuming time series 
analysis in order to make their predictions of temporary asset price changes. The market 
opinion is then simply reflected on the macroeconomic level by the average of these two 
expectations generating mechanisms formed by means of the weight \(\alpha_s, 1 - \alpha_s\) in each case 
(long-term bonds and equities). In line with the dominant view of currently prevailing 
economic theory we consider the lazy agents as the stupid ones and the ambitious agents as 
the clever ones.

Due to their myopic perfect foresight clever agents of course perform better than the stupid 
one and thus will have a higher overall rate of return than this latter group. Yet, since 
they will change their behavior in later parts of their lives (due to changing habits and 
obligations of people that get older) they will only temporarily outperform the market and 
accumulate wealth at a higher pace. The overall effect of the existence of these two groups 
of people among the pure asset holders is that markets would adjust their prices according 
to the interest rate differentials perceived by the stupid agents, but that the existence of
clever agents and their perfect short-run expectations works such that the former interest rate differentials are transformed and corrected to some extent into the directions the clever agents see them to be. This is due to the redirection of portfolio demands as they come from this second group of agents which are not made explicit here. Adaptive expectations that are too high with respect to actual changes in asset prices are thereby made less severe in their impact on asset demands and resulting asset price changes.

In order to justify this approach to expectations formation further we now insert this average expectation on asset price or exchange rate changes into the corresponding price formation rule which in the case of long-term bonds, for example, implies the following final form for the adjustment of bond prices:

$$\dot{p}_b = \frac{\beta_{p0}}{1 - \beta_{p0}(1 - \alpha_s)}[(1 - \tau_c)r^t + \alpha_s \pi_{bs} - ((1 - \tau_c)r + \xi)]$$

together with equation (78) for $\pi_{bs}$. Increases in the population and the weight $1 - \alpha_s$ of the clever ones among the asset owners (with their time-consuming establishment of myopic perfect foresight), starting from $\alpha_s = 1$ therefore increases the volatility of bond prices and the difficulties to predict them perfectly, since it increases the adjustment speed with which the interest rate differentials as viewed by stupid agents (weighted by their fraction in the total population of asset owners) is transferred into bond price changes.

There is an absolute upper limit with respect to this increase in volatility which is represented by the critical proportion $\alpha_s^*$ of 'lazy' asset owners given by:

$$\alpha_s^* = (\beta_{p0} - 1)/\beta_{p0} < 1.$$ 

At $\alpha_s^*$ the speed of adjustment of bond prices has become infinite while it is still finite (and working into the right direction) for all admissible $\alpha_s$ that are larger than $\alpha_s^*$. Decreasing the number of lazy asset owners therefore is bounded by this critical value $\alpha_s^*$ where the young workaholics must finally lose sight of the true behavior of asset prices and the exchange rate.

We do not however investigate in this paper the adjustments that may take place in the share $\alpha_s$ of time series based expectations according to some switching mechanism between the two groups of asset owners here considered, but assume instead that this proportion is constant in time at a 'balancing age' where asset owners switch from ambitiousness to laziness. Endogenous changes in this dividing line and also other reasons for such a switch should be incorporated at a later stage.\textsuperscript{44}

Whatever the outcome of such a discussion may be, we thus here simply assume that there is a mechanism at work that creates heterogeneous asset owner behavior and heterogeneous expectation formation of a type and extent that prevents the model converging to situations of overall myopic perfect foresight in the financial markets. Rather a situation is reached where asset price reactions to interest rate不同ials are still normal with respect to direction and are finite.

Of course, these considerations of long-term bond price dynamics apply to the dynamics of equities in the same way and as the above module of asset prices shows also to the dynamics of the exchange rate and the expectations mechanisms there assumed. In all three cases we

\textsuperscript{44}See Brock and Hommes (1997), Chiarella and Khomin (1999) and Sethi (1996) for examples of analysis of the implication of such switching mechanisms.
assume therefore a lower limit for the proportion or market share of ‘lazy’ or ‘stupid’ asset owning households given by

$$\alpha^*_s = (\beta_s - 1)/\beta_s$$

where $x$ stands for the asset market under consideration. Note that we must assume that $\alpha_s$ is larger than all three critical ratios that are generated on the three considered asset markets in order to have normal reactions of asset prices and exchange rates on all three markets.

We stress that all interest rate comparisons are made with respect to gross levels of these rates (not net of taxes at the rate $\tau_e$), i.e., the tax rate $\tau_e$ does not show up in these laws of motion for interest rates, asset prices and the rate of exchange. Taking net rate in the place of gross ones would only complicate the above formulae without any change in substance.

This closes the description of the behavior we assume for the asset markets of the economy. Powell and Murphy (1997) assume only ‘clever’ agents or ‘workaholics’ to exist in their formulation of asset market behavior and assume in addition that adjustment speeds of asset revaluations are always infinite, leading them to the usual interest rate parity conditions as for example for the comparison of domestic and foreign long-term bonds: $(1 - \tau_e)\tau^{r^d} + \hat{e} = (1 - \tau_d)\tau^r$. Their model thus is based on the limiting case of myopic perfect foresight in the asset markets which leads them to the then usual jump variable technique as assumed representation of the forward-looking behavior of agents and restricts the dynamics of the model to its stable manifold (thereby removing all local instability from sight). We do not follow this procedure in our formulation of the dynamics of the model, which only partly incorporates forward looking behavior on the asset markets, whose dynamical implications are – as will be seen – radically different from those with complete myopic foresight on the asset markets and perfect adjustments of asset price (no interest rate differentials).

The consequence is of course that we will have differentiated rates of return at each moment in time, without formulating a full portfolio approach to take account of the non-uniformity of these rates. In sum, we must state that the description of the asset market adjustment processes represents the module of our model where future improvements are needed the most. At present ‘causality’ runs from the interest rate policy of the central bank (with respect to short-term debt which is not traded internationally) to adjustments in the rate of interest of long-term debt to adjustments of the exchange rate (in view of internationally traded long-term bonds) to changes in the rate of profit expected by firms. There are also adjustments in equity prices based on the rate of return for long-term bonds but these adjustments do not feed back into the model due to the lack of wealth effects and more advanced financing rules for firms (and due to the hierarchy chosen for the adjustment of asset prices). With the exception of the latter type of dynamics we have effects of the above changes in asset markets on business fixed investment and housing investment, but this is basically all that relates the real and the financial part of our economy. Note here also that the monetary authority is steering the short-term rate of interest basically from an anti-inflationary perspective and that it can do so to some extent since short-term debt is not traded internationally. Of course, it has to accept than the consequences that result from the adjustments of the long-term rate of interest and nominal exchange rates.
Powell and Murphy’s (1997, p.291ff.) list of consolidated equations is very brief with respect to the representation of this module. We have a conventional (Cagan) form of money demand function, eq. 18 of the consolidated list, which is based on gross national expenditure (GNE), which is eq. 97 of the consolidated list. This function can be explicitly solved for the short-term rate of interest since it is linear in logarithms and it provides an interest rate steering condition that is based on the ratio between money supply and GNE solely. There is thus stock market equilibrium assumed with respect of the usual representation of the LM curve and liquidity preference theory, while we have assumed in sections 2 – 4 that there is no need for keeping transaction balances in the private sector of the economy and have replaced the above money supply oriented interest rate policy rule through a Taylor type rule that is no longer referring to the money supply, but is setting the short-term nominal rate of interest in a more direct and a more diversified way. In eq. 57 of their consolidated list, Powell and Murphy (1997) furthermore provide the definition of the growth rate of the money supply which is exogenous to the Murphy model. Finally, in view of our above module 6, describing the dynamics of the asset prices, Powell and Murphy (1997) assume in their consolidated list in eq.s 45, 50, 51 (see also their pages 250, 178, 246) that long-term domestic bonds are perfect substitutes for short term domestic bonds as are long-term foreign bonds. This together with the assumption of model consistent expectations (also for the rate of inflation that is used by investors) provides them with an equation for the term structure of interest rates and the uncovered interest rate parity condition. We have assumed in our module 6, that asset prices do not immediately adjust to these two types of conditions and that there are heterogeneous expectations of asset holders with respect to asset returns. In the limit, where all the adjustment speeds that are involved in this module are set equal to infinity we would however obtain the rational expectations solution of equilibrated asset markets used in the Murphy model, but would then either have to apply the conventional jump variable technique of the rational expectations approaches or study the limit process just mentioned in its definitely complicated details.

4.7 External accounts and foreign country data

The next module 7. describes the various items that appear in the balance of payments Z, nominal net exports $NX^n$, nominal net (international) interest payments $NFX^n$ and nominal net capital exports $NCX^n$. Concerning nominal net interest payments, which are normally interpreted as net ‘factor’ exports $NFX^n$ and which need not cross borders and thus need not appear as an item in the current account, we have in fact assumed that they do cross borders. They are fully present in the calculation of the disposable income of wealth owners and also in the current account of the balance of payments Z. We stress again that the balance of payments must be balanced in our model due to assumed behavior of asset holders with respect to the domestic supply of debt and equities and the international adjustments that residually follow from them.

7. Balance of Payments:

\[

NX^n = EX^n - IN^n = ep^n X - ep^{n,wd}
\]

\[

NFX^n = e(1 - \tau^e)B^e_1 + e(1 - \tau_c)B^e_c
\]

\[

NCX^n = e\ddot{B}^e_1 - \dot{B}^e_1 / \ddot{r}
\]

\[

Z = NX^n + NFX^n - NCX^n = 0
\]
Powell and Murphy’s (1997, p.291ff.) list of consolidated equations contains the balance of payments description as equation 74 (see also their page 258) and it is based on a more diversified list of interest payments and other transfers besides the trade account as far as the current account is concerned, including equity investment returns (where equities are represented by the capital stock items owned by foreigners) as profit payments based on the gross operating surpluses of firms. The capital account on the other hand exhibits New Borrowing Overseas (NBO) and equity investment inflow, represented as net investment in the domestic capital stock. There is also a balancing item ABL in this equation, which may be due to various statistical discrepancies, but also contain elements of foreign exchange market operations of the central bank as they occur in actual economies. A question we have with respect to this representation of the balance of payments is to what extent the average rates of return here used are consistent with the assumption of rational expectations and asset market equilibrium rates of interest considered in the preceding module of our model. We do not have international trade in equities in our model yet, and thus can ignore this item in the balance of payments as well as eq. 75 of the consolidated list of the Murphy model which provides the accumulation identity for foreign owned business fixed capital.

Module 8. finally provides the data needed from the ‘foreign’ economy in the simplest form possible. It is assumed that the modeling of the foreign economy is based on the same qualitative principles we used for the description of the domestic economy and that it is inflation-free, exhibits a constant rate of growth and a constant rate of interest on long-term bonds.

8. Foreign Country Data:

\[
\begin{align*}
    r^{i*} & = \text{const.} \quad \text{(world interest rate)} \\
    \tau_c^* & = \text{const.} \quad \text{(foreign tax rate = } \tau_c \text{ by assumption)} \\
    \gamma & = \text{const.} \quad \text{(world growth rate)} \\
    p^* & = \text{const.} \quad \text{(world price level of the export good)} \\
    p^*_m & = \text{const.} \quad \text{(world price level of the import good)}
\end{align*}
\]

Powell and Murphy’s (1997, p.291ff.) list of consolidated equations here finally contain the given short- and long-term interest rates of the world market, eqs 70 and 71 of their consolidated list, which are determined as a weighted average of the respective rates for the US, Japan, Germany and the UK. We consider in our model the rest of the world as one country and thus do not have to use such average concepts in our model. Furthermore, as we allow only for international trade of long-term bonds for the moment we only use the foreign long-term rate of interest as a datum for our model and do not consider its relationship to the short-term foreign rate which in fact is the one that appears in the uncovered interest rate parity condition of the Murphy model, while there long-term foreign rate appears in the form of a moving average in the calculations that concern their representation of the balance of payments.

There are various exchange rate identities in the Murphy model, see the eqs 46, 47, 48, and 72 which all boil down to one equation for the exchange rate e in the model of this paper. Finally, there is the definition of the given foreign rate of inflation which in this case is not a weighted average of the countries that are confronted with the Australian economy in the formulation of the Murphy model. We do not consider inflation in the world economy (as well as in the domestic economy) in the present formulation of our model, but simply assume given US-dollar world prices for export and import commodities.

50
This closes the description of the extensive or structural form of the model of a small open economy and its detailed comparison with the structure of the Murphy model for the Australian economy.

We stress once again that the short-run of the model is Keynesian throughout which means that supply bottlenecks can either be avoided through appropriate buffers or have to be added still for larger deviations of the economy from its steady state behavior as described in Chiarella, Flaschel, Groh, Köper and Semmler (1999a).²⁴⁵

Summarizing our comparison with the Murphy model as presented in Powell and Murphy (1997) we can state that model basically differ in the range of assets they allow in the financial part of the economy where we use a disequilibrium approach to asset market dynamics and expectations while the Murphy model rests on interest rate parity conditions coupled with perfect foresight of investors, both with respect to financial as well as real investment which is a limit case of the approach we have adopted. It may be that this limit case is the only convincing case of the situations we allow for asset market dynamics as far as a pure flow treatment of these markets is concerned. In our view this would imply that the asset market module of our model must be replaced by a full portfolio approach in later reformulations of the model as it is presented in Franke and Semmler (1997).

The largest difference is the difference in the treatment of production as far as formal difficulties are concerned. Powell and Murphy (1997) have to solve an eight dimensional nonlinear equation system in their treatment of the objectives of firms on the background of their nested CES-technology while we have only explicit linear expressions for the same procedures in the case of a fixed proportions technology of the same type. We will show in Chiarella, Flaschel, Groh, Köper and Semmler (1999a), however, how the approach chosen for the Murphy model, its so-called neoclassical heart, can be integrated into the model of the present paper and that the qualitative dynamic behavior of the model remains the same.

Further differences concern household behavior where we will show in Chiarella, Flaschel, Groh, Köper and Semmler (1999a) how the representative life cycle approach of the Murphy model can be integrated into our heterogeneous household framework. There we will also allow again for the conventional type of money holding and a Cagan type money demand function for the two types of households we consider. There are also many further, but generally minor differences to the Murphy model which will not be investigated here. In view of this we would nevertheless claim, even at the present stage of the investigation, that the model of this paper and the Murphy model are very similar in spirit, though of course different in purpose still, with respect the weights these two model types give to theoretical or applied considerations.

²⁴⁵see Chiarella, Flaschel, Groh and Semmler (1999) for the details of such an extension which explicitly includes the possibility of and the reaction to supply bottlenecks.
5 Outlook: Intensive form, reduced dynamics and numerical simulations

We have introduced and discussed in this paper in great detail an integrated macrotheoretic model of monetary growth for a small open economy in extensive or level form which had many features in common with the macroeconometric model for the Australian economy as presented in Powell and Murphy (1997). Our aim in this paper was to provide a starting model which can provide macrofoundations to applied models of the Murphy type, in the twofold sense that all budget equations of all agents that are considered are spelled out in their details and in their consequences (the assumed behavior of economic agents of course being consistent with these budget restrictions) and secondly that there is a fully specified steady state solution to be used as consistency check and as reference path for the dynamics implied by the model.

For the moment we have used fairly conventional macro descriptions for the behavior of households, firms and the government without demonstrating how they can and have been microfounded in the literature. Specific microfoundations may be provided later on and may change some of the modules we have presented here to a certain degree, but we expect that they will not change the general outlook and type of investigation of the presented description of labor-, goods-, and asset market dynamical (disequilibrium) adjustment processes. In our view, macrofoundations (or macroperspectives) come first (before micro-foundations), since they provide the overview across the modules of the structure to be studied in their dynamic interdependence, while microfoundations are needed later on to obtain hopefully a firmer basis and more convincing formulation of the modules used in the initial macrostructure.

Our next steps in pursuing the project of macrofoundations and macro analysis in this way further will aim at obtaining first of all a thorough presentation of the intensive or state variable form of the model (explaining its characteristics in detail also on this level). We then calculate (with respect to real magnitudes) on this basis a uniquely determined interior steady state solution of the model and study the comparative dynamic implications it gives rise to. This will lead us to a 34 dimensional nonlinear dynamical model and its steady state solution which in this general form is difficult to understand with respect to the many economic features it contains and which generally can only be investigated from the numerical point of view.

In order to approach the understanding of such large disequilibrium growth models in a systematic way we shall simplify them in various ways in a sequence of subsequent papers. We make use there of a core 18D model that is obtained from the general version by suppressing certain secondary feedback structures of the full 34D dynamics. This 18D model can be further reduced to a basic 6D Keynes-Metzler-Goodwin type real dynamical model of a closed economy as it was introduced in Chiarella and Flaschel (1999a, Ch. 6). Starting from this model type various routes of extending it back to the 18D structure will then be investigated and compared in their numerical behavior, see Chiarella, Flaschel and Zhu (1999a,b) for details.

Next, from the theoretical point of view, we shall isolate all the partial feedback mechanisms that are contained in the 18D core case in order to discuss their stabilizing or destabilizing
potential from a theoretical point of view. This will add extra insights to the numerical investigations already made and will often allow us to predict how the full 18D model behaves when some of these feedback mechanisms become more pronounced, see Chiarella and Flaschel (1999f) for details. In this way we will arrive at a method of understanding of large theoretical (but small applied) macrodynamical models which is quite new, since these models have rarely been studied in the literature from the theoretical perspective, see Barnett and He (1998) for an exception. It is our opinion that there is urgent need for such investigations of the dynamical features of applied or applicable integrated macrodynamical models and that tools are now indeed available for the achievement of progress on this frontier. We expect that one outcome of this analysis will be that applied structural disequilibrium models of monetary growth will exhibit a rich menu of attractors (points, limit cycles, quasi periodic orbits and also more complex ones) and also interesting transient behavior towards such attractors that will severely question the narrow, but still prevailing view of only steady state attractors, as far as the deterministic part of published macroeconometric models is concerned. In our view this understanding will drastically change the way such models are conceived and utilized in theory as well as in applications in the future.

References


Appendix: Notation

The following list of symbols contains only domestic variables and parameters. Foreign magnitudes are defined analogously and are indicated by an asterisk (*). To ease verbal descriptions we shall consider in the following the 'Australian Dollar' as the domestic currency (A$) and the 'US Dollar' ($) as a representation of the foreign currency (currencies).

A. Sthatically or dynamically endogenous variables:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>Output of the domestic good</td>
</tr>
<tr>
<td>Y_d</td>
<td>Aggregate demand for the domestic good</td>
</tr>
<tr>
<td>Y_p</td>
<td>Potential output of the domestic good</td>
</tr>
<tr>
<td>Y_dp</td>
<td>Normal sales of the domestic good</td>
</tr>
<tr>
<td>Y_n</td>
<td>Normal output of the domestic good</td>
</tr>
<tr>
<td>Y_e</td>
<td>Expected sales for the domestic good</td>
</tr>
<tr>
<td>Y_w, Y_c</td>
<td>Nominal disposable income of workers and asset-holders</td>
</tr>
<tr>
<td>Y_f</td>
<td>Income of firms</td>
</tr>
<tr>
<td>L_1</td>
<td>Population aged 16 – 65</td>
</tr>
<tr>
<td>L_2</td>
<td>Population aged 66 – ...</td>
</tr>
<tr>
<td>L_0</td>
<td>Population aged 0 – 14</td>
</tr>
<tr>
<td>L_d</td>
<td>Total employment of the employed</td>
</tr>
<tr>
<td>L_w</td>
<td>Total employment of the work force of firms</td>
</tr>
<tr>
<td>L_g</td>
<td>Total government employment (= public work force)</td>
</tr>
<tr>
<td>L_f</td>
<td>Work force of firms</td>
</tr>
<tr>
<td>V^* (V_f)</td>
<td>(Normal) Employment rate of those employed in the private sector</td>
</tr>
<tr>
<td>a_t</td>
<td>Participation rate of the potential work force</td>
</tr>
<tr>
<td>V = L_d / L</td>
<td>Rate of employment (V the employment–complement of the NAIRU)</td>
</tr>
<tr>
<td>C_w(C_w*)</td>
<td>Real (equilibrium) goods consumption of workers</td>
</tr>
<tr>
<td>C_h(C_h*)</td>
<td>Real (equilibrium) goods consumption of asset owners</td>
</tr>
<tr>
<td>C = C_w + C_h</td>
<td>Total goods consumption</td>
</tr>
<tr>
<td>C^d</td>
<td>Supply of dwelling services</td>
</tr>
<tr>
<td>C_d^h</td>
<td>Demand for dwelling services</td>
</tr>
<tr>
<td>I</td>
<td>Gross business fixed investment</td>
</tr>
<tr>
<td>I_h</td>
<td>Gross fixed housing investment</td>
</tr>
<tr>
<td>I^a (I^na)</td>
<td>Gross (net) actual total investment</td>
</tr>
<tr>
<td>I</td>
<td>Planned inventory investment</td>
</tr>
<tr>
<td>N</td>
<td>Actual inventories</td>
</tr>
<tr>
<td>N_d</td>
<td>Desired inventories</td>
</tr>
<tr>
<td>r</td>
<td>Nominal short-term rate of interest (price of bonds p_b = 1)</td>
</tr>
<tr>
<td>r^l</td>
<td>Nominal long-term rate of interest (price of bonds p_b = 1/r^l)</td>
</tr>
<tr>
<td>p_b = p_b*</td>
<td>expected appreciation in the price of long-term domestic bonds</td>
</tr>
<tr>
<td>p_r</td>
<td>Required rate of interest</td>
</tr>
<tr>
<td>p_e</td>
<td>Price of equities</td>
</tr>
<tr>
<td>p_e = p_e*</td>
<td>expected appreciation in the price of equities</td>
</tr>
<tr>
<td>S^n</td>
<td>Total nominal savings</td>
</tr>
<tr>
<td>S^n_p</td>
<td>Nominal savings of households</td>
</tr>
<tr>
<td>S^n_f</td>
<td>Nominal savings of firms (= p_y Y_f, the income of firms)</td>
</tr>
<tr>
<td>S^n_g</td>
<td>Government nominal savings</td>
</tr>
<tr>
<td>T^a(T)</td>
<td>Nominal (real) taxes</td>
</tr>
<tr>
<td>G</td>
<td>Real government expenditure</td>
</tr>
<tr>
<td>p_s</td>
<td>Expected short-run rate of profit of firms</td>
</tr>
<tr>
<td>p_a</td>
<td>Actual short-run rate of profit of firms</td>
</tr>
<tr>
<td>p_l</td>
<td>Expected long-run rate of profit of firms</td>
</tr>
<tr>
<td>p_h</td>
<td>Actual rate of return for housing services</td>
</tr>
<tr>
<td>p_h^l</td>
<td>Expected long-run rate of return for housing services</td>
</tr>
</tbody>
</table>
\( K \)  
Capital stock

\( K_h \)  
Capital stock in the housing sector

\( w^b \)  
Nominal wages including payroll tax

\( w \)  
Nominal wages before taxes

\( w^u \)  
Unemployment benefit per unemployed

\( w^r \)  
Pension rate

\( p_v \)  
Price level of domestic goods including value added tax

\( p_v \)  
Price level of domestic goods net of value added tax

\( p_x \)  
Price level of export goods in domestic currency

\( p_m \)  
Price level of import goods in domestic currency including taxation

\( p_h \)  
Rent per unit of dwelling

\( \pi^i = \hat{\pi}^e \)  
Expected rate of inflation

\( \epsilon = \hat{\epsilon}^e \)  
Expected rate of change of the exchange rate

\( L \)  
Labor supply

\( B \)  
Stock of domestic short-term bonds  (index d: stock demand)

\( B^w \)  
Short-term debt held by workers

\( B^c \)  
Short-term debt held by asset owners

\( B^l \)  
Stock of domestic long-term bonds, of which \( B^l_1 \) are held by domestic asset-holders  (index d: demand)

\( B^l_2 \)  
by foreigners  (index d: demand)

\( B^l_2 \)  
Foreign bonds held by domestic asset-holders  (index d: demand)

\( E \)  
Equities  (index d: demand)

\( W^n, W \)  
Nominal and real domestic wealth

\( n \)  
Natural growth rate of the labor force  (adjustment towards \( \tilde{n} \))

\( n_{l} \)  
Rate of Harrod neutral technical change  (adjustment towards \( \tilde{n}_{l} \))

\( X \)  
Exports

\( J^d \)  
Imports

\( NX^n = px - e_{p_m^*} J^d \)  
Net exports in terms of the domestic currency

\( NFX^n \)  
Net nominal factor export payments  (in A$)

\( NCX^n \)  
Net nominal capital exports  (in A$)

\( \tau_w \)  
tax rate on wages, pensions and unemployment benefits

\( \tau_m \)  
tax rates on imported commodities

\( \tau^n \)  
total taxes per value unit of capital

\( g_k, g_k \)  
Desired and actual rate of growth of the capital stock \( K \)

\( g_h, g_h \)  
Desired and actual rate of growth of the housing capital stock \( K_h \)

\( d \)  
Actual public debt / output ratio

B. Parameters of the model

\( \delta \)  
Depreciation rate of the capital stock of firms

\( \delta_h \)  
Depreciation rate in the housing sector

\( \alpha^j_1 \)  
All \( \alpha \)-expressions  (behavioral or other parameters)

\( \beta^z_i \)  
All \( \beta \)-expressions  (adjustment speeds)

\( \gamma \)  
Steady growth rate in the rest of the world

\( \hat{U} \)  
Normal rate of capacity utilization of firms

\( \hat{U}_h \)  
Normal rate of capacity utilization in housing

\( \kappa_{u}, \kappa_{p} \)  
Weights of short- and long-run inflation  \((\kappa_u\kappa_p \neq 1)\)

\( \kappa \)  
\( (1 - \kappa_u\kappa_p)^{-1} \)

\( y^p \)  
Output-capital ratio

\( x^p \)  
Export-output ratio

\( l^p \)  
Labor-output ratio

\( j^p \)  
Import-output ratio

---

\(^{46}\)The parameters of the nonlinear extensions of the model are described later on.
\( p^*_m \)  
World market price of import commodities

\( p^*_e \)  
World market price of export commodities

\( d \)  
Desired public debt / output ratio

\( \xi \)  
Risk and liquidity premium of long-term over short-term debt

\( \xi_e \)  
Risk premium of long-term foreign debt over long-term domestic debt

\( \tau_c \)  
Tax rates on profit, rent and interest

\( \tau_v \)  
Value added tax rate

\( \tau_p \)  
Payroll tax

\( c_1 \)  
Propensity to consume goods (out of wages)

\( c_2 \)  
Propensity to consume housing services (out of wages)

C. Further notation

\( \dot{x} \)  
Time derivative of a variable \( x \)

\( \ddot{x} \)  
Growth rate of \( x \)

\( \tau_o, etc. \)  
Steady state values

\( y = Y/K, etc. \)  
Real variables in intensive form

\( m = M/(p_vK), etc. \)  
Nominal variables in intensive form

GBR  
Government Budget Restraint