Towards Applied Disequilibrium Growth Theory: II

Carl Chiarella
Peter Flaschel

ISSN: 1036-7373
Towards Applied Disequilibrium Growth Theory:
II. Intensive form and steady state analysis of the model

Carl Chiarella
School of Finance and Economics
University of Technology, Sydney
Sydney, Australia

Peter Flaschel
Faculty of Economics
University of Bielefeld
Bielefeld, Germany

December 1, 1999
Towards Applied Disequilibrium Growth Theory:
II. Intensive form and steady state analysis of the model

Carl Chiarella
School of Finance and Economics
University of Technology, Sydney
Sydney, Australia

Peter Flaschel
Faculty of Economics
University of Bielefeld
Bielefeld, Germany

Abstract

In this paper we investigate further the 34D applied structural model whose extensive form we introduced in Chiarella and Flaschel (1999c). Here we express the model in terms of intensive form state variables, thereby abstracting from the underlying growth trend. We explain the dynamic (and static) laws of the model directly in terms of the intensive form variables, and then determine the steady state and its characteristics. Finally we show how a small number of simplifying assumptions, concerning in particular consumption of asset holders and some secondary adjustment processes, reduce the 34D model to an 18D core model. It is this latter core model whose detailed structure, steady state characteristics and dynamical behavior will become the object of study in the remaining papers of this sequence.
1 Introduction

In this paper we derive and investigate the 34D intensive (state variable) form of the applied structural model of disequilibrium growth we have introduced and discussed on its originally extensive form level in great detail in Chiarella and Flaschel (1999c). We will represent the resulting 34D dimensional dynamical system from various perspectives, providing compact intensive form representations of the real and the financial sector of this economy in table form and also in form of a system of national accounts. We will then discuss to some extent the economic contents of the obtained laws of motion from their intensive form perspective, thereby showing that the model can be understood from the outset on this intensive form level.

Presenting the system from these various perspectives serves the purpose of making the reader acquainted with the notation and the relationships that apply on the intensive form level of the model. We hope that this increases the readability of the laws of motion for quantities (including rates of growth), for prices (including wages, asset prices, and also expectations), financial asset accumulation and feedback fiscal and monetary policy rules to be presented and discussed in section 3 of the paper. Section 4 then calculates the (up to the determination of nominal variables) uniquely determined steady state solution of this dynamical system and briefly considers its comparative dynamic properties which are generally very simple in nature. We then go on and show that the dimension of the dynamics can be significantly decreased by only a few simplifying assumptions (leading us from 34D to 18D dynamics) whereby we obtain what we will call the 18D core dynamics of our approach to disequilibrium growth.

We shall briefly compare these dynamics in section 6 of the paper with the fourteen equations second order system of Bergstrom, Nowman and Wymer (1994), a prominent example from the literature on continuous time macroeconometric model building and testing. We then use an approach as the one by Barnett and He (1998), who reconsider the 14 equation model just mentioned from the numerical perspective, in order to also study the numerical properties of our 18D core dynamics in particular with respect to the role played by speeds of price and quantity adjustment. In the present paper, however, we shall for the time being only use eigenvalue calculations based on one-parameter changes in order to see which adjustment speeds (and their corresponding feedback chains) are stabilizing in the full 18D dynamics, and which are destabilizing when they are increased. In future work we will also calculate as in Barnett and He (1998) bifurcation boundaries in two-parameter spaces (which bound the regions of local asymptotic stability of the system) and will then show that the occurring bifurcations are essentially of the Hopf type (which at present is only a conjecture based on earlier work on such disequilibrium growth dynamics).

In a concluding section we will finally briefly describe the ways in which the model introduced in Chiarella and Flaschel (1999c) will be investigated, both analytically and numerically, in a series of further papers that are in preparation, see our work cited in the list of references, thereby indicating what can be done at present in the analysis of reasonably detailed applicable macroeconomic models of disequilibrium growth.

---

1See also Bergstrom (1996) for a brief survey on this literature.
2 The Real and the Financial Structure on the Intensive Form Level

The following two tables provide a survey of the structure of the economy to be investigated in the following and they do this on the basis of what has been presented and discussed in Chiarella and Flaschel (1999c) with respect to the extensive structural form of a general disequilibrium monetary growth model by transferring this discussion to the intensive form level and related steady state calculations. This paper therefore continues the analysis begun in Chiarella and Flaschel (1999c) by showing that this model type has a well-defined intensive-form state variable representation and also a basically (up to the level of nominal variables) uniquely determined interior steady state or balanced growth path solution.

2.1 The structure of the real sector of the economy

Let us start with a presentation of the variables that comprise the real part of the economy to be considered which, as already stated, are all recalculated here in per unit of capital form as far as the side of quantities is concerned, plus in efficiency units in the case of labor, and also in efficiency units in the case of wage rates, since these variables also would exhibit a positive trend otherwise (since they rise with labor productivity on average). Price levels, however, are at present without trend in the considered model, since it is assumed that the central bank follows an interest rate policy rule with a zero target rate of inflation, which restricts the steady state solution of the dynamics to zero.

2See the appendix to this paper for the employed notation.

3In order to clarify the notation used and the contents it represents the reader should therefore utilize this original presentation of the model.
Table 1 describes the real sector of the considered economy. We have a labor market, three commodity markets and the housing market. Domestic production \( y = Y/K \), per unit of capital, concerns one good that is only domestically used (for all private consumption \( c_w + c_c \), all investment \( g_h, g^d, I/K \), also into housing, and all government consumption \( g = G/K \) and which uses up all the imports \( j^d \) as intermediate goods) and one good that is only used for exports \( x \). There is thus only a single commodity used in domestic absorption – up to the housing services \( c^d_h \) demanded by workers. We denote the demand for this domestically produced and absorbed commodity by \( y^d(= Y^d/K) \).

Our model exhibits three domestic sectors: Households, firms and the government, but with heterogeneous agents in the household sector, workers and (pure) asset holders, the former supplying their labor \( l^e \) (measured here in efficiency units) at the gross wage level \( w^{be} \) (which includes payroll taxes) and the latter the housing services \( c^h \) for the workers. Firms produce a non-traded domestic and an exported commodity and employ labor \( l^e_f \) (with varying rates of utilization \( l^e_f \)) and imports \( j^d \) (besides their capital stock \( K \)) for these purposes, and invest into fixed business capital \( g^d_h \) (per unit of capital) and inventories \( I/K \). Government finally provides public consumption goods \( g \), pays rents \( w^{re} \) and unemployment benefits \( w^{ue} \) and also employs part of the workforce \( l^{de}_g \). There is endogenous growth \( n \) of the potential labor force \( L_1 \), of the capital stock \( K \), by \( g^d_h - \delta \) and of the stock of housing \( K_h \), by \( g^d_h - \delta_h \), (supplied at price \( p_h \) for rental services) and also actual change of inventories \( \nu = N/K \) that is different from their desired rate of change \( I/K \).

\[^4\dot{x} \text{ the growth rate of a variable } x.\]
2.2 The structure of the financial sector in intensive form

Let us next consider the financial part of the economy. Note that all stock variables $B, ..., E$ (and their rates of change) appearing here are measured relative to the gross value of the capital stock $p, K$ based on prices $p_0$ that include value added tax. They are then denoted by lowercase Latin letters (and by $\varepsilon$ in the case of equities $E$).

<table>
<thead>
<tr>
<th></th>
<th>Money</th>
<th>Short-term Bonds</th>
<th>Long-term Bonds</th>
<th>Equities</th>
<th>Foreign Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workers</td>
<td></td>
<td>$\dot{B}_w b_w[= \frac{B_w}{p, K}]$</td>
<td>$\dot{B}_l l_l[= \frac{B_l}{p, K}]$</td>
<td>$\dot{E}_e[= \frac{E_e}{p, K}]$</td>
<td>$\dot{B}_l l_l[= \frac{B_l}{p, K}]$</td>
</tr>
<tr>
<td>Asset holders</td>
<td>$-$</td>
<td>$\dot{B}_c b_c[= \frac{B_c}{p, K}]$</td>
<td>$\dot{B}_l l_l[= \frac{B_l}{p, K}]$</td>
<td>$\dot{E}_e[= \frac{E_e}{p, K}]$</td>
<td>$-$</td>
</tr>
<tr>
<td>Firms</td>
<td>$-$</td>
<td>$-$</td>
<td>$\dot{B}_l l_l[= \frac{B_l}{p, K}]$</td>
<td>$\dot{E}_e[= \frac{E_e}{p, K}]$</td>
<td>$-$</td>
</tr>
<tr>
<td>Government</td>
<td>$-$</td>
<td>$\dot{B}_b[= \frac{B_b}{p, K}]$</td>
<td>$\dot{B}_l l_l[= \frac{B_l}{p, K}]$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>Prices</td>
<td>1</td>
<td>$1 [\tau]$</td>
<td>$p_0 = 1/\tau^l$</td>
<td>$p_e$</td>
<td>$e p_e^* = e \cdot 1/\tau_1^*$</td>
</tr>
<tr>
<td>Expectations</td>
<td>$-$</td>
<td>$-$</td>
<td>$\pi_b = \pi_b^*$</td>
<td>$\pi_e = \pi_e^*$</td>
<td>$\epsilon = \epsilon^*$</td>
</tr>
<tr>
<td>Stocks ($/p, K$)</td>
<td>$-$</td>
<td>$b = b_w + b_c$</td>
<td>$b^l = b_l^l + b_l^*$</td>
<td>$\varepsilon$</td>
<td>$b^*_l$</td>
</tr>
<tr>
<td>Growth</td>
<td>$-$</td>
<td>$\dot{B}_b, \dot{B}_w, \dot{B}_c$</td>
<td>$\dot{B}_l, \dot{B}_l^i$</td>
<td>$\dot{E}$</td>
<td>$\dot{B}_l^*$</td>
</tr>
</tbody>
</table>

Table 2: The financial part of the economy.  
(Foreign country data: $\tau_1^*, \tau_c^* = \tau_c$)

The first column in the above table shows that we do not consider money holdings in the model of this paper, see Chiarella and Flaschel (1999c) for details. At present there are only (four) interest-bearing financial assets in our model that can be held by the (pure) asset owners and by the workers of our economy (as shown in the above table). As in the Keynes-Metzler model of monetary growth of closed and open economies, see Chiarella and Flaschel (1999a,b), we here assume, in order to start with a simple representation of financial flows, that bonds are only issued by the government, that firms use only equity financing and pay out expected earnings as dividends, and that there exist also long-term bonds issued by the 'foreign government'. Financial flows between the sectors of our economy are therefore very narrowly defined. Note that we allow for savings out of wages in the present model (in a Kaldorian way) and that workers save only in the form of short-term debt (interest-bearing saving deposits held at the local branches of the central bank). All other assets (plus the remainder of short-term debt) are exclusively held by the (pure) asset holders of our model. We stress that this formulation has served the purpose of simplifying the budget.
constraints of the agents in Chiarella and Flaschel (1999c), but should be extended in future reformulations of the model.

This is the basic structure we assume for our economy which will be further explained in the next section from the viewpoint of national accounting before we present and discuss the intensive form of the model of Chiarella and Flaschel (1999c).

2.3 The Structure of the Economy from the Viewpoint of the System of National Accounts

We shall consider in this subsection the production accounts, income accounts, accumulation accounts and financial accounts of the four internal agents in our economy: firms, asset holders, workers, and the government (including the monetary authority). These accounts – plus the balance of payments – provide basic information on what is assumed for these four sectors and which of their activities are still excluded from the present theoretical framework of a small open economy.

2.3.1 The sector of firms

We start with the accounts of the firms – shown in table 3 – which organize production \( y \) (per unit of capital), employment \( l_f^e \), in efficiency units, of their workforce \( l_f^{we} \) and gross business fixed investment \( g_f \) (per unit of capital) and which use (in the present formulation of the model) only equities \( e \) (per value unit of capital) as financing instrument (no debt in the form of bank loans or bonds issued by firms). There are value added taxes \( \tau_v \) on consumption goods, import taxes \( \tau_m \) and payroll taxes \( \tau_p \) with respect to hours worked \( l_f^e \), but no further taxation in the sector of firms and there are also no subsidies paid to this sector.

Firms build dwellings – which are of the same type as all other domestic (non-traded) production – and sell them to the asset holders (as investors) and thus have no own investment occurring in the housing sector. They sell consumption goods to workers, asset holders and the government, export goods to the world economy, organize fixed gross investments with respect to their capital stock (as well as voluntary inventory changes \( \mathcal{I}/K \) with respect to finished goods) and experience involuntary inventory changes \( N/K = (y - y^d)/\nu \) (per unit of capital) due to the deviation of aggregate demand \( y^d \) from output \( y \) (which is based on expected sales \( y^e \) and planned inventories \( \mathcal{I}/K \) (everything per unit of capital).

Firms use up all imports as intermediate goods which thereby become part of the unique homogeneous good that is produced for domestic purposes. They have replacement costs with respect to their capital stock, pay indirect taxes and wages including payroll taxes. Their accounting profit is therefore equal to expected profits (based on sales expectations and paid out as dividend to equity owners) and retained profits (equal to planned inventories). As

\footnote{The fifth agent, the foreign economy, is represented by the balance of payments at the end of this subsection and is confined to steady state behavior in the present form of the model. All demands of this foreign sector are indexed by \( * \), while its supply of long-term bonds \( B_2 \) to domestic residents is indexed by \( 2 \).}
is obvious from the narrow income account of firms, firms thus only save an amount equal to their intended inventory changes. The accumulation account is self-explanatory as is the financial account which repeats our earlier statement that the financial deficit of firms is financed by the issuing of new equities solely.

Note that all investment is valued (and performed) net of value added tax and thus at producer prices $p_y$ in the place of the consumer prices $p_v = (1 + \tau_v)p_y$. Indirect taxes (value added taxes)\(^8\) thus only fall on consumption activities and not on gross investment (of any kind, i.e., here also not on housing investments and the inventory investment of firms). There are furthermore no direct (capital) taxes in the sector of firms, neither on property nor on profits, since our model will be formulated in such a way that all expected profits are distributed to asset holders and since there are no taxes on windfall profits (unexpected retained earnings – or losses – of firms that help to finance investment). Note however that the wages $w^{be}$ (in efficiency units) paid by firms include payroll taxes $\tau_p w^e$ (for unemployment insurance, health and other social insurance, and retirement pensions) and that wage income $w^e$ of workers is taxed at the rate $\tau_w$ in addition. Note finally that the accumulation account of firms is based on realized magnitudes and thus refers to their intended inventories $I/K$ only indirectly (via the expected rate of profit $\rho^e$, to be distinguished from the actual rate of profit $\rho^a$, which includes windfall profits or losses, due to unfulfilled sales expectations $g^e$ of firms, $\bar{N}/K = \bar{N}N/K, N/K = \nu$).

### Production Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imports $e p_m^* j^d / p_v$</td>
<td>Consumption $c_w$</td>
</tr>
<tr>
<td>Depreciation $\delta p_y / p_v$</td>
<td>Consumption $c_c$</td>
</tr>
<tr>
<td>Indirect Taxes $\tau_v (c_w + c_c + g^d)p_y / p_v + \tau_m e p_m^* j^d / p_v$</td>
<td>Consumption $g^d$</td>
</tr>
<tr>
<td>Wages (including payroll taxes) $w^{be+p+e}$</td>
<td>Exports $p_x x / p_v$</td>
</tr>
<tr>
<td>Profits $\rho^a p_y / p_v + (I/K) \cdot (p_y / p_v) = \rho^a p_y / p_v + \bar{N} v p_y / p_v$</td>
<td>Gross Investment $g_k / p_v$</td>
</tr>
<tr>
<td></td>
<td>Durables (Dwellings) $g_k k_k p_v / p_v$</td>
</tr>
<tr>
<td></td>
<td>Inventory Investment $\bar{N} v p_y / p_v$</td>
</tr>
</tbody>
</table>

\(^8\)There is however a tax on the imports made by the firms.
Income Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividends $\rho^p p_v / p_v$</td>
<td>Profits $\rho^p p_v / p_v$</td>
</tr>
<tr>
<td>Savings $S^p_\tau / (p_v K) = (I/K)(p_v/p_v)$</td>
<td></td>
</tr>
</tbody>
</table>

Accumulation Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Investment $g_v p_v / p_v$</td>
<td>Depreciation $\delta p_v / p_v$</td>
</tr>
<tr>
<td>Inventory Investment $\hat{N} v p_v / p_v$</td>
<td>Savings $S^\tau_\tau / (p_v K)$</td>
</tr>
<tr>
<td>Financial Deficit $FD / (p_v K)$</td>
<td></td>
</tr>
</tbody>
</table>

Financial Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Deficit $FD / (p_v K)$</td>
<td>Equity Financing $p_e \hat{E} \epsilon$</td>
</tr>
</tbody>
</table>

Table 3

Note with respect to table 3 that profit rates are calculated per net value of the capital stock $p_v K$, since there is no value added tax $\tau_v p_y$ on all purchases intended for investment purposes, while the relative variables and the intensive form of the model is calculated by definition per gross value of the capital stock $p_v K$, $p_v = (1 + \tau_v) p_y$, which implies that the ratio $p_v / p_u$ has to be used in various places in order to correct for such differences in the valuation of the capital stock. Note also that we do not always supply short-cut expressions for the intensive form variables, in those cases where such a lower case representation is not need in the presentation of the laws of motion of the model in section 3. Note finally that we use $\epsilon$ in the place of $e$ for denoting $E/K$ since $e$ is used to denote the nominal exchange rate of our model.

2.3.2 The sector of households: Asset holders

Table 4 gives the accounts of the sector of asset-holders for whom we already know that the investment in housing as well as the supply of housing services has been exclusively allocated
to this sector. The production account in this regard shows the actual sale (not the potential sale) of housing services (= demand for housing services by assumption) which is divided into replacement costs and actual earnings or profits on the uses side of the production account.

Income of asset holders comes from various sources: interest payments on short- and long-term domestic bonds and on long-term foreign bonds (net of tax payments which must be paid abroad), dividend payments of firms (based on their expected profit) and profits from housing rents. All domestic profit income is subject to tax payments at the rate $\tau_c$ and after tax income by definition is divided into the consumption of domestic commodities (including houses, but not housing services) and the nominal savings of asset owners.

The accumulation account shows the sources for gross investment of asset-holders in the housing sector, namely depreciation and savings, the excess of which (over housing investment) is then invested into financial assets as shown in the financial account. Note here that short-term bonds are fixed price bonds $p_b = 1$ (which are perfectly liquid), while long-term bonds have the variable price $p_b = 1/r^t$ (and fixed nominal interest payments of one unit of money per period) which shows that they are of the type of consols or perpetuities (the same holds true for imported foreign bonds, which are of long-term type solely).

**Production Account of Households (Asset Owners including Housing Investment):**

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation</td>
<td>Rent $p_h c_h^d/p_v$</td>
</tr>
<tr>
<td>$\delta_h k_h p_u/p_v$</td>
<td></td>
</tr>
<tr>
<td>Earnings</td>
<td></td>
</tr>
<tr>
<td>$\Pi_h/p_v K$</td>
<td></td>
</tr>
</tbody>
</table>

**Income Account of Households (Asset Owners):**

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax payment $\tau_c r b_c$</td>
<td>Interest payment $r b_c$</td>
</tr>
<tr>
<td>Tax payment $\tau_c b_1^t$</td>
<td>Interest payment $b_1^t$</td>
</tr>
<tr>
<td>Taxes $\tau_c (p_h c_h^d/p_v - \delta_h k_h p_u/p_v)$</td>
<td>Interest payment $e(1 - \tau_c^*) b_2^t$</td>
</tr>
<tr>
<td>Tax payment $\tau_c p_y/p_v$</td>
<td>Dividend payment $\rho^s p_v/p_v$</td>
</tr>
<tr>
<td>Consumption $c_c$</td>
<td>Earnings $\Pi_h/p_v K$</td>
</tr>
<tr>
<td>Savings $S_c^u/p_v K$</td>
<td></td>
</tr>
</tbody>
</table>

---

Footnote: And are thus not perfectly liquid, since there is no 'money back' guarantee here for the sector of asset owners as a whole.
Accumulation Account of Households (Asset Owners):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Investment $g_kk \times p_y/p_o$</td>
<td>Depreciation $\delta_kk \times p_y/p_o$</td>
</tr>
<tr>
<td>Financial Surplus $FS/p_o K$</td>
<td>Savings $S^*_o/p_o K$</td>
</tr>
</tbody>
</table>

Financial Account of Households (Asset Owners):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term bonds $B_c d_c$</td>
<td>Financial Surplus $FS/p_o K$</td>
</tr>
<tr>
<td>Long-term bonds $p_c B[l_b]$</td>
<td></td>
</tr>
<tr>
<td>Foreign Bonds $e_p B_z d_z$</td>
<td></td>
</tr>
<tr>
<td>Equities $p_e E \epsilon$</td>
<td></td>
</tr>
</tbody>
</table>

Table 4

2.3.3 The sector of households: Workers

The next set of accounts, the ones of worker households, are fairly simple and easy to explain and are given in table 5. First, there is no production account in this sector. Income of the members of the workforce, which may be employed, unemployed or retired, thus derives from wages, unemployment benefits or pension payments where $l^e = \alpha_l l^w$ denotes the total number of persons in the current workforce ($l^w$ the part that is employed) and $\alpha_l l^w$ the number of retired people which have access to pension funds ($\alpha_l=\text{const. the participation rate of the potential workforce } l^s$). To this we have to add the interest income on saving deposits (short-term bonds) which is taxed at the general rate used for financial asset income. All wage type incomes are subject to taxation at the rate $\tau_w$ and are by definition divided into nominal consumption (consumption goods and housing services) and savings. Note here that the employment $l^w$ of the employed $l^w$ can differ from their normal employment which is measured by $l^w$, the number of persons that are employed (reduced to efficiency units and per unit of capital). Note also that wages $w$ (in efficiency units) are calculated net of payroll taxes.

We assume in the following that workers have a positive savings rate and that they hold their savings in the form of short-term bonds solely which is mirrored here in the accumulation and finance account in a straightforward way.
Production Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Income Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Taxes</strong> $\tau_w \left[w^c l^d e + w^u e (l^e - l^u e) + w^r e \alpha i_t^2 \right] + \tau_c r b_w$</td>
<td><strong>Wages</strong> $w^e l^d a$</td>
</tr>
<tr>
<td>Consumption $c_w + p_h q_h^d/p_v$</td>
<td><strong>Unemployment benefits</strong> $w^u e (l^e - l^u e)$</td>
</tr>
<tr>
<td></td>
<td><strong>Pensions</strong> $w^r e \alpha i_t^2$</td>
</tr>
<tr>
<td></td>
<td><strong>$rb_w$ Interest payments</strong></td>
</tr>
<tr>
<td><strong>Savings</strong> $S^a_w/p_v K$</td>
<td><strong>$S^a_w/p_v K$</strong></td>
</tr>
</tbody>
</table>

Accumulation Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial Surplus</strong> $FS/p_v K$</td>
<td><strong>Savings</strong> $S^a_w/p_v K$</td>
</tr>
</tbody>
</table>

Financial Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Short-term bond accumulation</strong> $\hat{B}_{w b_w}$</td>
<td><strong>Financial Surplus</strong> $FS/p_v K$</td>
</tr>
</tbody>
</table>

Table 5

2.3.4 The public sector: Government and Monetary Authority

There are finally the accounts of the fiscal and monetary authority (see table 6) which due to the many taxation schemes and transfer payments that are assumed are more voluminous than the preceding accounts – at least with respect to the income account. There is first however a fictitious production account where the supply of public goods is valued at production costs which consist of government expenditures for goods and labor.
The resources of government income consist of taxes on the various forms of workers' income (taxed at a uniform rate), of taxes on the various forms of profit, interest and rent income (again taxed at a uniform rate), payroll taxes, value added taxes and import taxes. Uses of this tax income of the government are interest payments, transfers to the unemployed and the retired persons and the costs of the mentioned government 'production'. In general all these uses of the tax income of the government will exceed its income so that there will result a negative amount of nominal savings $S_p^u/p_u K$ which balances the income account of the government.

There is no accumulation of real assets in the government sector which means that we only have to look into the financial account of the government to see how the excess of government outlays over government revenue is financed through short- or long-term debt. Note that there is some type of accounting money in the economy that however only fuels the economy during the transactions period, but does not appear as flow in the financial accounts of asset owners, and workers, and the government, but instead returns to the banking sector at the end of each transaction period $t$ by the settlement of all budget restrictions in the economy. In striking contrast to a cash in advance constraint we thus assume in this paper, following Chiarella and Flaschel (1999c), that agents can obtain all money they need for transaction purposes during the transaction period $t$ (as intra-day credit in one form or another), but that they have to satisfy their budget constraint at the end of each such period $t$ where money holdings are not needed and thus not present in them. Instead all liquid asset holdings concern the short term bonds of the government as some form of interest bearing saving deposit.

### Production Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government expenditure for goods $g^d = (1 - g)y^s$</td>
<td>Costless Provision of public goods</td>
</tr>
<tr>
<td>Government expenditure for services $w^{sfr}y^d$</td>
<td></td>
</tr>
</tbody>
</table>

11
Income Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest payment $rb$</td>
<td>Wage income taxation $\tau_w[w^{ide} + w^{ue}(l^e - l^{ue}) + w^{re} \alpha t_l^e]$</td>
</tr>
<tr>
<td>Interest payment $b^I$</td>
<td>Profit and interest taxation $\tau_c[p^s p_y/p + rb + b^I]$</td>
</tr>
<tr>
<td>Pensions $w^{re} \alpha t_l^e$</td>
<td>Rent income taxation $\tau_c(p_k c_n^d/p_u K - \delta_k k_h p_y/p_u)$</td>
</tr>
<tr>
<td>Unemployment benefits $w^{ue}(l^e - l^{ue})$</td>
<td>Payroll taxes $\tau_p w^{n} l^{de}$</td>
</tr>
<tr>
<td>Government consumption $g^d$</td>
<td>Value added tax $\tau_v(c_w + c_c + g^d)p_u/p_v$</td>
</tr>
<tr>
<td>Salaries $w^{ide} b^d_i$</td>
<td>Import taxes $\tau_m e p_m^d j^d/p_v$</td>
</tr>
<tr>
<td>Savings $S^n_g/p_u K$</td>
<td></td>
</tr>
</tbody>
</table>

Accumulation Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings $S^n_g/p_u K$</td>
<td></td>
</tr>
<tr>
<td>Financial Deficit $FD/p_u K$</td>
<td></td>
</tr>
</tbody>
</table>

Financial Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial deficit $FD/p_u K$</td>
<td>Short-term debt $\hat{B} b$</td>
</tr>
<tr>
<td></td>
<td>Long-term debt $p_b \hat{B} ^{bl} b^l$</td>
</tr>
</tbody>
</table>

Table 6

2.3.5 The balance of payments

We finally describe in table 7 the balance of payments of the economy under consideration. This will be done from the viewpoint of the foreign sector which is thus a fifth agent of the economic structure considered in this paper. The description of the behavior of this agent will however be confined to steady state behavior in the present investigation of the structural equations of the model.
The balance of payments:10

<table>
<thead>
<tr>
<th>Uses</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e\rho x/p_v$</td>
<td>$e\rho j^d/p_v$</td>
</tr>
<tr>
<td>$(1 - \tau_c)\hat{b}_2$</td>
<td>$(1 - \tau_c)\hat{b}_1$</td>
</tr>
<tr>
<td>$\hat{b}_1^*\hat{b}_1/r^t$</td>
<td>$e\hat{b}_2^*\hat{b}_2/r^t$</td>
</tr>
</tbody>
</table>

Table 7

This balance of payments shows the trade account (exports $x$ and imports $j^d$ per unit of capital), the international component of interest payments (to foreigners and from abroad) that are all assumed to cross borders and the outflow and the inflow of new capital (long-term bonds). Note that this account does not show any reserve changes of the central bank due to foreign exchange market operations. This is possible in the approach chosen in this paper, despite a temporarily given exchange rate $e$, since the supply of bonds of the government domestically and abroad of the equities of firms are channeled here into the savings decisions of households without readjustments, while the excess of domestic private savings is going into foreign bonds, which in turn implies that the balance of payments must be balanced without any intervention from the central bank, see Chiarella and Flaschel (1999b,c) for more details.

This concludes our intensive form description of the four accounts of the three typical sectors of a small open economy (here with heterogeneous agents in the household sector) plus a foreign sector that is here represented via the balance of payments solely.

3 The implied 34D Dynamics

In order to study the dynamics of our stylized disequilibrium growth model analytically and numerically it is necessary to reduce the equations of the model presented in Chiarella and Flaschel (1999c) to intensive or per (value) unit of capital form. This has already been indicated and discussed in the preceding section from the viewpoint of the system of national accounts by dividing all (nominal) level magnitudes through (the value of) the capital stock $K$ of firms (measured at consumer prices $p_v$) and by taking note of the fact that the model exhibits Harrod neutral technological change which means that all variables involving labor must be measured in efficiency units, i.e., they are to be multiplied with the term $\exp(n_h t)$ in addition to remove the trend in labor productivity from them. Note that this procedure must also be applied to the nominal wage $w$ which is to be replaced by the term $w^e = w/\exp(n_h t)$ since nominal wages $w$ will rise with labor productivity in the steady state and must therefore be detrended and replaced by the wage rate per efficiency unit of labor in order to get a variable that in principle allows for stationarity ($\dot{w}^e = \dot{w} - n_h$).

10 Viewed from the foreign economy and in terms of the domestic currency.
Note here again that the model has been formulated in a way that implies zero price inflation in the steady state if it is assumed that the target inflation rate of the central bank: $\bar{\pi}$, equals zero. This assumption will be made for the remainder of the paper.

The variables $w^e, p_y, p_h, etc.$ therefore need not be detrended any further, but represent state variables of the dynamical system to be formulated below. The first two laws of motion of these state variables are easily obtained from module 5b of Chiarella and Flaschel (1999c) by inserting the definitions for $V_f^w = t_f^d / t_f^w, U = y / y^p$ in particular and by replacing level variables by their intensive form measures. In the same way we obtain the dynamic laws for long-run inflationary expectations $\pi^l$ by making use of $\dot{Y}^p = \dot{K}$, and also the law of motion for the price of dwelling services $p_h$, see module 2b of Chiarella and Flaschel (1999c).

Note finally that the magnitude $c_c = C_c / K$, the consumption of asset holders of the domestic good per unit of capital, is a given magnitude in the steady state of the model (but not off the steady state), due to the assumption $\dot{C}_c = \gamma$ made in Chiarella and Flaschel (1999c).

### 3.1 The laws of motion

Let us start our presentation of the model in intensive form by considering the quantity dynamics it implies first:

1. **The Quantity Dynamics (7 laws of motion):**

   \[
   \begin{align*}
   \dot{y}^e &= \beta_{y^e}(y^d - y^e) + (\gamma - (g_k - \delta))y^e \\
   \dot{v} &= y - y^d - (g_k - \delta)v \\
   \dot{c}_w &= \alpha_{cw}^w (c^w / c_w - 1) + \alpha_{2w}^w (V - \bar{V}) + \gamma - (g_k - \delta), \\
   \dot{c}_h^d &= \alpha_{ch}^d (c^d / c_h^d - 1) + \alpha_{2h}^d (V - \bar{V}) + \gamma - (g_k - \delta), \\
   \dot{c}_c &= \gamma - (g_k - \delta), \\
   \dot{\hat{r}}^w &= \beta_{w} (V_f^w - \bar{V}_f^w) + \gamma - (g_k - \delta), \\
   \dot{\bar{V}} &= \beta_{0} (V - \bar{V})
   \end{align*}
   \]

These formulae are obtained from the extensive form presented in Chiarella and Flaschel (1999c) by the usual growth rate formula for intensive expressions, for example $Y^e / K = \dot{Y}^e - \dot{K}$ with $\dot{K} = g_k - \delta$ by reformulating such expression in terms of time derivatives whenever necessary. The dynamical laws for quantities describe sales expectations dynamics, actual inventory dynamics, three types of consumption demand dynamics for workers (domestic goods and housing services) and asset holders (domestic goods including houses), the dynamic employment policy of firms and finally the dynamics of the NAIRU rate of employment.

---

\(^{11}\)These dynamics, as well as the growth dynamics, will be considerably more complicated if substitution is allowed for in the production possibilities of firms, see Chiarella, Flaschel, Groh, Köper and Semmler (1999a,b) on this matter.
Next we describe the price dynamics as far as real markets are concerned:

2. Wage / Price Dynamics (4 laws of motion):

\[
\dot{w}^e = \beta_{w1}(V - \bar{V}) + \beta_{w2}(V_f^w - \bar{V}_f^w) + \kappa_w(\hat{p}_y + n_t) + (1 - \kappa_w)(\pi_l + \hat{n}_t) - n_t
\]

\[
\hat{p}_y = \beta_p(y/y^p - \bar{U}) + \kappa_p\dot{w}^e + (1 - \kappa_p)\pi_l
\]

\[
\pi_l = \beta_{\pi1}(\alpha_{\pi1}(\hat{p}_y - \pi_l) + (1 - \alpha_{\pi})(0 - \pi_l))
\]

\[
\hat{p}_h = \beta_h(\frac{\beta}{k_h} - \bar{U}_h) + \kappa_h\hat{p}_y + (1 - \kappa_h)\pi_l
\]

These equations for wage and price dynamics (including medium run inflationary expectations) and rental price dynamics are straightforward consequences of the laws of motion as they were formulated in Chiarella and Flaschel (1999c).

We have next the dynamics of asset prices, expectations about this dynamics and the dynamics of certain long-run concepts of interest and profits. Note that we here make use of Tobin's \( q = \frac{p_E}{p_R} \) as an aggregate expression for the joint dynamics of equity prices \( p_e \) and the number of equities \( E \). Due to the formulation of the model in Chiarella and Flaschel (1999c) it suffices to describe the dynamics of \( q \) in the intensive form of the model which moreover, due to the lack of wealth effects and the like, does not feed back into the rest of the dynamics. Note however that the expression for \( \hat{p}_e \) from Chiarella and Flaschel (1999c):

\[
\hat{p}_e = \frac{\beta_{pe}}{1 - \beta_{pe}(1 - \alpha_s)}[(1 - \tau_c)\rho^e/q + \alpha_s\pi_{es} - ((1 - \tau_c)\pi_l + \pi_b)],
\]

with aggregate expectations \( \pi_b \) being determined by \( \alpha_s\pi_{bs} + (1 - \alpha_s)\hat{p}_b \), has to be inserted into the law of motion for Tobin's \( q \) in order to get a description of these dynamics that is complete. Due to the isolated nature of these dynamics it is however not necessary here to go into these dynamics in more detail.

3. Asset Prices and Expectations (8 laws of motion):

\[
\dot{p}_b = \frac{\beta_{pb}}{1 - \beta_{pb}(1 - \alpha_s)}[(1 - \tau_c)\rho^e + \alpha_s\pi_{bs} - ((1 - \tau_c)r^l + \pi_b)], \quad p_b = 1/r^l
\]

\[
\pi_{bs} = \beta_{\pi bs}(\hat{p}_b - \pi_{bs})
\]

\[
\dot{q} = \hat{p}_e - \hat{p}_y + \frac{g_k - \delta + y - y^d - (\beta_n(\beta_n y^f - \nu) + \gamma\beta_n y^f)}{q} - (g_k - \delta)
\]

\[
\pi_{es} = \beta_{\pi es}(\hat{p}_e - \pi_{es})
\]

\[
\dot{\tilde{e}} = \frac{\beta_e}{1 - \beta_e(1 - \alpha_s)}[(1 - \tau_c^*)\rho^e + \alpha_s\epsilon_s - ((1 - \tau_c)r^l + \pi_b)]
\]

\[
\dot{\epsilon}_s = \beta_{\epsilon s}(\dot{\tilde{e}} - \epsilon_s)
\]

\[
\rho^1 = \beta_{\rho^1}\rho^1
\]

\[
\rho^1_h = \beta_{\rho^1_h}(\rho_n - \rho^1_h)
\]
We have first the law of motion for the price of long-term bonds \( p_b = 1/r^t \) which is here expressed in terms of the interest rate these bond prices (consols) represent. This interest rate adjusts in the direction of the risk free interest rate on short-term bonds after taxes \((1 - \tau_v)r\), augmented by a risk and liquidity premium \( \xi \) for long-term bonds. Note that we have removed the perfect foresight expectations from the left side of this adjustment equation which – as shown in Chiarella and Flaschel (1999c) – gives rise to the fraction in front of the shown formula. Note furthermore that only the law of motion of 'lazy' expectations is then needed in order to make this substructure determinate, but that aggregate expectations on these bond price changes \( \tau_b \) are needed in the subsequent laws of motion of asset prices.

Making use of the formula for the rate of change of equity prices \( \pi_e \) expected on average we can transform the law of motion for equity prices just as the law of motion for long-term bond prices and remove the explicit representation of 'ambitious' agents from it too – as shown in the representation of the \( p_e \)-dynamics. This law is again to be supplemented by the law of motion for the expectations of 'lazy' agents.

The next law of motion concerns Tobin's \( q \) which, as already shown, is measured by ratio between the value of equity stock and the producer price of the existing capital stock: \( q = \frac{p_e E}{p_y K} \). We have \( \dot{q} = \dot{p}_e - \dot{p}_y + \dot{E} - \dot{K} \) where the first two inflation rates have already been determined in the formulae preceding block 3. For the remaining expression \( \frac{\dot{E}}{\dot{K}} = \dot{E} - \dot{K} \) we get

\[
\frac{\dot{E}}{\dot{K}} = \dot{E} - \dot{K} = \frac{p_e \dot{E}}{p_y K} \cdot \frac{p_y K}{p_e E} - \dot{K} = [g_k - \delta + y - y^d - (\beta_n (\beta_n y^e - \nu) + \gamma \beta_n y^e)]/q - (g_k - \delta),
\]

which implies the law of motion for Tobin's \( q \) shown in module 3. Note again that the expression for \( \dot{p}_e \) can be inserted into the \( q \) dynamics and thus gives rise to one law of motion in Tobin's \( q \) solely.

The method used for the describing the dynamics of \( p_e \) also applies to the law of motion for the exchange rate \( e \) by removing again the correct expectations of the 'ambitious' agents from its right hand side after having inserted the expression for average expectations \( \epsilon = \alpha_s \epsilon_s + (1 - \alpha_s) \dot{e} \) into this formula. Again this is to be supplemented by the law that describes the evolution of the expectations of 'lazy' agents in the postulated adaptive way.

There follow the two laws of motion for expected long-run profitability: \( \rho' \), which is used in the investment equation for the capital stock, and \( \rho''_h \), which is used in the investment equation for the capital stock in housing. Both of these measures follow their short-run equivalents with some time delay.

Next we consider the laws of growth that apply to the economy under consideration:

4. Growth Dynamics (6 laws of motion):

\[
\begin{align*}
\dot{n} &= \beta_n (\bar{n} - n), \quad \ddot{n} = \ddot{n}(V, \bar{V}) \quad (20) \\
\dot{\epsilon} &= n + n_t - (g_k - \delta) \quad (21)
\end{align*}
\]
\[ \dot{g}_k = \beta_g (g_k^d - g_k) \]  
\[ \dot{k}_h = g_h - \delta_h - (g_k - \delta) \]  
\[ \dot{g}_h = \beta_g (g_h^d - g_h) \]  
\[ \dot{n}_t = \beta_n (\bar{n}_t - n_t) \text{, } \bar{n}_t = \bar{n}_t(g_k) \]

The growth equations represent the time rate of change of the so-called natural rate of growth, of law of motion for the full employment labor intensity (in efficiency units), the time rate of change of gross investment per unit of capital (also for the housing sector), the growth rate of the relative magnitude of the stock of houses to the capital stock employed by firms and finally the time rate of change of the rate of Harrod neutral technological change. There is no further comment needed with respect to the above presentations of the growth laws of the economy which again use the formula \( \dot{K} = g_k - \delta \) in the formulation of intensive expressions.

Next the dynamic feedback rules for government behavior are collected, concerning the steering of the short-term nominal rate of interest by the central bank, the rate wage taxation rule based on the evolution of government debt \( d \) and the motion of the tax rate on imports which is here used to establish a balanced trade account in the steady state.

\[ \dot{r} = -\beta_r (r + \xi - r^*_t) + \beta_\pi (p - \bar{p}) + \beta_y (y^p - \bar{U}) \text{, } \bar{p} = 0 \]  
\[ \dot{\tau}_w = \alpha_{\tau_1} (d/\bar{d} - 1) + \alpha_{\tau_2} \bar{d}, \quad d = \frac{b + b'/\bar{d}}{y^e} \]  
\[ \dot{\tau}_m = \alpha_{\tau_m} \frac{p_k x - p_m j^d}{p_x x} \]

Note with respect to module 5 that the aggregate accumulation of government bonds cannot be divorced from the real sector (even though wealth effects are not yet included in the model) due to the assumed wage income taxation rule whereby the government attempts to steer a certain ratio for government debt to a desired ratio \( \bar{d} \).

There remains the dynamics of aggregate and individual asset holdings which represents the most involved block in our dynamical system. We have already stressed that the individual allocation of government bonds (between workers and asset holders and also throughout the world) does not feed back into the remaining dynamics, since only total government bonds do matter in the present context due to the absence of wealth effects in consumption and due to the independence of consumption of workers and asset holders from their interest income. In the current version of the dynamics only the laws of motion for \( b, b' \) feed back into the real part of the dynamics via the wage tax collection rule of the government.

\[ \dot{b} = \alpha_b ^{(gy^e + rb + b')} - \dot{t}^n + (w^{ue}/p_v) (l^e - l^{we}) + (w^{re}/p_v) \alpha_l l^*_g + (w^{be}/p_v) l^{de} \]  
\[ -(\dot{p}_y + g_k - \delta)b \]  

6. Assets Dynamics (6 laws of motion):
\[ i^t = r^t(1 - \alpha^g) [gy^e + rb + b^t] \\
\quad - t^n + (w^{ue}/p_v)(l^e - l^{ue}) + (w^{re}/p_v)\alpha l_2^e + (w^{be}/p_v)l_g^{de} \\
\quad - (\bar{p}_y + g_k - \delta)b^t \\
\]
\[ b_w = y^D_w - c_w - \frac{p_h}{p_v} e^A - (\bar{p}_y + g_k - \delta)b_w, \quad \dot{b}_c = \dot{b} - b_w \]  
\[ \dot{b}_1^t = r^t(1 - \alpha^g) [gy^e + rb + b^t] \\
\quad - t^n + (w^{ue}/p_v)(l^e - l^{ue}) + (w^{re}/p_v)\alpha l_2^e + (w^{be}/p_v)l_g^{de} \\
\quad - (\bar{p}_y + g_k - \delta)b_1^t \\
\]
\[ \dot{b}_1^* = r^t(1 - \alpha^g) (1 - \alpha^g) [gy^e + rb + b^t] \\
\quad - t^n + (w^{ue}/p_v)(l^e - l^{ue}) + (w^{re}/p_v)\alpha l_2^e + (w^{be}/p_v)l_g^{de} \\
\quad - (\bar{p}_y + g_k - \delta)b_1^* \\
\]
\[ \dot{b}_2 = \dot{B}_2/(p_v K) - (\bar{p}_y + g_k - \delta)b_2, \quad \text{where we have} \] 
\[ \frac{\dot{B}_2}{p_v K} = r^t e\left[y^D_c - c_c - \frac{\dot{B}_c}{p_v K} - \frac{\dot{B}_1^t}{p_v K} - \frac{p_e E}{p_v} - \frac{p_y (g_h - \delta_h) k_h}{p_v}\right] \\
\quad = r^t e\left[y^D_c - c_c - (\alpha^g + (1 - \alpha^g)\alpha^g) [gy^e + rb + b^t - t^n] \\
\quad + (w^{ue}/p_v)(l^e - l^{ue}) + (w^{re}/p_v)\alpha l_2^e + (w^{be}/p_v)l_g^{de} \\
\quad - (p_y/p_v) [g_k - \delta - y - y^d - (\beta_n(\beta_n y^f - \nu) + \gamma(\beta_n y^f)] \\
\quad - (p_y/p_v) (g_h - \delta_h) k_h\right] \]

according to the flow budget constraint of asset holders.

This part of the dynamics is to some extent missing in the Murphy model, see Powell and Murphy (1997), with which we compared our model in detail in Chiarella and Flaschel (1999c), due to the lack of a complete treatment of the budget equations of the three sectors that form the basis of this model. As indicated above the laws of motion of the individual assets that are held by the household sector in our model do however not feed back into the rest of the dynamics, since they do not show up in the real part of the economy. Therefore, only the two laws of motion for short-term and long-term government debt are really needed at present in the discussion of the growth pattern and the fluctuations that may occur around them which are implied by the disequilibrium growth model under consideration.

Note with respect to the right hand sides of these stock accumulation equations that they are based on a fixed ratio \( \alpha^g \) describing the allocation between the short- and long-term financing of government debt done by the government (as in the Murphy model) and that the term in square brackets, the government budget equation:

\[ gy^e + rb + b^t - t^n + (w^{ue}/p_v)(l^e - l^{ue}) + (w^{re}/p_v)\alpha l_2^e + (w^{be}/p_v)l_g^{de} \]

in both cases represents the sum of government expenditure for goods, labor, interest and transfer payments to the unemployed and the retired persons minus \( t^n \), the sum of all taxes that are raised by the government (per value unit of capital). \(^{12}\)

\(^{12}\)Note that all wage concepts in the above intensive form of the GBR are in efficiency units (to allow for stationarity) and are deflated by consumer prices \( p_v \).
The total amount of debt financing is thus represented through the expression shown above, leading to \( \dot{b} + \frac{b_t}{r^t} \) in sum, and it is divided by means of the weights \( \alpha_2^2, 1 - \alpha_2^2 \), applied to the GBR in the above intensive form, into short-term financing \( \dot{b} \) and long-term financing \( \frac{b_t}{r^t} \). Note that \((\dot{p}_y + g_k - \delta)b\) and \((\dot{p}_y + g_k - \delta)\dot{b}^t\) have to be deducted from the resulting expression in addition, due to the fact that these bond variables are in intensive form and are thus divided by \( p_v K \).

The dynamics of real savings of workers per unit of capital, \( b_w = B_w / (p_v K) \), follow from the definition of this expression and from the definition of the disposable income of worker households and their saving plans, while the law of motion for short-term debt of asset holders per capital, \( b_e = B_c / (p_v K) \), is a simple consequence of the two laws of motion assumed for the expressions \( b, b_w \).

The next two dynamic laws for the distribution of new domestic long-term debt throughout the world basically flow again from the above shown budget restriction of the government (expressed in intensive form), since there is also a fixed proportion \( \alpha_2^1 \) assumed to apply with respect to the distribution of long-term debt between domestic and foreign asset holders. The expressions for the proportions of long-term financing that go to domestic and foreign residents, \( B_1, B_1^t \) and \( B_2, B_2^t \) are thus obtained by applying the weights \( \alpha_2^1, 1 - \alpha_2^1 \) to \( b \) and taking again note of the fact that now \((\dot{p}_y + g_k - \delta)\dot{b}_1^2\) and \((\dot{p}_y + g_k - \delta)\dot{b}_1^t\) have to be deducted from the resulting expressions (due to the intensive form formulation) in the place of the former expression \((\dot{p}_y + g_k - \delta)b\).

There remains the law of motion of foreign assets held by domestic residents which due to the definition of the intensive variable \( B_2 = B_2^t / (p_v K) \) basically demands the determination of the variable \( \dot{B}_2^t / (p_v K) \) in terms of intensive expressions. This task is solved residually by referring to the fact that \( \dot{B}_2 \) is given by the disposable income of asset holders minus their consumption minus all other asset accumulation that these households undertake. This provides the last law of motion shown in the above block of asset accumulation equations, which is thus purely residual in nature.

### 3.2 Static relationships

There are a variety of definitions and static relationships used in the above collection of the laws of motion of our disequilibrium model of monetary growth. These abbreviations are collected in the next six blocks of the intensive presentation of the model and are generally immediate consequences of the corresponding equations in extensive form presented in Chiarella and Flaschel (1999c).

#### 7. Output and Demand on the market for goods (including Housing Services):

\[
\begin{align*}
y^d &= c_w + c_e + g_k + g_h k_h + g y^e \quad (35) \\
y &= y^e + \beta_n (\beta_n y^e - \nu) + \gamma \beta_n y^e \quad (36) \\
x &= x y y \quad (37) \\
j^d &= j y y \quad (38)
\end{align*}
\]
\[ c^e_w = c_1(1 - \tau_w)[\omega^e l^de + \omega^{ue}(l^e - l^{ue}) + \omega^r \alpha l^e_2]/p_v \]  
\[ c^h_o = c_2(1 - \tau_w)[\omega^e l^de + \omega^{ue}(l^e - l^{ue}) + \omega^r \alpha l^e_2]/p_h \]  

Note with respect to block 7 of the static equations that all variables are obtained here by dividing the extensive expressions by \( K \), giving rise for example to \( k_h = K_h/K \). The same procedure applies to block 8 of the algebraic equations underlying our dynamic model:

8. Employment, Labor Supply and Retired Persons:

\[ l^le_f = l_y y \text{ see block 1. for the law of motion for } l^we_f \]  
\[ l^le_g = l^we_g = \alpha_g g^e \]  
\[ l^le = l^le_f + l^le_g : V^w_f = l^le_f/l^e \]  
\[ l^we = l^we_f + l^we_g : V = l^we/l^e \]  
\[ l^e_2 = (L_2(0)/L_1(0))l^e/\alpha_l \]

Note here again that all magnitudes concerning labor inputs and supply are expressed in efficiency units – due to the technological condition \( L^d_f = l_y \exp(-n_t)Y \) that represents Harrod neutral technical change. When multiplied with wages measured in efficiency units the term \( \exp(-n_t) \) just cancels from the resulting expression and gives the corresponding wage payments in both efficiency units and original levels. Note finally, that we need initial conditions in order to relate the sizes of the intensive expression for potential labor supply and retired people.

9. Desired Growth Rates of the Capital Stocks (Firms/Dwellings):

\[ g^d_k = \alpha_1^k((1 - \tau_c)\rho^l - r^r) + \alpha_2^k(r^l - (r + \xi)) + \alpha_3^k(y/y^p - \bar{U}) + \gamma + \delta \]  
\[ g^d_h = \alpha_1^h((1 - \tau_c)\rho^l - r^r) + \alpha_2^h(r^l - (r + \xi)) + \alpha_3^h(c^d_k/k_h - \bar{U}_h) + \gamma + \delta_h \]

The growth rates of the capital stocks for fixed investment of firms and housing investment of asset holders (here shown as gross rates) are an immediate consequence of their original formulation in Chiarella and Flaschel (1999c), and simply state that these ratios are assumed to be influenced by long-run profitability measures, by the interest rate spread and by rate of capacity utilization. The same immediate correspondence to what has been introduced in Chiarella and Flaschel (1999c) holds true for the following definitions of rates of return needed for the dynamical laws or just for the discussion of the steady state of the model.
10. **Rates of Return on Real and Financial Assets:**

\[ \rho^e = y^e - \delta + (p_x/p_y)x - (w^e/p_y)j^e - (p_m/p_y)j^d \]

\[ y^{dp} = \frac{\bar{U}yp}{1 + \gamma \beta_n}, \quad y^n = \bar{U}yp \]

\[ \rho^n = y^{dp} - \delta + (p_x/p_y)x_y y^n - w^{be}y^n - (p_m/p_y)j_y y^n \]

\[ \rho^r = (1 - \tau_c) \pi - \pi' \]

\[ \rho_h = (p_h/p_y)c^h/k_h - \delta_h \]  

The following definitions of the various prices we use in our model in addition to the prices that appear as state variables in the dynamics are also an immediate consequence of their definitions in the extensive form of the model in Chiarella and Flaschel (1999c) when account is again taken of the fact that wages have to be expressed in efficiency units in order to get their stationarity in the steady state: \( w^e = w/\exp(nt) \).

11. **Consumer Prices, Gross Wages and Transfers:**

\[ p_x = e p^*_x \]

\[ p_m = (1 + \tau_m) e p^*_m \]

\[ w^{be} = (1 + \tau_p) w^e \]

\[ w^{ue} = \alpha^u w^e \]

\[ w^{re} = \alpha^r w^e \]

\[ p_v = (1 + \tau_v)p_y \]  

Finally the short-cut expressions for the intensive forms of disposable incomes and taxes are easily obtained from their extensive form analogs and are of the form shown below. Note that the expression for disposable income of workers and asset holders are only needed in their bond accumulation equations and are not yet involved in the consumption decisions of the two types of households considered as the model is currently formulated.

12. **Disposable Incomes and Government taxes per value unit of capital:**

\[ y^D_w = (1 - \tau_w)[w^e l^{de} + w^{ue}(\ell^e - l^{ue}) + w^{re} \alpha l^2_2]/p_v + (1 - \tau_c)rb_w \]

\[ y^D_{w1} = (1 - \tau_w)[w^e l^{de} + w^{ue}(\ell^e - l^{ue}) + w^{re} \alpha l^2_2]/p_v \]

\[ y^D_c = (1 - \tau_c)[\rho^e(p_y/p_v) + rb_c + b'_1 + (p_h/p_v)c^h_k - (p_y/p_v)\delta_k k_h] + e(1 - \tau_c)b_2^2 \]

\[ t^n = \tau_w[w^e l^{de} + w^{ue}(\ell^e - l^{ue}) + w^{re} \alpha l^2_2]/p_v + \tau_p(w^e/p_v)^l^{de} + \tau_v(y^d - g_k - g_h k_h)(p_y/p_v) + \tau_c[\rho^e(p_y/p_v) + rb + b'_1 + (p_h/p_v)\rho_h k_h] + \tau_m e(p_m/p_v)j^d \]
In closing the discussion of the intensive form of our disequilibrium model of monetary growth we stress again that the law of motion for equity prices \( p_e \) is not needed, since it can be substituted into the law of motion for Tobin’s \( q \). It can thus be removed from explicit representation, since there is here no feedback from Tobin’s \( q \) on the real part of the economy, due to the lack of wealth effects in the present version of the model.

4 Steady State Analysis

In this section of the paper we show that there is an, up to the level of nominal variables, uniquely determined economically meaningful balanced growth path or steady state solution of our model which provides us with a useful reference path for the dynamical evolutions implied by the model, which may or may not converge to this steady state solution, not even if long-run moving averages are used in the place of the temporary positions the economy will run through.

The calculation of this interior, economically meaningful, steady state\(^{13} \) of the full model (which up to the level of nominal magnitudes for wages and goods prices is uniquely determined) is in many respects simple due to the given growth rate of the world economy and the given interest rate (on consols) abroad. Note that we only consider expressions for the total supply of domestic bonds in the following, and not their distribution at home and abroad which can be easily obtained from the savings decisions of workers and pure asset owners.

To simplify subsequent presentations of the dynamics of the model and also its steady state solution somewhat we assume in the remainder of this paper for the consumption of asset owners \( C_c = 0 \) and for the liquidity premium applied to long-term debt \( \xi = 0 \). These two assumptions do not restrict the dynamical behavior of the system in an important way.

The first set of steady state conditions presented below concerns the growth rates of our small open economy:

\[
\begin{align*}
\gamma &= g_k - \delta = g_k^d - \delta \quad \rightarrow g_k = g_k^d = \gamma + \delta \\
\gamma &= g_h - \delta_h = g_h^d - \delta_h \quad \rightarrow g_h = g_h^d = \gamma + \delta_h \\
\gamma &= \bar{n}(\bar{V}) + \bar{n}_t(g_k) = \bar{n}(\bar{V}) + \bar{n}_t(\gamma + \delta) \quad \rightarrow \bar{V}, \quad n = \bar{n}, n_t = \bar{n}_t
\end{align*}
\]

These equations state that capital (and thus also output) will grow with the external rate \( \gamma \), to which also the natural rate of growth adjusts. This means that the steady state value of NAIRU rate of employment \( \bar{V} (= V) \) has to adjust such that \( \gamma = \bar{n}(\bar{V}) + \bar{n}_t(g_k) \) holds. This determines a unique NAIRE \( \bar{V} = V \) for the long-run of the model under suitable – but simple – assumptions on the function \( \bar{n} \).

The next set of steady state conditions concerns inflation and expected inflation (for all prices that exist in our model, except wage rates) and also the various rates of interest and

---

\(^{13}\)where we thus neglect as alternative steady state positions all zeros which can be obtained mathematically from the growth law formulations that our model employs. Note also the steady state depends parametrically on the initial conditions for \( \frac{L_0}{L_1(0)} \), \( P_v \), \( \frac{G_0}{L_1(0)} \) that characterize the initial composition of the labor force, the initial output price level (including value added taxes) and a relative expression for the consumption of asset owners.
profit of our model:

\[
\begin{align*}
\hat{p}_v &= \hat{p}_y = \hat{p}_h = \pi^l = \pi_{bs} = \pi_b = \pi_{es} = \pi_e = \epsilon_s = \epsilon = 0 \\
\rho_i^* &= \rho^l = \rho^r = r = 1/p_h \\
\rho_i^* &= \rho^e = \rho^l = \rho_h = \rho_h^l \\
\end{align*}
\]

(65) (66) (67)

These conditions state that there is no steady state inflation and thus no nonzero expectations of it, which is due to the interest rate policy rule of the central bank. Furthermore, all (expected) rates of return are equalized in the steady state and determined by the world rate of interest on long-term bonds \( r_i^* \).

The next block concerns the steady state determination of various quantities, of the steady state ratio of government debt to aggregate demand and of the tax rate on imports:

\[
\begin{align*}
y &= y^P \bar{U}, \quad c^d_h = k_h \bar{U}_h \\
l^{de}_f &= l^{we}_f = l_y y \\
l^{de}_g &= l^{we}_g = \alpha_g y^e \\
l^{we} &= l^{de} = l^{we} + l^{we}_g \\
l^e &= l^{de}/\bar{V}, \quad [V = \bar{V}], \quad l^e_1 = l^e/\alpha_l, \quad l^e_2 = (L_2(0), L_1(0))l^e_1 \\
x &= x_y y, \quad j^d = j_y y \\
d &= (b + b_i/r_i^*)y^e = \bar{d} \\
\tau_m &= \frac{p^*_z x - p^*_m j^d}{p^*_m j^d} \\
\end{align*}
\]

(68) (69) (70) (71) (72) (73) (74) (75)

These equations still depend on the steady state value of \( y^e \) – which will be given below – and they reveal certain supply side influences on the steady behavior of our economy.

Further steady state relationships on the side of quantities are:

\[
\begin{align*}
y^e &= \frac{y}{1 + \gamma \beta_n} \\
y^d &= y^e, \quad \nu = \beta_n y^e \\
c^o &= c_1 y_{w_1}, \quad c^o_h = c_2 p_v y_{w_1}/p_h \\
y_{w_1} &= (1 - \tau_w) [w^{ide} + w^{we}(r^e - r^{we}) + w^e \alpha_l l^e_2]/p_v \\
\end{align*}
\]

(76) (77) (78) (79)

By using the distribution laws for government bonds we furthermore get:

\[
\begin{align*}
b &= \alpha_b \bar{d} y^e \\
b - 1 &= \rho_i^* (1 - \alpha_b \bar{d} y^e) \\
\end{align*}
\]

(80) (81)

from which the individual distribution of bonds can be derived if desired.

Next, one can determine the following nominal steady state expressions on the basis of an (arbitrarily) given price level \( p_v \). This indeterminacy of the general price level of domestically
produced goods is due to the fact that the central bank has adopted an interest rate policy rule and allows for intraday deviations from the budget constraints of the two types of households, the firms and the government and that therefore, in our present model, money is not held as cash balance by the agents of our economy.

\[
p_v = \text{indetermined and thus a parameter} \quad (82)
\]

\[
p_y = \frac{p_v}{(1 + \tau_v)} \quad (83)
\]

\[
w^{be} \text{ via } r^*_p = y^e - \delta - \frac{w^{be,l}}{p_y} : w^{be} = \frac{y^e - \delta - r^*_p}{l_y y} p_y \quad (84)
\]

\[
w^e = \frac{w^{be}}{(1 + \tau_p)} , w^{ue} = \alpha w^e , w^{re} = \alpha^r w^e \quad (85)
\]

The equation for the determination of gross wages in efficiency units holds since we have balanced trade in the steady state (see the above), and thus no influence of the international trade in goods on the rate of profit in the steady state. Note here that net real wages \( \omega^e \) (excluding payroll taxes, but before wage taxation), measured by in terms of consumer prices \( p_v \) and efficiency units, are given by

\[
\omega^e = \frac{w^e}{p_v} = \frac{1}{(1 + \tau_v)(1 + \tau_p)} \frac{y^e - \delta - r^*_p}{l_y y}
\]

and thus do not depend on the arbitrarily determined nominal price level \( p_v \). This observation also applies to all other real magnitudes implying that there are no real effects of shocks which lead to a different consumer price level \( p_v \) in the long-run.

The equations \( c_2^h = \bar{U}_h k_h \) and \( r^*_p = p_h c_2^h/(p_y k_h) - \delta_h \), on the one hand, and

\[
c_2 y^D_{w1} = p_h c_2^h/p_y
\]

\[
y^e = c_1 y^D_{w1} + \gamma + \delta + (\gamma + \delta_h) k_h + g y^e,
\]

on the other hand,\(^{14}\) allow us next to determine the price ratio \( p_h/p_y \) and \( k_h , y^D_{w1} , \tau_w \) as follows:

\[
p_h/p_y = (r^*_p + \delta_h)/\bar{U}_h \quad (86)
\]

\[
k_h = \frac{y^e(1 - g) - \gamma - \delta}{c_1 (r^*_p + \delta_h)/(c_2 (1 + \tau_v)) + \gamma + \delta_h} \quad (87)
\]

\[
y^D_{w1} = p_h \bar{U}_h k_h/(c_2 p_v) \quad (88)
\]

\[
\tau_w = 1 - \frac{w^{lde} + w^{ue}(1 - l^{we}) + w^{re} \alpha l^e}{p_v} \quad (89)
\]

Note also that this determination of \( y^D_{w1} \) and \( k_h \) must be used above for the determination of consumption plans per unit of capital.

There remains the determination of the steady state value of the exchange rate \( \epsilon \),\(^{15}\) since the price of long term bonds \( p_0 = 1/r^*_p \) has already been determined and since the price of equities does not matter in the core 34D dynamics of the model.\(^{16}\) The calculation of the

\(^{14}\)where the last one stands for goods-market equilibrium in the steady state

\(^{15}\)which in turn determines the prices \( p_{x1} , p_{m} \).

\(^{16}\)The same holds true for the disposable income of asset holders \( y^D_e \).
steady state exchange rate is economically seen complex, but mathematically still simple. Mathematically it is provided by the following implicit equations for the variables $t^r, e$ on the basis of the expression for $t^c$ shown below:

\[ 0 = gy^e + rb + b' - t^w - t^c + \left[ w^{ue}(l^e - l^{we}) + w^{re}c_1l_2 + w^{be}_g l^{de}\right]/p_v - \gamma b/\alpha_0^g \]
\[ 0 = \tau_w y^D_{w1}/(1 - \tau_w) + \tau_p(w^e/p_v)l^{de} + \tau_v(p_y/p_v)(y^d - g_k - g_hk_h) + \tau_m p^*_m j^d/p_v - t^r \]

where

\[ t^c = \tau_v[p^*(p_y/p_v) + r^*_t b + b' + (p_h/p_v)c^h_h - (p_y/p_v)\delta_hk_h] \]
\[ = \tau_v[(p^* + r^*_t k_h)/(1 + \tau_v) + r^*_t b + b'] \]

which give rise to:

\[ t^r = gy^e + rb + b' - t^c + \left[ w^{ue}(l^e - l^{we}) + w^{re}c_1l_2 + w^{be}_g l^{de}\right]/p_v - \gamma b/\alpha_0^g \] (90)
\[ e = \frac{t^r - [\tau_w y^D_{w1}/(1 - \tau_w) + \tau_p(w^e/p_v)l^{de} + \tau_v(p_y/p_v)(y^d - g_k - g_hk_h)]}{\tau_m p^*_m j^d/p_v} \] (91)

We see that the long-run rate of exchange is neither determined in the market for goods, nor through the trade balance nor by international capital flows, but is a complicated expression of many parameters and steady state values of the model and is in particular heavily dependent on the form of the GBR and its components.

Note finally that the dynamics of equity prices implies that $q = 1$ must hold true in the steady state, i.e., $p_e E = p_y K$ and that the steady state distribution of long-term bonds can be derived from block 6. of Chiarella and Flaschel (1999c), while the steady state value of $b_w$ (and $b_e$) follows by setting eq. (31) equal to zero. This gives rise to:17

\[ b_w = \frac{(1 - c_1 - (p_h/p_v)c_2)y^D_{w1}}{\gamma - (1 - \tau_v^*)r^*_t}, \]

due to the difference that exists between our definitions of $y^D_{w1}$ and $y^D_{w2}$.

Summarizing, we see that the steady state of the considered economy heavily depends on the data assumed to apply to the rest of the world (generally in a fairly straightforward and simple way) and that the steady values of the wage taxation rate and in particular the exchange rate are complicated functions of the parameters and various other steady state solutions of the dynamics. Furthermore, since the steady rate of profit $\rho^e$ and thus also the real wage rate $\omega^e = w^e/p_v$, see the above expressions, are determined through the foreign rate of interest, we get for the real wage $\omega^{ne}$ (after taxes, at consumer prices and in efficiency units) the expression:

\[ \omega^{ne} = (1 - \tau_w)\omega^e = \frac{1 - \tau_w}{(1 + \tau_v)(1 + \tau_p)} \frac{y^e - \delta - r^*_t}{l_y} \]

17This solution implies by economic reasoning that $\gamma > (1 - \tau_v^*)r^*_t$ should hold true in the world economy (because of the particular interest income policy of workers).
which implies that all increases in the three tax rates shown (value added tax, payroll taxes and (endogenously determined) wage income taxes) fall on real wages of workers (measured in this paper by the consumer price of domestic goods solely). There is however also an influence of the value added tax on overall interest rate income per value unit of capital, besides of course a dependence of this income on the capital taxation rate $\tau_c$.

Output levels (per unit of capital) and also steady employment (per unit of capital and measured in efficiency units) are basically determined through supply side considerations (technology and desired steady rates of capacity utilization), but the former also depend on government expenditures $g$ per unit of capital and thus will be changing with this ratio.

We stress again that the external growth rate $\gamma$ determines the NAIRU rate of employment via

$$\gamma = \bar{n}(\bar{V}) + \bar{n}_t(\gamma + \delta)$$

which implies that an increase in $\gamma$ will increase the NAIRU rate of employment $\bar{V}$ if $\bar{n}' > 0, \bar{n}_t \in (0, 1)$ holds.

The above has also shown that there is no inflation occurring in the steady state of 34D dynamics as far as goods and asset prices are concerned. It is of course important to also consider economies which allow for (moderate) price inflation in the steady state, in the place of stationary price levels that here prevail in the steady state. Just as rates of growth, the steady levels of interest and profit rates are here all fixed and given through the external rate of interest $r^*_f$ so that there is no possibility that the considered country may exhibit an extraordinary level of profitability in the steady state.

5 The 18D Core Dynamics of the Model

In order to get (as a starting point for our dynamical investigations of the model in subsequent papers) a dynamical system that is, on the one hand, as close as possible in spirit to the one of the general model of this paper and, on the other hand, also as low dimensional as possible, we consider in this section a simplified structure for our Keynesian disequilibrium dynamics of monetary growth which reduces its 34D representation to the following 18D core dynamics. This reformulation makes use of the assumptions $[C_z = 0, \xi = 0]$ already used in the preceding section (there used to simplify slightly the calculation of the interior steady state of the model), and it removes furthermore certain delayed adjustment processes from the considered dynamics, which gives the dynamic model an outlook that is not too far away from the theoretical models introduced and analyzed in Chiarella and Flaschel (1999a, Ch.s 6,7, 1999b, part III). Furthermore, the natural rate of growth, of Harrod neutral technological change, and of employment are assumed as constant in the following. Finally, the parameter $\alpha_{\tau_{c1}}$ is set equal to zero, implying that there is only a proportional influence of government debt on the wage taxation rate (and no longer an additional derivative one).
5.1 The laws of motion

1. The Quantity Dynamics:

\[
\begin{align*}
\dot{y}^e &= \beta_y \left( y^d - y^e \right) + \left( \gamma - (g_k^d - \delta) \right) y^e \\
\dot{\nu} &= y - y^d - (g_k^d - \delta) \nu \\
\dot{\bar{I}}_f^{we} &= \beta_l \left( l_f^{de} - \bar{V}_f^{we} \right) + \left[ \gamma - (g_k^d - \delta) \right] \bar{I}_f^{we}
\end{align*}
\]  

2. The Price Dynamics:

\[
\begin{align*}
\dot{\bar{w}}^e &= \beta_{w_1} \left( \bar{w}_{f}^{we} / \bar{w}_f^{de} - \bar{V} \right) + \beta_{w_2} \left( l_f^{de} / \bar{I}_f^{we} - \bar{V}_f \right) + \kappa_w \bar{p}_y + (1 - \kappa_w) \pi^l \\
\dot{\bar{p}}_y &= \beta_p \left( y / y^p - \bar{U} \right) + \kappa_p \bar{w}^e + (1 - \kappa_p) \pi^l \\
\dot{\pi}^l &= \beta_{\pi} \left( \alpha_{\pi} (\bar{p}_y - \pi^l) + (1 - \alpha_{\pi}) \left( 0 - \pi^l \right) \right) \\
\dot{\bar{p}}_h &= \beta_h \left( \frac{c_{ch}}{\bar{c}_h^{we}} - \bar{U}_h \right) + \kappa_h \bar{p}_y + (1 - \kappa_h) \pi^l
\end{align*}
\]

There is no direct change of the dynamics with respect to the various types of price adjustment rules of the model, on the market for labor, goods and housing services (including the expectations formation mechanism for medium run inflation rate of the domestically produced good), but an indirect one due to the following assumptions on natural growth and employment (which are now considered as given: \( \gamma = n + n_i = \bar{n} + \bar{n}_t, V \) all constant).

3. The Growth Dynamics:

\[
\begin{align*}
\dot{\bar{v}}^e &= \gamma - (g_k^d - \delta) \\
\dot{\bar{k}}_h &= g_h^d - \delta_h - (g_k^d - \delta)
\end{align*}
\]

In block 3 we have assumed that the natural rates of growth and of technical change, \( n, n_i \), are given exogenously and equal in sum to \( \gamma \). Furthermore actual accumulation rates are assumed to be adjusted with infinite speed to their desired targets and thus are no longer represented as lagged adjustment rules. We have furthermore removed from this block 3 of the dynamics the adjustment equations for the rates of return \( \rho_h, \rho_t \) by assuming that
these rates are adjusted with infinite speed to their short-run equivalents, now used in the corresponding behavioral equations in their place.

Next, we present the set of equations that represent the dynamics of asset accumulation and asset prices that are needed for the analysis of the real part of the dynamics of the model:

4. Asset Market Dynamics:

\[
\begin{align*}
\dot{b} & = \alpha_b \left[ (gy^c + rb + b' - t^w - t^c + (w^{ue}/p_v)(t^e - t^{ue}) + (w^{re}/p_v) \alpha_t l^m + (w^{be}/p_v) \alpha_b l^m \right] \\
\dot{b}' & = r'(1 - \alpha_b^c) \left[ g y^c + rb + b' - t^w - t^c + (w^{ue}/p_v)(t^e - t^{ue}) + (w^{re}/p_v) \alpha_t l^m + (w^{be}/p_v) \alpha_b l^m \right] \\
\dot{p}_b & = \frac{\beta_{p_s}}{1 - \beta_{p_b}(1 - \alpha_s)} \left[ (1 - \tau_c) r_I + \alpha_s \pi_{b_s} - (1 - \tau_c) r_I \right], \quad r_I = \frac{1}{p_b} \\
\dot{\pi}_{b_s} & = \beta_{p_s} (\dot{p}_b - \pi_{b_s}), \quad \pi_b = \alpha_s \pi_{b_s} + (1 - \alpha_s) \dot{p}_b \\
\dot{\epsilon} & = \frac{\beta_{\epsilon_s}}{1 - \beta_{\epsilon_s}(1 - \alpha_s)} \left[ (1 - \tau_c) r_I^* + \alpha_s \epsilon_s - (1 - \tau_c) r_I^* + \pi_b \right] \\
\dot{\epsilon}_s & = \beta_{\epsilon_s} (\dot{\epsilon} - \epsilon_s)
\end{align*}
\]

The price adjustment rules in block 4 concern the nominal value of long-term bonds \( p_b \) and the nominal exchange rate \( e \) and they are based (as explained in Chiarella and Flaschel (1999c) and in this paper) on heterogeneous expectations of the pure wealth owners of this model. Note that – as in the larger model – the dynamics of equity prices and of Tobin’s \( q \) are not needed in the investigation of the core dynamics of the model.

Next, the dynamic policy rules are presented which are basically the same as in the larger dynamics. Note however that we have removed the derivative term from the right hand side of the wage rate dynamics.

5. The Feedback Policy Rules:

\[
\begin{align*}
\dot{r} & = -\beta_{r_1} (r - r_I^*) + \beta_{r_2} (\dot{p}_y - 0) + \beta_{r_3} (y/y^p - \bar{U}) \\
\dot{\tau}_w & = \alpha_{\tau_{w_1}} (d/d - 1), \quad d = \frac{b + b'}{y^e} \\
\dot{\tau}_m & = \alpha_{\tau_{m}} \frac{p_x x - p_m j^d}{p_x x}
\end{align*}
\]
as well as of employment. Note that the housing services sector feeds back into this core dynamics of the model through the investment demand for dwellings (on the market for domestic goods) and through its rate of return which is in particular determined by the law of motion for the rent price of housing services.

5.2 Static Relationships

As abbreviations and static relationships we now have the following reduced and modified list of equations underlying this 18D dynamical system.

1. Output and Demand:
   \[ y^d = c_w + g^d + g^d_kh + gy^e, \]  
   \[ c^o_w = c_{1Yw1}, \]  
   \[ c^d_{0h} = p_vc_2y^D_{w1}/ph, \]  
   \[ y = y^e + \beta_n(\beta_n y^e - \nu) + \gamma \beta_n y^e \]  
   \[ x = x_y y \]  
   \[ j^d = j_y y \]  

2. Employment and Labor Supply:
   \[ t^d_f = l_y y \]  
   \[ t^d_g = l_w^e = \alpha_g y^e \]  
   \[ t^d_e = t^d_f + t^d_g \]  
   \[ t^w_e = t^w_e + t^w_g \]  
   \[ l^e_2 = (L_2(0)/L_1(0))l^e = (L_2(0)/L_1(0))l^e/\alpha_l \]  

3. Growth Rates of the Capital Stocks of Firms and Asset Owners:
   \[ g^d_k = \alpha^k_1((1 - \tau_c)(\rho^e - \rho^r)) + \alpha^k_2(r^l - r) + \alpha^k_3(y/y^p - \bar{U}) + \gamma + \delta \]  
   \[ g^d_h = \alpha^h_1((1 - \tau_c)(\rho_h - \rho^r)) + \alpha^h_2(r^l - r) + \alpha^h_3(\frac{c^d_{kh}}{kh} - \bar{U}h) + \gamma + \delta_h \]  

4. Rates of Return:
   \[ \rho^e = y^e - \delta + (p_x/p_y)x - (w^{be}/p_y)t^d_f - (p_m/p_y)j^d \]  
   \[ \rho^r = (1 - \tau_r)r^l - \pi^l \]  
   \[ \rho_h = (p_h/p_y)c^d_{h}/kh - \delta_h \]  

5. Prices, Wages and Transfers:
   \[ p_x = ep^e_x \]  
   \[ p_m = (1 + \tau_m)ep^e_m \]  
   \[ w^{be} = (1 + \tau_p)w^e \]  
   \[ w^{we} = \alpha^w w^e \]  
   \[ w^{re} = \alpha^r w^e \]  
   \[ p_u = (1 + \tau_u)p_y \]
6. Disposable Income of workers and taxes per value unit of capital:

\[
\begin{align*}
D_{w_1} &= (1 - \tau_w)[w^e l^{de} + w^{ue}(l^c - l^{ue}) + w^{re}\alpha_t l^v] / p_v \\
D_w &= y_{w_1} + (1 - \tau_c) rb_w \\
t^w &= \tau_w[w^e l^{de} + w^{ue}(l^c - l^{ue}) + w^{re}\alpha_t l^v] / p_v + \tau_p(w^e / p_v) l^{de} \\
&+ \tau_v(p_y / p_v) (y^d - g^d_h - g^d_k) + \tau_m e g_m^d / p_v \\
t^c &= \tau_c[\rho^c(p_y / p_v) + rb + b^t + (p_h / p_v) c_h - (p_y / p_v) \delta_h] 
\end{align*}
\]

The steady state of the model is the same as before, see the preceding section. The dimension of this dynamical system can be further reduced, to dimension 16, if the housing sector is removed from the model.

6 Outlook: Feedback structures and stability issues

We have presented a structural mode of disequilibrium growth which is fairly complete with respect to markets, sectors and agents. We believe it is sufficiently detailed to capture the essential dynamic features of modern macroeconomies, whilst at the same time abstracting from the welter of detail that inevitably must characterize large scale macroeconometric models such as the Fair model of the US economy of the Murphy model of the Australian economy.

We considered the model from the point of view of national accounts, and then discussed the development of its extensive form and finally expressed its dynamic structure in terms of intensive form state variables. We saw in particular that in intensive form we are dealing with a 34-D dynamical system. We further found that with a small number of further assumptions (concerning consumption of asset holders and certain secondary delayed adjustment processes) the dynamics reduce to an 18-D dynamical system which we call the core model.

These turn out to be eight main partial feedback mechanisms contained in the 18D core model. First, the labor and goods market interaction whose tendency to become destabilizing is determined by an interplay between wage and price flexibility. Second, the expected sales and inventory accumulation interaction. The tendency to instability of this mechanism is determined by the relative values of speeds of adjustment of expected sales and inventory changes. Third, the dynamics of the housing sector is determined largely by the strength of investment into this sector and the speed of adjustment of the prices for housing services. This partial feedback mechanism is always stabilizing. Fourth, the dynamic interaction between the level of economic activity and the nominal interest rate (the so-called Keynes effect). The stabilizing/destabilizing tendency of this mechanism is very dependent on the sensitivity of the nominal interest rate to the price level. We note that when considered in conjunction with the Taylor-style interest rate rule used in our model this mechanism is by and large stabilizing. Fifth, the inflation / expected inflation mechanism. This is essentially a destabilizing effect (associated with the names of Mundell and Cagan) determined by the interplay of speeds of adjustment of prices and inflationary expectations. Sixth, the bond and stock market dynamics (originally considered by Blanchard (1981)) first in isolation and
then their interaction, are driven by rates of return and expectations feedback. The mainly destabilizing tendency of this mechanism is driven by the speeds of adjustment of bond prices and expectations of bond price inflation. Seventh, the dynamics of the government budget restraint. This mechanism, when considered in isolation, is stabilizing provided the rate of growth of government debt is restricted in certain ways. Eight, the exchange rate/expected exchange rate (Dornbusch) mechanism where stabilizing/destabilizing tendencies essentially depend upon the relationship between the speed of adjustment of exchange rates and exchange rate expectations.

The 18D core model of this paper will be studied in a series of further papers, see Chiarella and Flaschel (1999d), Chiarella, Flaschel and Zhu (1999a,b), Chiarella, Flaschel, Groh, Köper and Semmler (1999a,b). These papers discuss the feedback structure of the 18D core dynamics (discussed above) in great detail with respect to partial stabilizing or destabilizing feedback chains there present. They also present further numerical investigations of the model both from the local and the global point of view (adding also extrinsic nonlinearities to achieve global boundedness) and extending the theoretical basis of the disequilibrium growth model employed so far (by allowing for smooth input and output substitution and other flexibilities).

Prototype subdynamics of this 18D system which are often discussed in the literature in isolation, and which we have briefly discussed above, will be derived and analyzed in Chiarella and Flaschel (1999d). This paper will provide more insights into the stability properties of the 18D dynamics by investigating the partial feedback chains it contains (often well-known from comparative static analysis) with respect to the partial dynamics to which they give rise. The interaction of these partial dynamics will then be studied in Chiarella, Flaschel and Zhu (1999a), and further papers, there basically from the numerical point of view as in Barnett and He (1998) who – as we have seen in section 7 – use a 14D second order dynamical representation of the UK economy for studying the bifurcation loci to which such a model type can give rise.

7 References


31


Appendix: Notation

The following list of symbols contains only domestic variables and parameters. Foreign magnitudes are defined analogously and are indicated by an asterisk (*). To ease verbal descriptions we shall consider in the following the 'Australian Dollar' as the domestic currency (A$) and the 'US Dollar' ($) as a representation of the foreign currency (currencies).

A. Statically or dynamically endogenous variables:

\[
\begin{align*}
    y & \quad \text{Output (per K) of the domestic good} \\
    y^d & \quad \text{Aggregate demand (per K) for the domestic good} \\
    y^n & \quad \text{Potential output (per K) of the domestic good} \\
    y^n_\pi & \quad \text{Normal sales (per K) of the domestic good} \\
    y^n_\phi & \quad \text{Normal output (per K) of the domestic good} \\
    y^n_\pi & \quad \text{Expected sales (per K) for the domestic good} \\
    y^D, y^D_c & \quad \text{Real disposable income (per K) of workers and asset-holders} \\
    l_{16}^f & \quad \text{Population aged 16 - 65 in efficiency units (EU: \( \times \exp(n_{it}) \), per K)} \\
    l_{65}^f & \quad \text{Population aged 66 - ... in EU (per K)} \\
    l_{0}^f & \quad \text{Population aged 0 - 14 in EU (per K)} \\
    l_{de}^e & \quad \text{Total employment of the employed in EU (per K)} \\
    l_{de}^e & \quad \text{Total employment of the work force of firms in EU (per K)} \\
    L_g & = L_g^d & \quad \text{Total government employment in EU (per K)} \\
    l_{w}^e & \quad \text{Work force of firms in EU (per K)} \\
    l_{w}^e & \quad \text{Total active work force}
\end{align*}
\]
Employment rate of those employed in the private sector

Participation rate of the potential work force

Rate of employment (\( V \) the employment–complement of the NAIRU)

Real (equilibrium) goods consumption of workers (per K)

Real (equilibrium) goods consumption of asset owners (per K)

Total goods consumption (per K)

Supply of dwelling services (per K)

Demand for dwelling services (per K)

Gross business fixed investment (per K)

Gross fixed housing investment (per K)

Gross (net) actual total investment (per K)

Planned inventory investment (per K)

Actual inventories (per K)

Desired inventories (per K)

Nominal short-term rate of interest (price of bonds \( p_b = 1 \))

Nominal long-term rate of interest (price of bonds \( p_b = 1/r^t \))

Expected appreciation in the price of long-term domestic bonds

Required rate of interest

Price of equities

Expected appreciation in the price of equities

Total nominal savings (per \( p_b K \))

Total nominal savings (per \( p_b K \))

Nominal savings of households (per \( p_b K \))

Nominal savings of firms (\( = p_b Y_f / p_b K \), the income of firms) per \( p_b K \)

Government nominal savings (per \( p_b K \))

Nominal (real) taxes \( p_b K, K \)

Real government expenditure (per K)

Expected short-run rate of profit of firms

Actual short-run rate of profit of firms

Normal operation rate of profit of firms

Expected long-run rate of profit of firms

Actual rate of return for housing services

Expected long-run rate of return for housing services

Capital stock

Capital stock in the housing sector (per K)

Nominal wages including payroll tax (in EU)

Nominal wages before taxes (in EU)

Unemployment benefit per unemployed (in EU)

Pension rate (in EU)

Price level of domestic goods including value added tax

Price level of domestic goods net of value added tax

Price level of export goods in domestic currency

Price level of import goods in domestic currency including taxation

Rent per unit of dwelling

Expected rate of inflation (over the long run)

Exchange rate (units of domestic currency per unit of foreign currency: AS$/\)$

Expected rate of change of the exchange rate

Labor supply (per K)

Stock of domestic short-term bonds (index d: stock demand) (per \( p_b K \))

Short-term debt held by workers (\( = B / p_b K \))

Short-term debt held by asset owners (per \( = B_c / p_b K \))

Stock of domestic long-term bonds, of which \( b_d \) are held (\( = D_d / p_b K \)) by domestic asset-holders (index d: demand)

and \( b_d^K \) by foreigners (index d: demand)
The text provides a list of variables and their definitions, organized in a natural way. It includes:

- $B^i$: Foreign bonds held by domestic asset-holders
- $E$: Equities (index d: demand)
- $n$: Natural growth rate of the labor force
- $n_t$: Rate of Harrod neutral technical change
- $\tau_m$: Tax rates on imported commodities
- $x$: Exports (per $K$
- $j^d$: Imports (per $K$
- $n_x = (p_x x - ep^*_m, j^d)/p$: Net exports in terms of the domestic currency
- $nfx$: Net factor export payments
- $ncx$: Net capital exports
- $\tau_w$: Tax rate on wages, pensions and unemployment benefits
- $d$: Actual public debt / output ratio

**B. Parameters of the model**

- $\delta$: Depreciation rate of the capital stock of firms
- $\delta_h$: Depreciation rate in the housing sector
- $\alpha$: All $\alpha$-expressions (behavioral or other parameters)
- $\beta$: All $\beta$-expressions (adjustment speeds)
- $\gamma$: Steady growth rate in the rest of the world
- $\bar{U}$: Normal rate of capacity utilization of firms
- $\bar{U}_h$: Normal rate of capacity utilization in housing
- $\kappa_{w}, \kappa_p$: Weights of short- and long-run inflation
- $\kappa = (1 - \kappa_{w}\kappa_p)^{-1}$
- $y^p$: Output-capital ratio
- $x_v$: Export-output ratio
- $l_v$: Labor-output ratio (labor in efficiency units)
- $j_v$: Import-output ratio
- $d$: Desired public debt / output ratio
- $\xi$: Risk and liquidity premium of long-term over short-term debt
- $\xi_e$: Risk premium of long-term foreign debt over long-term domestic debt
- $\tau_c$: Tax rates on profit, rent and interest
- $\tau_v$: Value added tax rate
- $\tau_p$: Payroll tax
- $c_1$: Propensity to consume goods (out of wages)
- $c_2$: Propensity to consume housing services (out of wages)