Follow the Leader
Simulations on a Dynamic Social Network

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Abstract
A social network in which individuals seek to be early adopters of a popular trend has as its equilibrium structure a (unique) leader and hierarchy of followers. The static analysis describing this equilibrium fails to offer a pathway for the equilibrium to emerge from an initially unstructured environment. A dynamic model with evolving social links is employed to test for and understand the emergence of the equilibrium structure. Simulations of the model reveal the importance of adaptation to the observed environment. Different adjustment processes are explored to understand emergence hierarchies conforming to the equilibrium. The emergence of structure not conforming to equilibrium or autarky for certain settings offer insight into those adjustment processes and environments that fail.

Keywords: Dynamic Network, Social Interaction, Consumer Choice, Simulation

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1. Introduction

This exercise models decisions by a population for a product in which tastes are uncertain and possibly endogenous to the process. Opinion leaders (Gurus) arise naturally from the population and contribute to the agents’ decisions, possibly by shaping tastes.

Consider three scenarios under which consumers seek input from outside experts:

1. The consumer has a fixed set of preferences but imperfect information concerning the available product options. The expert offers information or advice that helps the consumer buy the product that maximizes his exogenous utility. The environment could be modeled as one in which the consumer possesses a highly noisy signal of the utility he or she would derive from each product and the expert, through nature or the expenditure of resources, can reduce the noise associated with each product.

2. The consumer possesses some innate preferences over the available products, but can be influenced by the opinion of others (peers) and experts. The “expert” may be someone possessing both better information than the general public about the consumer options (as in case I), but also someone who can provide advice that shapes preferences of consumers through education or charisma. This situation offers a clear opportunity for feedback between the advice provided by the expert and the choices made by the population of consumers should the expert wishes to be an opinion leader with a wide range of influence.

3. The consumer has no innate preferences. The consumer’s tastes are fully fashioned by the influence of peers and experts. In this case, the expert shapes opinion, but need not have any special advantage in evaluating the options. Success is derived from the expert’s influence over the general population’s choice.

To motivate the endogeniety of preferences, consider the purchase of wine. A wine expert plays an integral role in the purchasing decisions of many consumers. The wine expert can be a professional who makes a living as a wine critic, rating and reporting about wines, it may be the helpful proprietor or employee at a preferred wine retailer, or it may be an acquaintance of the consumer who is perceived to be better informed about wines.
Preferences for certain products, including wine, are clearly subjective. For many such products, the consumer may be able to recognize difference, but may seek guidance in ranking these differentiated products. It seems quite reasonable that a consumer, informed by an expert that a particular bottle of wine is of high quality, will then adapt his definition of “good” to accommodate the flavors experienced. The fact that a particular description of the wine’s character is provided, for example “Red purple hue. Rich, ripe dark fruit aromas follow through with concentrated and mouthcoating flavors of black cherry and tobacco. Finishes with very rich tannins and well-integrated oak,” fails to explain why this particular set of flavors deserves the particularly high ranking or why “Brilliant garnet-ruby red hue. Cherry, strawberry and cedar aromas. A medium-bodied palate leads to a simple, clean finish with bright fruit, soft tannins and balanced acidity,” scores lower. Consumers who follow the advice may very well learn (rather than innately believe) that black cherry and tobacco with tannins and oak are superior flavors to cherry, strawberry and cedar.

Consider the ranking of the chateaus of the Bordeaux region of France. In 1855, Napoléon III assigned wine merchants the task of ranking wines for the upcoming Exposition Universelle de Paris. The merchants selected roughly 200 top chateaus of the region, bestowing upon them the label “Classified Growth.” Further indication of quality was designated by dividing the group into five categories, “first growth” (the best) through “fifth growth.” Original chateaux selection and ranking was based on reputation and the price of their wines. The list has remained largely fixed to the present. More importantly, so has the perception of quality despite numerous changes in ownership, management, production methods, technology, and the expansion of grape growing and wine production to many new regions of the world. Until recently, wine critics have been accused of adjusting their pallet each year in order to grant the most favorable reviews to the wines produced by the first growth chateaux. Such behavior by critics and the acceptance of such behavior by consumers suggests that preferences are fungible.

1 Wineanswers.com online review of Lagrange 2000 St. Julien (score: 93 points, exceptional) http://www.tastings.com/scout_wine.lasso?id=165665
2 Wineanswers.com online review of Cave de Chusclan 1999 Chateau de Gicon, Côtes-du-Rhône Rouge (score: 84 points, recommended) http://www.tastings.com/scout_wine.lasso?id=165925
3 This original ranking could be deemed a reflection of some exogenous measure of quality. The price was believed to be an accurate reflection of the wine’s quality. Marketing, technology, and economies of scale were fairly homogenous. Alternatively, consistent with this paper, this original list could be considered arbitrary.
4 The only change was in 1973 when Château Mouton-Rothschild was elevated from a second growth to a first growth vineyard.
Today’s wine critics are perceived as applying a more consistent standard to their wine tasting, but the ability of consumers to learn to enjoy what they are advised to appreciate through acquisition of an educated palate remains.

The rise of Robert Parker Jr. as a leading wine critic, whose influence is recognized as dominant in the industry, raised alarm among other wine experts and among the wine production industry. The concern is that Parker’s dominant position imposes uniformity in consumption and tastes. Consider case I, homogeneous tastes among the wine consuming public explains Parker’s dominance. In this scenario, Parker’s dominance over other wine critics reflects Parker’s ability to offer the most accurate signal on wine quality. With homogeneous exogenous preference, the concern of other the industry is unfounded. Consumers are instead well served by Parker’s advice. Under case II, consumers’ tastes are malleable to conform to Parker’s advice. The uniformity of the educated palate based on Parker’s recommendations and the social rewards to conforming to these recommendations overwhelms heterogeneity in innate preferences. In case III, the reward to drinking wine is from drinking wine that is recognized by the consumer and his or her peers as high quality wine. Parker’s ratings serve as a coordinating device that improves the social payoffs to by setting a uniform standard. The heterogeneity that existed in Parker’s absence reflected a failure to coordinate rather than heterogeneity in preferences.

There are a number of areas of commerce, for example in financial planning and investment strategies, where increased homogeneity in behavior, particularly if it is the result of social influences dominating innate heterogeneity, can be to the industry’s (and the economy’s) detriment.

This project focuses on issues concerning the social phenomenon by which an agent in the population becomes a leader influencing the choice of others in the population. Within this examination is the question of whether the leadership position and structure of the social network of the followers is stable. Subsequent examinations will explore feedback relationship between a leader and his or her followers and other social phenomenon related to this environment.

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5 A number of profiles of Parker exist, including *The Atlantic Monthly* (2000).
Social networks have recently received considerable attention in economics for explaining aggregate social behavior. In these models, individuals may be influenced globally by the aggregate behavior of the population or locally by their neighbors, a subset of the population who serve as a reference group. This project incorporates aspects of both. Economic incorporation of local interactions in decision making include Schelling (1971) and (1973) and Katz and Shapiro (1985). The network is a social structure by which discrete decisions are made or new products or technologies are adopted. The utility of a choice depends on the choices of those in an agent’s peer group. Because of social interactions, products that garner no particular consumer preference, and may even be perceived to be inferior, can grow to dominate consumer choice. Social interactions that produce a majority payoff have been used to explain the persistent use of the QWERT typewriter key arrangement, the dominance of VHS over Beta video tapes and DOS over Macintosh operating systems. Analysis of these models has been facilitated by employment of tools from statistical mechanics to explore stability, multiple equilibria, and switching.

The reward structure for this project moves away from the simple majority or minority payoff to take timing into account. Agents are rewarded for being early adopters of a trend that subsequently becomes popular. Financial or social reward come to those who can gain a reputation for being a predictor (or setter) of trends. “Wine geeks just love bragging rights. They get kudos from their peers when they get a high-score wine first or get it cheaper” (Bialek quoted in Los Angeles Magazine, 1998(Dec)). Because the agents seek reward through early adoption, timing of an agent’s decision must be modeled as an endogenous component of the social network, a clear deviation from the traditional mean field examinations of local interaction. Further, agents are rewarded based on aggregate behavior rather than the local behavior of the individual’s peer group. Brock and Durlauf (2001) also models the individual as seeking to conform to the social norm, but in an environment in which the payoff is based purely on the aggregate behavior.

The traditional examination of social interactions in economic decisions is based on a fixed social network. Regular structures are convenient for deriving aggregate properties of the network. The level of connectedness determines the rate at which information disperses through the network, which can impact the leader and follower structure.
Dynamic models such as Watts (2001), Bala and Goyal (2000), Jackson and Watts (2002), and Kirman et al (2007) are concerned with evolution and convergence. Bala and Goyal (2000) examine the evolution of a network in which agents create links with individuals with whom the benefit exceeds the cost of maintaining the link. Each agent \( i \) offers a direct link benefit (that is uniform), but also the benefit gained by providing indirect access to those with whom agent \( i \) is linked. Thus, the benefit of linking to an individual is endogenous to how connected that individual is to others in the population. One stable attracting network configuration is a star formation where one agent serves as a hub through which all agents connect.

Networks are also a vehicle for information transmission. Directed networks (where the link between two agents is established by one of the linked pair) have been used to examine advertising and marketing, as in Dutta and Jackson (2000). Undirected networks (where the link requires the support of both agents in a linked pair) also serve as a mechanism of information dispersion, as in Ellison and Fudenberg (1995) where word-of-mouth communication can lead to conformity in behavior.

Information, evolution, convergence, and stability are all features of the proposed project, but the payoff structure and its dependence on the timing of a decision substantially and substantively alter the role played by the social connections, thus altering the process of formation and self organization. The directional path through which information is transmitted gives the network a hierarchical structure with a leader and follower. That the directed links are established by agents seeking information rather than by advertisers looking to push information is another important distinction affecting the formation of the network and is employment by the agents.

Kirman et al (2007) point out two relevant deficiencies in the earlier dynamic models. In setting the mechanism by which links are updated, one deficiency is that each link, existing or potential, has a value known to the agent. This knowledge required that the agent know the existing structure of the full social network and the mapping from the network structure to payoffs. A second concern is with the artificial process determining the order with which agents update their links. The developed model does not suffer from this deficiency.
Like this examination, Chang and Harrington (2005) consider an evolutionary network driven by an experience weighted attractor. Their population is heterogeneous with some better suited to innovation and other better suited to dissemination of ideas. They examine the endogenous social network that arises to connect these different types of researchers that results in a symbiotic relationship between innovators and those who are able to facilitate in the propagation of the innovator’s ideas. For the current investigation, agents must be allowed to adapt to their environment in a manner to improve their payoff. Simulations will be employed to characterize the behavior of the population.

2. Model

Goldbaum (2009) examines the steady state property of a static model with full information and full rationality. The following model environment is taken from Goldbaum (2009).

The population consists of \(L\) agents. In each time period, indexed \(t \in \{1, 2, 3, \ldots\}\), each agent, indexed \(i \in \{1, 2, 3, \ldots, L\}\) chooses from among \(K\) available options, indexed \(k \in \{"a", "b", "c", \ldots, K\}\). Agent \(i\)’s choice at time \(t\) is captured by the notation by \(k_{i,t}\). Think of each set of \(k_i\) as a new product offering within a product category. Each period is subdivided into rounds, indexed \(r \in \{1, 2, 3, \ldots, R\}\). As developed below, individual agents employ a variety of strategies to select between the available options, leading to different rounds of adoption.

Selection between the different options is motivated by three concerns. The agent wishes to make a choice consistent with his or her own preferences across the different options. The agent also wishes to make a choice that conforms to the popular choice of the population. Finally, in conforming to the popular choice, the agent wishes make his or her choice ahead of the crowd. To this end, the choice made by individual \(i\) at time \(t\) earns a utility payoff according to the formula,

\[
\pi_{i,t} = u(k_{i,t}) + J(N^{i^T}_{k_{i,t}}) + T(N^{T}_{k_{i,t}}). 
\]

(1)

The first term, \(u(k_{i,t})\) is the individual’s innate private utility derived from the choice. Its value is specific to the individual and is unaffected by the choices made by others in the population. The \(J(N^{i^T}_{k_{i,t}})\) component captures the conformity of choice with others in the
population. The function input, \(N_{ki,t}^{J}\), represents the total number of agents in period \(t\) to have chosen the particular option \(k\) the same as agent \(i\). It is unaffected by the round of adoption. The desired attributes for \(J\) has \(J(x) \geq 0\), \(J(1) = 0\) and \(J'(x) \geq 0\).

The \(T(N_{ki,t}^{T})\) element of the payoff rewards the agent for being an early adopter of a subsequently popular trend. The function input \(N_{ki,t}^{T}\) represents total number of agents in period \(t\) who adopt choice \(k\) that coincides with that of agent \(i\) in rounds subsequent to agent \(i\)'s adoption. If \(N_{ki,t}^{r}\) is the number of agents who adopt choice \(k_{i}\) by round \(r\) in period \(t\), then \(N_{ki,t}^{T} = N_{ki,t}^{R} - N_{ki,t}^{C}\) and \(N_{ki,t}^{J} = N_{ki,t}^{R}\). The desired attributes for \(T\) are \(T(x) \geq 0\), \(T(0) = 0\), and \(T'(x) > 0\). A linear structure for \(J\) and \(T\) produce a constant marginal benefit for each new conforming agent and each new follower,

\[
J(N_{ki,t}^{J}) = a_{J}(N_{ki,t}^{J} - 1) \quad \text{(2)}
\]

\[
T(N_{ki,t}^{T}) = a_{T}N_{ki,t}^{T} \quad \text{(3)}
\]

The coefficients \(a_{T}\) and \(a_{J}\) differentiate the payoff to agent \(j\) for being an early adopter from the payoff for making a popular choice. Meaningful incentives require \(0 < a_{J} < a_{T}\).

Agent \(i\) maintains \(d\) non-redundant links to other agents of the population. If agent \(i\) maintains a link to agent \(j\), then \(j\) is one of agent \(i\)'s “friends.” The links are one-way in that agent \(i\) who maintains a link to agent \(j\) may choose to imitate agent \(j\) but the reverse is not true unless agent \(j\) independently happens to maintain an link to agent \(i\).

The individual agents have the option to act independently when in selecting \(k_{i,t}\). If they do so, they will choose in the first round of the period, as there is nothing to be gained by waiting. Alternately, they may employ a strategy to imitate the choice made by another in the population. Employing this strategy results in individual \(i\) making a decision in the round following the round they observe the action of the person they imitate.

This particular investigation is interested in isolating the social influence on individual decision strategies and the social structure that emerges to support those decisions. To that end \(u(k_{i,t}) = 0\) is presumed for the remainder of this investigation.
Goldbaum (2009) establishes that for every agent in the population, there exists a hierarchical structure based on that agent as the sole leader in the population. Under the assumption of full information about the network, the hierarchy is a Nash equilibrium.

With the objective of understanding the emergence of a leader and hierarchy formation from an unstructured social network, many of the assumptions of the static model are relaxed. Instead of engaging in rational strategic behavior, agents choose among the available strategies probabilistically, updating these probabilities over time based on individual observations and experiences.

Each period starts with each agent choosing whether to act independently or through imitation. With probability \( \theta_{i,t} \), \( 0 \leq \theta_{i,t} \leq 1 \), agent \( i \) chooses to act independently. If acting independently, agent \( i \) chooses among the available options with probability,

\[
\Pr(k_{i,t} = k) = p_{i,t}^k
\]

with \( \sum_{k=1}^{K} p_{i,t}^k = 1 \). In the absence of innate preferences, \( p_{i,t}^k = 1/K \forall k,i,t \).

With probability \( 1 - \theta_{i,t} \), agent \( i \) choose to imitate one of the other agents in the population. Associated with each of agent \( i \)’s \( d \) links is a weight, \( w_{i,t}^j \). The weight \( w_{i,t}^j \) is the probability that agent \( i \) imitates agent \( j \), conditional on having chosen to imitate, so that

\[
\Pr(\text{agent } i \text{ imitates agent } j) = (1 - \theta_{i,t})w_{i,t}^j
\]

with \( \sum_{j} w_{i,t}^j = 1 \).

Prior to the first round, each agent chooses either to act independently or has chosen a friend to imitate for the period. In round 1 of period \( t \), those who intend to act independently make their choice. In round two, those who have chosen to imitate those who acted in the previous round do so. The process continues for \( R \) rounds as trends disseminate through the population via the network of imitators.
In the final round, any agents yet to make a choice considers the population and choose option \( k \) with a probability \( p^{R}_{k,t} \) that depends on the popularity of the option observed in the previous round,

\[
\Pr(k^{R}_{t,i} = k \mid k^{R-1}_{i,t} \text{ unassigned}) = \frac{\exp(\rho N^{R-1}_{k,t})}{\sum_{k} \exp(\rho N^{R-1}_{k,t})}.
\] (5)

The parameter \( \rho \) is the intensity of choice, capturing how sensitive the residual population is to the relative popularity. Setting \( \rho \to \infty \) produces full adoption of the most popular choice. Setting \( \rho < \infty \) can be interpreted as bounded rationality or as the result of noise in the perception of relative popularity of the choice among the population. The agents choose without considering the population for \( \rho = 0 \).

At the end of each period, the agents know \( N^{r}_{k,t} \) for each \( k \) and each \( r \). The agent also knows the round of adoption of each friend. The agent does not have knowledge of the network of links employed, preventing the agent from correctly computing the payoff that would have been received had they chosen differently. Instead, they simply take the number of adopters in each round as given and compute the payoff of imitating each friend based on these populations.

After the last round of the period, each agent adjusts the probability, \( \theta_{i,t} \), and weights, \( w^{i}_{t} \), based on the perception of how successful each option would have been during the just completed period. For the strategy actually employed, the agent knows the actual payoff generated by the strategy.

Similarly, if they imitated a friend, they need to speculate what the payoff would have been to acting independently. Each agent has chosen one of the options as the one they would have adopted had they acted independently. Naively, they take the payoff of having chosen that option in the first period as their speculative payoff. This can generate “regret” for those that would have chosen a subsequently popular choice for that period. Let \( z^{i}_{t} \) represent the payoff earned by agent \( i \) by imitating agent \( j \) in period \( t \), whether actual or speculative.
The probabilities $\theta_{i,t}$ and $w_{i,t}^{j}$ are adjusted according to the algorithm derived from the *Experience-Weighted attraction* (EWA) learning rule suggested by Camerer and Ho (1999). Let $A_{i,t}^{j}$ be a cumulative performance measure associated with agent $i$’s imitation of agent $j$. A separate performance measure is maintained for each friend of agent $i$. The value of $A_{i,t}^{j}$ is updated according to

$$A_{i,t}^{j} = (\phi_{A}A_{i,t-1}^{j,N} + (\delta_{A} + (1 - \delta_{A})I(s_{i,t}, j))\pi_{A}(z_{i,t}^{j}) ) / N_{A,t}.$$  

This algorithm allows considerable flexibility. Briefly, $\pi_{A}(z_{i,t}^{j})$ is the period $t$ realization of performance associated with agent $i$ selecting to imitate agent $j$. The term $(\delta_{A} + (1 - \delta_{A})I(s_{i,t}, j))$ is a mechanism for down-weighting the unobserved speculative payoff associated with the friends who were not selected by $i$ in the current period. Thus, the update of $A_{i,t}^{j}$ gets full weight if $j$ was actually chosen by $i$ in that period and has the diminished weight of $\delta$, $0 \leq \delta_{A} \leq 1$, if not. Setting $\phi_{A} < 1$ decreases the weight of an observation with the passage of time. Finally, $N_{A,t}$ follows a process

$$N_{A,t} = \rho_{A}N_{A,t-1} + 1.$$  

For $N_{A,0} = 1$ and $\rho_{A} = 1$, then $N_{A,t} = t$. Combine these with $\phi_{A} = 1$ and (6) becomes a formula for creating an equal-weighted average of all previous observations. Various setting of $N_{A,0}$, $\rho_{A}$, and $\phi_{A}$ can be used to achieve a variety of weighted combinations of the previously observed $\pi(z_{i,t}^{j})$. See Camerer and Ho (1999) for more detail.

The weight that agent $i$ assigns to his link to agent $j$ is based on the relative performance measure. The power distribution is one option suggested by Camerer and Ho and is one of two primary mechanisms examined in this paper,$^{6}$

$$w_{i,t}^{j} = \frac{A_{i,t}^{j,k}}{\sum_{k=1}^{k}A_{i,t}^{j,k}}.$$  

$^{6}$ Other options explored were the exponential distribution

$$w_{i,t}^{j} = \exp(\lambda A_{i,t}^{j}) / \sum_{k} \exp(\lambda A_{i,t}^{k}).$$
Notice that the weights are based on (a non-linear function of) relative performance. Better performing friends have a higher weight, but all friends with a non-zero performance measure have a non-zero probability of being chosen (for finite $\lambda$). Simulations that set weights according to (8) will be referred to as EWA simulations.

A modification to the traditional EWA that allows a higher performing friend attracts greater weight at the expense of poorly performing friends is also examined. The alternative employs a modification to the $K$-choice Replicator Dynamic process proposed by Branch and McGough (2008) to model the evolution of discrete choice selection. Let $\bar{A}_i$ be the average performance across agent $i$’s friends, weighted by current link weights, $\bar{A}_i = \sum_j w_{ij} A_{ij}$. Further, let a friend’s performance fall into the sets “good” or “bad” according to

$$
\hat{d} = \{1, \ldots, d\}, \quad Gd = \{ j \in \hat{d} \mid A_{ij} \geq \bar{A}_i \}, \quad Bd = \{ j \in \hat{d} \mid A_{ij} < \bar{A}_i \}
$$

The weights evolved according to the process

$$w_{ij} = w_{ij} + \begin{cases} r(A_{ij} - \bar{A}_i) w_{ij} & \text{for } j \in Bd \\ x \sum_{j \in Gd} r(A_{ij} - \bar{A}_i) w_{ij} & \text{for } j \in Gd \end{cases} \quad (9)$$

with

$$x_j = \frac{\zeta / |Gd| + A_{ij} - \bar{A}_i}{\zeta + \sum_{j \in Gd} (A_{ij} - \bar{A}_i)}$$

and $r(x) = \tanh(\lambda_{rd} x / 2)$.

The modification from Branch and McGough is to use the weighted average, $\bar{A}_i$, rather than the simple equal weighted average performance. Without this modification, there is no mechanism for the highest performing friend to attract weight from other “good” friends (particularly once all of the “bad” friends reach zero weight).7

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7 The process when employing weighted averaging converges to place all weight on the highest performing friend. If that process converges before relative position in the hierarchy stabilizes, the agent could settle on a best friend that is eventually realized to be suboptimal. It might be natural to leave such a situation be but to introduce a mechanism to allow the agent to adjust weights in such a case, the agent can check for the event $\max_{j} (w_{ij}) = 1$ and $\max(A_{ij}) > \bar{A}_i$, in which case the conditions for “good” and “bad” become

$$
\hat{d} = \{1, \ldots, d\}, \quad Gd = \{ j \in \hat{d} \mid A_{ij} \geq \bar{A}_i \}, \quad Bd = \{ j \in \hat{d} \mid A_{ij} < \bar{A}_i \}
$$
Payoffs are transformed into performance measures by normalizing by the highest payoff friend,

\[ \pi(z'_{i,j}) = z'_{i,j} / \max_k (z_{i,k}^k). \]  

(10)

Similarly, \( \theta_{i,t} \) is updated according to

\[
B'_{i,t} = (\delta_B B'_{i,t-1} N_{B,t-1} + (1-\delta_B) I(s_{i,t}, j)) \pi_B (y'_{i,t}) / N_{B,t},
\]

\[ \theta_{i,t+1} = \theta_{i,t} + \begin{cases} \tanh(\lambda_z (B''_{i,t} - B'_{i,t}) / 2)(1-\theta_{i,t}) & \text{for } B''_{i,t} \geq B'_{i,t} \\ \tanh(\lambda_z (B''_{i,t} - B'_{i,t}) / 2)\theta_{i,t} & \text{for } B''_{i,t} < B'_{i,t} \end{cases}, \]

(11)

(12)

\( j = 0,1 \) and \( N_{B,t} = \rho_B N_{B,t-1} + 1 \). The payoff \( y^0_{i,t} \) is simply the payoff to acting independently, \( y^0_{i,t} = z^\text{ind}_{i,t} \) while the payoff \( y^1_{i,t} \) is the payoff to imitating. The latter is taken by the agent to be the best payoff offered by one of his or her friends, \( y^1_{i,t} = z^\text{max(f)}_{i,t} = \max_h (z^h_{i,t}) \).

3. Analysis of the social network

Two concepts emerge from the fully rational steady state analysis; the “efficiency” of a hierarchy and the “optimal” of a hierarchy. Briefly, given a leader, there is an efficient hierarchy by which each remaining agent in the population connects to the leader directly or indirectly using the shortest round possible. For the individual follower, there may be multiple routes to the leader. The efficient tier for the individual in the hierarchy is the highest tier attainable by the agent given the leader and their list of friends.

The efficiency of a hierarchy is measured based on the cumulative distance of individual agents from their efficient tier. Let \( \delta_{i,t} \) represent the tier occupied by agent \( i \) in period \( t \). Let \( \delta^*_{i,t} \) indicate the efficient tier based on the current leader at time \( t \). Efficiency, \( E \), is thus measured as

\[ E = \sum_i \delta_{i,t} - \delta^*_{i,t} \]

(13)

with \( E = 0 \) representing a fully efficient hierarchy and values of \( E > 0 \) represent deviations from efficiency.

\[ Gd = \{ j \in \hat{A} \mid A'_{i,j} \geq \overline{A}''_{i,j} \}, \quad Bd = \{ j \in \hat{A} \mid A'_{i,j} \leq \overline{A}''_{i,j} \} \]

for as long as the event remains true.
While each individual is the leader of a hierarchy that is a Nash equilibrium to the static model, individuals are not all equally robust as leaders. A natural leader is the agent(s) in the population who has the greatest number of incoming links. In a symmetric network for which each agent has $d$ incoming links, there is no natural leader. In a randomly generated network, but there is a distribution of incoming links around the average of $d$. The hierarchy is considered optimal if the agent with the greatest number of incoming links emerges as the leader.\(^8\)

The optimality of the leader is measured by the difference in the number of potential fans of the current leader relative to the maximum number of potential fans of the individuals in the population. Let $f_i$ be the number of agents with links to agent $i$ and let $f_{0,t}$ be the number of links to the leader at time $t$. The optimality of the leader, $O$, is thus

$$O = \max(f_i) - f_{0,t}$$

(14)

4. Simulations

Each simulation starts with a population of individuals linked through a social network. The baseline initiation is to randomly assign the links, though symmetrically structured social networks are examined as well. The baseline is to assign the initial weights equally, $w_{i,j} = 1/d$ and $\theta_{i,j} = 0.5$. The other baseline parameters of the simulation are reported in Table 1.

Two types of figures are employed to display the findings. The time-series figure plots a time-series of a number of model-generated variables. $N(\text{ind})$ is the number of agents choosing to act independently, $\text{MaxFans}$ is the number of imitators of the agent with the highest number of fans that period, and $N(r=tr)$ is the number of agents who make their selection in the final round. Also included are the measures of optimality and efficiency.

The hierarchical figure plots the social structure as it exists in the final period of the simulation. Across the top of the figure are the $K$ choices, labeled using capital letters. Individual agents, labeled using numbers, appear in rows below the choices, based on the round of adoption. An arrow from an individual directly to one of the choices represents a choice by an

\(^8\) Other designation of a “natural leader” could be considered. The proposed definition is both reasonable and readily verifiable.
individual who has chosen to act independently for the period. An arrow from one agent to another represents a path of imitation with the arrow originating from the imitator.

4.1 Emergence of a leader

The emergence of a leader and a hierarchy of followers from the initially unstructured social network is quite robust. An example of the process of emergence is plotted in Figure 1. Figure 2 depicts the resulting social hierarchy. In this example, agent #2 emerges from the population as the leader and a hierarchy, based on agent #2’s leadership, forms. In round 1 of each period subsequent to formation, agent #2, occupying tier 0 of the hierarchy, randomly selects one of the 12 options available for the period. In round 2, those in tier 1 of the hierarchy imitate agent #2. By round 5, agent #2’s choice for the period has disseminated through the entire population.

An individual’s emergence is path dependence. In the early periods of the simulation, success by the individual is the result of random events that, in the absence of any adjustment, are transitory. For a leader to emerge, it must be that others in the population respond to the lucky individual’s success by, for example, increasing the weight, \( w_{i,j} \), if they are linked to someone successful and decreasing their own \( \theta_{i,j} \) in response to the higher payoff offered by imitation. The process of observation and adjustment allows the successful agent to become empowered by his or her followers, eventually generating a hierarchy. Success breeds further success.

4.2 Efficiency in the hierarchy

Efficiency in the hierarchy depends on the individual’s ability to find and settle on the friend offering the shortest route to the leader.

Given the emergence of a leader, asymptotic efficiency in the hierarchy requires that individuals asymptotically settle on the best friend offering the most direct link to the leader (or friends if multiple friends offer equal length shortest paths). To achieve this, the random component of their decision process must converge to zero, as can be accomplished employing
the replicator dynamic process with infinite memory, eventually assigning $w_{ij}^t = 0$ to those friends offering inefficient routes to imitate to the leader.

As a leader emerges, the consistently superior performance offered by the efficient hierarchy leads to its eventual adoption with the probabilities of alternative strategies driven to zero by each individual. As a Nash equilibrium, there is no incentive for individual deviations from employing the strategies that produce the efficient hierarchy that has emerged.

Individual adjustment can be very slow and still generate enough adjustment over time to allow eventual emergence. Figure 3 is generated from a simulation with $\lambda_{RD} = 0.01$.

Alternatively, allocating weight according to the EWA process fails to produce an efficient hierarchy. Because weights on inferior friends do not converge to zero, individual follower to occasionally choose an inefficient friend. Doing so disrupts the efficiency of the choices of others in the hierarchy. These simulations tend to produce a constant shuffling of those in the lower tiers of the hierarchy. Figure 4 plots the time-series output from a EWA simulation. The leadership position is stable, but those in the lower tiers of the hierarchy shuffle positions each period. The resulting hierarchy, displayed in Figure 5, tends to have a greater number of tiers as individuals end up employing inefficient paths.

The only reason to be caught without making a choice before the final round is for the agent to have been caught in a closed circle of links, for example when two agents link with each. With so much shuffling in the population, this situation arises when, say, agent $i$ higher up in the hierarchy links to an individual farther down in the hierarchy who is connected to the leader through agent $i$. This undecided population choose one of the $K$ options in the final round for the period according to (5).

4.3 Optimality of the leader

In the baseline simulation employing replicator dynamics, a leader will typically emerge within the first 20-50 periods. While being a natural leader with a high number of links directed at the individuals increases the probability of emerging as a leader, other factors dominate the process of emergence. The random events that determine initial choices and their success tend to
swamp the benefit of being a natural leader. An individual who persistently acts independently in the early periods of the simulation has a considerably increased probability of emerging as the leader. If all $\theta_{i,0}$ values are initialized at 0.5, then persistent independent actions is initially luck that evolves into a learned behavior as followers are attracted, empowering the leader. When $\theta_{i,0}$ is initialized randomly, the eventual leader tends to be among those with the largest starting value for $\theta_{i,0}$.

It is better for a non-leader to imitate than to act independently. With infinite memory in the process governing $\theta_{i,t}$, individuals learn this lesson once a leader has emerged. For the leader, $\theta_{i,t} \rightarrow 1$ quickly while $\theta_{j,t} \rightarrow 0$ for all non-leaders, making them into followers. Once a leader has emerged in simulation, whether a natural leader or not, that individual retains the leadership position throughout. As a Nash equilibrium, there is no single deviant behavior that will remove the leader, whether or not movement among the followers in the hierarchy remains present.

A finite memory in the $\theta_{i,t}$ process tends to prevent convergence of $\theta_{i,t} \rightarrow 0$ for followers but still allows $\theta_{i,t} \rightarrow 1$ for the leader. In the extreme, with $\phi_2 = 0$, the individual is adaptive, using the previous period’s payoffs as the basis for the current period’s $\theta_{i,t}$. As a result, each period, a population of these naively delusional individuals are present to act independently, though the individuals populating this group change each period. Recall that the follower evaluates the value of acting independently employing a naïve process speculating on the payoff of a particular choice would have made if acting independently. As a result, each period, approximately $(L-1)/K$ individual followers speculatively choose the same. These individuals mistakenly believe that they are able to foresee trends and therefore choose to act independently in the following period.

Figure 6 and Figure 7 are from a simulation with $\phi_2 = 0$. From Figure 7, agent #44 is the stable leader while a number of individual agents act independently (dragging their followers

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Given this finding, it might be natural to define the agent with the highest $\theta_{i,0}$ as the “natural leader” rather than defining it based on the number of potential fans. This individual is predisposed to leadership through his or her behavior whereas the individual with a greater number of incoming links has a social advantage.
with them). In this period, agent #35 happens to choose the same as the leader and will thus choose to act independently again next period (and may even attract a new set of followers), but the remaining individuals acting independently will return to imitation next period. They will be replaced by a new set of individuals from the population of followers who speculatively choose “F” this period.

While good for creating efficient hierarchies, the robustness of the leader generated from the replicator dynamic process means that once a leader has been established, no events arise in simulation to disrupt that position allowing a new leader to emerge. Optimality of the leader tends to be poor under replicator dynamics.

While poor at generating efficiency, the EWA process is better able to occupy the leadership position with a leader of greater fitness than is the replicator dynamics. Observe in Figure 4 that the emergent leader is a natural leader with an optimality measure of zero. Incorporating a short memory in \( \theta_{i,s} \) by setting \( \phi_2 = 0 \), combines with the movement in the social structure under EWA creates enough mixing to help the natural leader emerge as the original leader or to depose an existing sub-optimal leader, allowing a more fit leader to emerge. This can be seen in Figure 8 as agent #77 is replaced by agent #49 (in period \( t = 116 \)), who is replaced by agent #75 (in period \( t = 239 \)), increasing fitness with each new leader.

The persistence in random behavior allows greater exploration of the payoff space and eliminates the Nash equilibrium efficient hierarchy as a long-run outcome. The more appropriate analysis then is one based on random behavior such as a Bayesian equilibrium.

### 4.5 Non-emergence of a hierarchy

A leader can fail to emerge from the population if agents fail to respond to an individual’s successes. The transitory event that produced success for an agent in one period will be gone the next. If there is inadequate adjustment to the social network in response to a moment of success, then the opportunity to build on individual success is lost. In this situation, there is no social mechanism that will generate a leader. There are two way to prevent emergence within the context of the model. The first is to slow the agents’ response to the success that they observe. The second is to blind them to other’s success.
In the replicator dynamics process, the rate of adjustment to $w_{ij}^t$ is determined by

$$\lambda_{RD} \in [0, \infty)$$

with $\lambda_{RD} = 0$ resulting in no change and increasing adjustment towards the better performing friend for greater $\lambda_{RD}$. Simulations with $\lambda_{RD} = 0$, a leader emerges but the hierarchy based upon the leader is itself transient as individuals often fail to link to the leader, directly or indirectly, as seen in Figure 9.

The parameters $\delta_A$ and $\delta_B$ capture how much attention the agent pays to the potential payoff of those strategic options not followed in the give period. Setting $\delta_A < 1$ downweights the speculative performance of friends not imitated while setting $\delta_B < 1$ downweights the speculative payoff to the un-chosen strategy of imitation versus independent choice.

To build a network of fans, an agent needs to be visible. If an agent is lucky enough to choose a trend early, he or she needs to be observed doing so in order to attract fans. Setting $\delta_A < 1$ reduces the successful agent’s exposure. Setting $\delta_A = 0$ or close to zero prevents the emergence of a leader and thus the formation of a social structure. Without exposure, there is no response and thus no emergence. In order to prevent emergence, $\delta_A$ had to be set very low. Though the process is slowed compared to the baseline simulation, a value of $\delta_A = 0.1$ still generates an emergent efficient hierarchy.

Setting $\delta_B = 0$ or near zero leads to a stable pool of agents acting independently. They do so suboptimally, settling on a strategy based on insufficient information. Figure 10 results from setting $\delta_B = 0.01$.

### 4.6 Multiple Hierarchies

Without innate preferences, the only reward is social and the larger the group one associates with, the greater the reward. Analysis finds that the equilibrium generally consists of a single leader/follower hierarchy. With few exceptions, a larger hierarchy attract individuals away from a smaller hierarchy. Simulation supports this finding with the following special case exception.

A social network forcing followers to form long chains in order to link to the leader can invite the population to divide into multiple smaller hierarchies. The agent at the bottom of a
long chain depends on every agent between him or her and the leader to maintain a link to the leader. While there is randomness in friend selection, for example as persists under the EWA, then it is quite likely that someone in the middle of the chain will choose to imitate one of his own fans, causing a closed circle with no link to the leader. Those in the hierarchy unable to consistently link to the leader find it advantageous to form small, more stable hierarchies, as captured in Figure 14 for which the initial social structure was a ring with each agent linked to the three agents on either side.

4.4 Small number of options

From Goldbaum (2009), a sufficient and necessary condition for full participation in the hierarchy by the entire population is that

\[(K - 1) a_j \geq a_j (L - 2)/(L - 1).\]  \((15)\)

Too few \(K\), and those at the bottom of the hierarchy will act independently in an attempt to earn the leader’s payoff by coincidently choosing the same as the leader. The cost to independent action is to sacrifice the certainty of the conformity payoff. Too little \(a_j\) and the sacrifice is not sufficient to deter independent action. Large \(K\) and large \(a_j\) discourage independent action.

For violations of (15) There is a population \(L'\) increasing in \(K\) and \(a_j\) who participate in the hierarchy, \(0 < L' < L\). The remainder of the population, \(L - L'\), optimally choose to act independently.

Choose \(K\) and \(a_j\) such that (15) is violated and one of two possible outcomes arise. Large \(K\) but small \(a_j\) tends to generate a hierarchy that is initially too large, with the number of participants exceeding \(L'\). Overtime, the hierarchy slowly sheds participants from the lowest existing tier, as can be seen in Figure 11 and Figure 12 where \(K = 6\) and \(a_j = 0.1\) so that \(L' = 3\). The process clearly has a long way to go before reaching a hierarchy of size \(L'\), but the process takes time because it is only once an agent realizes that he or she is in the lowest tier of the existing efficient hierarchy that it becomes apparent that imitating is not optimal.
A small $K$ tends to result in no hierarchical formation. With so few options, there is little need to coordinate to earn a substantial conformity payoff. There is thus no mechanism by which a leader can emerge from the population, as seen in Figure 13, for which $K = 4$. Interestingly, Agents 22 and 86 have $\theta_{i,r}$ values of 0.102 and 0.014 respectively, suggesting a small but persistent population of followers, though each rotates between his or her friends in deciding whom to imitate, since all of their friends act independently. This rotation explains the lack of an emergent leader.

4.7 Friend Selection

Introduced to the model is the ability to discard existing links to establish new links. If the weight on the link from agent $i$ to agent $j$ drops below a threshold, $w$, agent $i$ will drop agent $j$ from his list of friends and randomly select a new friend. Two mechanisms for finding new friends are examined, selecting randomly from the full population of non-friends and limiting the agent to select from the friends of his top ranked friends.

Allow individuals in the population to replace low performing friends with new friends and the hierarchy will eventually collapse to a single leader and a large single layer of direct imitators. The payoff to the leader is unchanged, but this structure changes substantially the followers’ payoff. In the extreme, if the whole population of followers occupies tier 1, then only the leader has followers and the remainder of the population earns only $J(L)$.

The simulation producing Figure 15 and Figure 16 allows individuals to drop the lowest performing friends with $w_{t,r} < w$ with $w = 0.2 / d$. Under this criterion, the friend is dropped if he or she substantially underperforms with a weight that is only 20% of the average weight. Notice that not all agents link directly to the leader, with individuals still occupying tiers 2, 3, and 4. These agents, with all of their friends occupying the tier immediately above them, have no friends meeting the replacement criteria. Setting $w = 1 / d$ produces a similar outcome, but now it is a search problem as it becomes increasingly difficult to find a new friend not occupying tier 1. The simulation producing Figure 17 restricts individuals to choosing a new friend from the friend list of their highest performing friend, substantially increasing the number of individuals in tier 1.
Efficiency and optimality are no longer compelling issues when the links of the network evolve. Whether or not the initial leader was a natural leader, he or she eventually attracts direct links from much of the population, surpass any other agent’s initial endowment of incoming links.

5. Conclusions

Simulations of an evolving social network in which individuals adjust strategy in an effort to earn rewards associated with early adoption of subsequently popular trends do tend to produce a leader/follower hierarchical social structure, as is the equilibrium for the setting being simulated. The emergent hierarchy is efficient when individuals respond and adjust towards high performing strategies, driving the probability of following an inefficient strategy to zero. Such a process favors stability as well as efficiency and as a result tends to propel an individual from the population towards leadership based on luck rather than advantage.

An adjustment process that allows individuals to occasionally pursue inferior strategies will produce less efficient hierarchies and fluidity in the hierarchy. Through this instability, a more fit leader, measured by the number of social links, tends to emerge.

The hierarchy emerges because early transitory events become embedded in the social network to favor those with early success. An individual’s good luck leads to adjustment in the social network that perpetuates the success by attracting followers and followers lead to greater success.

Certain environments can hinder the process of hierarchy formation. If individuals are less observant of the individual success around them, then they fail to respond adequately, thereby denying the successful individuals the opportunity to perpetuate his or her success through social evolution. Stubbornness on the part of individuals when adjusting strategies will also undermine the social process that produces hierarchies.

The uniqueness of the hierarchy can also be disrupted if followers cannot rely on the social structure of the hierarchy to provide a reliable link by which they can imitate the leader. In this case, the population will organize into smaller, more stable hierarchies.
Mindful of the Kirman et al (2007) criticism that endogenous network models tend to require unreasonably high knowledge regarding the full social network, hierarchy formation is achieved with no knowledge of the existing or potential social structure. Individuals pursue successful strategies based only on individual observation of the success of their friends and strategies resulting in equilibrium social structure. The agents do not think strategically about how to go about becoming a leader. Such forward thinking strategic behavior is likely to disrupt the social process that produces emergence and will be the basis for further examination.

The fact that individual agents take the leader as given is important to the existence and stability of the hierarchical equilibrium. Given a leader, the individual chooses a behavior that maximizes his or her expected payoff based on the available links or chain of links to the leader. The agent reacts to a given environment. In a dynamic setting, if the agent believes his or her actions may influence who holds the leadership position, then forward looking strategic behavior may disrupt or alter an existing social hierarchy or affect its formation.]
Bibliography


Table 1: Exogenous parameter setting used in the simulations

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<thead>
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<td>No friend selection</td>
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Figure 1: Time-series of population characteristics. Replicator Dynamics, $K = 12$, $w = 0$, $\phi_A = 0.5$, $\phi_B = 1$, $\lambda_{RD} = 1$. $N(\text{ind})$ = number of agents acting independently; MaxFans = maximum number of fan linking to any one agent; $N(r=tr)$ = number of agents who do not decide until the final round (an indication that the agent decided to imitate in the round, but did not have a direct or indirect link to someone who acted independently through the social network); Optimality = number of potential fans of the leader – maximum number of potential fans across the population; Efficiency is the cumulative number of tiers each agent is below his efficient position. The leader emerges as sole leader, the only agent acting independently. Soon after, the efficiency of the hierarchy increases to zero. The leader is not a natural leader. Once established, there are no disruptions to the hierarchy.
Figure 2: Structure of the social hierarchy in the final period (t = 500). Replicator Dynamics, $K = 12, \lambda = 1, w = 0, \phi_A = 0.5, \phi_B = 1, \lambda_{RD} = 1$. Efficient hierarchy based on Agent #2, who emerged from the population as the leader.
Figure 3: Time-series of population characteristics. Replicator Dynamics, $K = 12$, $w = 0$, $\phi_A = 0.5$, $\phi_B = 1$, $\lambda_{RD} = 0.01$. \( N(\text{ind}) \) = number of agents acting independently; \( \text{MaxFans} \) = maximum number of fan linking to any one agent; \( N(r=tr) \) = number of agents who do not decide until the final round (an indication that the agent decided to imitate in the round, but did not have a direct or indirect link to someone who acted independently through the social network); \( \text{Optimality} \) = number of potential fans of the leader – maximum number of potential fans across the population; \( \text{Efficiency} \) is the cumulative number of tiers each agent is below his efficient position. The leader emerges as sole leader, the only agent acting independently. Organization of the hierarchy towards efficiency progresses slowly, but does arise. The leader is not a natural leader. Once established, there are no disruptions to the leadership.
Figure 4: Time-series of population characteristics. Experience Weighted Attractor, $K = 12$, $w = 0$, $\phi_\beta = 0.5$, $\phi_\gamma = 1$, $\lambda = 0.1$. $N(\text{ind}) =$ number of agents acting independently; MaxFans = maximum number of fan linking to any one agent; $N(r=tr) =$ number of agents who do not decide until the final round (an indication that the agent decided to imitate in the round, but did not have a direct or indirect link to someone who acted independently through the social network); Optimality = number of potential fans of the leader – maximum number of potential fans across the population; Efficiency is the cumulative number of tiers each agent is below his efficient position. The leader emerges as sole leader, the only agent acting independently. Efficiency does not arise in the hierarchy. There is fluidity in the position of the followers in the hierarchy. The leader is a natural leader.
**Figure 5**: Structure of the social hierarchy in the final period (t = 500). Experience Weighted Attractor, $K = 12$, $w = 0$, $\phi_d = 0.5$, $\phi_B = 1$, $\lambda = 0.1$. Agent #73 is a natural leader. The hierarchy is not efficient.
Figure 6: Time-series of population characteristics. Replicator Dynamics, $K = 12$, $w = 0$, $\phi_A = 0.5$, $\phi_B = 0$, $\lambda_{RD} = 1$. $N(\text{ind})$ = number of agents acting independently; MaxFans = maximum number of fan linking to any one agent; $N(r=tr)$ = number of agents who do not decide until the final round (an indication that the agent decided to imitate in the round, but did not have a direct or indirect link to someone who acted independently through the social network); Optimality = number of potential fans of the leader – maximum number of potential fans across the population; Efficiency is the cumulative number of tiers each agent is below his efficient position. The leader emerges as sole leader, but each period sees a fraction of the follower population acting independently. Efficiency emerges in that when in the imitating, each agent uses the most efficient chain of links to the leader. The leader is not a natural leader. Once established, there are no disruptions to the leadership.
Figure 7: Structure of the social hierarchy in the final period (t = 2000). Experience Weighted Attractor, $K = 12$, $w = 0$, $\phi_a = 0.5$, $\phi_b = 0$, $\lambda_1 = 3.5$. Evolving pool of independent actors believing themselves able to forecast the next trend.
Figure 8: Time-series of population characteristics. Experience Weighted Attractor, $K = 12$, $w = 0$, $\phi = 0.5$, $\phi_b = 0$, $\lambda_i = 3.5$. $N(\text{ind})$ = number of agents acting independently; MaxFans = maximum number of fans linking to any one agent; $N(r=tr)$ = number of agents who do not decide until the final round (an indication that the agent decided to imitate in the round, but did not have a direct or indirect link to someone who acted independently through the social network). Optimality = number of potential fans of the leader – maximum number of potential fans across the population; Efficiency is the cumulative number of tiers each agent is below his efficient position. In most periods, there is one leader. There are two changes in leadership (at $t=116$ and $t=239$).
Figure 9: Time-series of population characteristics. Replicator Dynamics, $K = 12$, $w = 0$, $\phi_A = 0.5$, $\phi_B = 1$, $\lambda_{RD} = 0$. $N(\text{ind}) =$ number of agents acting independently; MaxFans = maximum number of fan linking to any one agent; $N(r=tr) =$ number of agents who do not decide until the final round (an indication that the agent decided to imitate in the round, but did not have a direct or indirect link to someone who acted independently through the social network). Optimality = number of potential fans of the leader – maximum number of potential fans across the population; Efficiency is the cumulative number of tiers each agent is below his efficient position. A leader emerges but the hierarchy depends on the random realizations of links.
**Figure 10:** Time-series of population characteristics. Replicator Dynamics, $K = 12$, $w = 0$, $\phi_A = 0.5$, $\phi_B = 1$, $\lambda_{RO} = 1$, $\delta_B = 0.01$. $N(\text{ind})$ = number of agents acting independently; MaxFans = maximum number of fan linking to any one agent; $N(\text{r=tr})$ = number of agents who do not decide until the final round (an indication that the agent decided to imitate in the round, but did not have a direct or indirect link to someone who acted independently through the social network). Optimality = number of potential fans of the leader – maximum number of potential fans across the population; Efficiency is the cumulative number of tiers each agent is below his efficient position. A lack of information leads a population of agents to suboptimally act independently.
Figure 11: Time-series of population characteristics. Replicator Dynamics, $K = 6$, $w = 0$, $\phi_x = 0.5$, $\phi_y = 1$, $\lambda_{RD} = 1$. $N(\text{ind})$ = number of agents acting independently; MaxFans = maximum number of fan linking to any one agent; $N(r=\text{tr})$ = number of agents who do not decide until the final round (an indication that the agent decided to imitate in the round, but did not have a direct or indirect link to someone who acted independently through the social network); Optimality = number of potential fans of the leader – maximum number of potential fans across the population; Efficiency is the cumulative number of tiers each agent is below his efficient position. The leader emerges as sole leader. The leader is not a natural leader. Once established, there are no disruptions to the leadership, but the hierarchy dissipates as those in the lowest existing tier decide to act independently.
Figure 12: Structure of the social hierarchy in the final period (t = 2000). Replicator Dynamics, $K = 6$, $w = 0$, $\phi_s = 0.5$, $\phi_B = 1$, $\lambda_{RD} = 1$. Stable (and growing) pool of independent actors from the lowest tiers of the efficient hierarchy under agent #40.
Figure 13: Structure of the social hierarchy in the final period (t = 500). Replicator Dynamics, $K = 4$, $w = 0$, $\phi_a = 0.5$, $\phi_b = 1$, $\lambda_{RD} = 1$. No leader emerges. A couple of followers exist.
Figure 14: Structure of the social hierarchy in the final period (t = 500). Experience Weighted Attractor, $K = 12$, $w = 0$, $\phi_d = 0.5$, $\phi_\beta = 1$, $\lambda_{RD} = 1$. Initial social structure as a ring, each agent linked to three neighbors on each side. Multiple hierarchies exist as followers prefer to stable links to small networks to unreliable links to larger networks.
Figure 15: Structure of the social hierarchy in the final period (t = 500). Replicator Dynamics, $K = 12$, $\gamma = 0.2/d$, $\phi_A = 0.5$, $\phi_B = 1$, $\lambda_{RD} = 1$. 
Figure 16: Time-series of population characteristics. Replicator Dynamics, $K = 12, \ w = 0.2 / d$, $\phi_A = 0.5, \phi_B = 1, \lambda_{BD} = 1$. $N(\text{ind}) = \text{number of agents acting independently}; \text{MaxFans} = \text{maximum number of fan linking to any one agent}; N(r=\text{tr}) = \text{number of agents who do not decide until the final round (an indication that the agent decided to imitate in the round, but did not have a direct or indirect link to someone who acted independently through the social network)}. \text{The leader emerges as sole leader. Once established, there are no disruptions to the leadership. Followers form direct links to the leader. Tier 1 stabilizes with under 37 members.}$
**Figure 17:** Time-series of population characteristics. Replicator Dynamics with new friends from friends of best friend, $K = 12$, $w = 1/d$, $\phi_s = 0.5$, $\phi_b = 1$, $\lambda_{RD} = 1$. $N(\text{ind}) = \text{number of agents acting independently};$ MaxFans = maximum number of fan linking to any one agent; $N(r=\text{tr}) = \text{number of agents who do not decide until the final round (an indication that the agent decided to imitate in the round, but did not have a direct or indirect link to someone who acted independently through the social network).}$ Optimality = number of potential fans of the leader – maximum number of potential fans across the population; Efficiency is the cumulative number of tiers each agent is below his efficient position. The leader attracts over 80 direct incoming links.