Local Interactions in a Market with Heterogeneous Expectations

Mikhail Anufriev[1] Andrea Giovannetti[2] Valentyn Panchenko[3]

[1,2]University of Technology Sydney

[3]UNSW Sydney, Australia

Computing in Economics and Finance 2018
Milano

THE SCENE

- Many *non-specialist* investors allocate money into pension funds

- Funds can be classified into a number of types, e.g.,
  - value (fundamental)
  - momentum (chartists)

- Investors are able to switch funds
  - Every period investors receive performance reports for the past period
  - Investors talk to their *network* of friends and may switch to a different fund if it performed better

LITERATURE

Behavioral Asset Pricing

- Brock and Hommes (JEDC, 1998)
  - model with switching based on past performance
- Panchenko, Gerasymchuk and Pavlov (JEDC, 2013)
  - local interaction in BH asset pricing model
  - no switching if all neighbors are of the same type
  - analytic solution for random graph, assuming homogeneous degree
  - simulations for small world model

Spread of behaviors on networks:

- Lopez-Pintado (2008, GEB)
  - related to S-I-S model on network (e.g., Vespignani, 2001)
  - two types of agents: Susceptible agent becomes Infected if the number of Infected neighbours crosses a threshold

CONTRIBUTION

1. Network is characterized by degree distribution $P(k)$
2. We can study the market dynamics for general classes of networks:
   - regular random networks (same degree k)
   - Poisson networks
   - scale free (power-law) networks
3. Analytical results using mean field approximation, finer approximation than in Panchenko et al. (2013)
4. Our approach can handle broad classes of diffusion mechanisms

BASELINE FRAMEWORK: ASSET PRICING MODEL

Brock and Hommes, 1998, JEDC

- Large population of traders $\mathcal{N} = \{1, 2, ..., N\}$ trading combination of risk-free bond with return R and risky asset z with stochastic dividend y_t
- $t = 1, 2, ...$
- **Agents.** Myopic optimizers with CARA preferences in wealth $W_{i,t}$:

$$\max_{z_{i,t}} U_{i,t+1}(W_{i,t+1}) \quad \Leftrightarrow \quad \max_{z_{i,t}} \left\{ E^h_{t-1}\left[W_{t+1}\right] - \frac{a}{2} V\left[W_{t+1}\right] \right\}$$

- Agent i selects the trading type $h_i \in \mathcal{H}$ to form expectations on p_t.
- **Market Clearing** (zero supply of risky asset):

$$\sum_{h \in \mathcal{H}} n^h_t \underbrace{\frac{E^h_{t-1}\left[p_{t+1} + y_{t+1}\right] - Rp_t}{a\sigma^2}}_{\text{individual mean-variance demand}} = 0 \Rightarrow p_t = \frac{1}{R} \sum_{h \in \mathcal{H}} n^h_t E^h_{t-1}\left[p_{t+1}\right] + \bar{y}$$

- Given the **fundamental price** p^*, define $x_t = p_t - p^*$:

$$x_t = \frac{1}{R} \sum_{h \in \mathcal{H}} n_t^h E_{t-1}^h \left[x_{t+1} \right]$$

- **Trading Types**: $h \in \mathcal{H} \equiv \{f, c\}$

$$E_{t-1}^f \left[x_{t+1} \right] = 0 \qquad E_{t-1}^c \left[x_{t+1} \right] = g \cdot x_{t-1}$$

$$\underbrace{n_t^c \equiv n_t}_{\text{proportion of c-type}} \qquad \underbrace{n^f = 1 - n_t}_{\text{proportion of f-type}}$$

- **Dynamics**

$$\begin{cases} x_t &= \dfrac{g}{R} \cdot n_t x_{t-1} \\ n_t &= \Delta_t = \dfrac{e^{\beta \pi_{t-1}^c}}{e^{\beta \pi_{t-1}^f} + e^{\beta \pi_{t-1}^c}} \\ \pi_t^h &= z_{t-1}^h (x_t - R x_{t-1}) - c^h \end{cases}$$

Information Network

- As in Panchenko et al. (2013) performance of types h is only *locally* observable:
  - for switching you need information from agents of other types
  - agent i gathers information from k_i other agents - neighbours
  - $P(k)$ can capture cognitive overload, inattention

Timeline

1. Agents survey their neighborhood
2. Agents select their type $h_{i,t}$ (based on past performance)
3. Demand for risky asset is generated
4. Price p_t determined via a Walrasian market clearing
5. Agents portfolios are updated, dividend realizes.
6. Agents observe performance of their strategies

MEAN-FIELD APPROXIMATION

- Nodes are homogeneous conditional on their own degree k
- Neighbors types are *independent* from each other

A random link in the network points to a chartist with probability

$$\theta_t = \sum_k k n_{k,t-1} \cdot P(k)/\langle k \rangle,$$

where $n_{k,t-1}$ fraction of chartists for agents with degree k, and $\langle k \rangle = \sum_k k \cdot P(k)$ average degree of the network

SWITCHING

Let a counts the number of chartists in neighborhood $k_{i,t}$.

1. $F_t(k,a)$: probability of $f \to c$ given (k,a)
2. $R_t(k,a)$: probability of $c \to f$ given (k,a)

(Possibly $R_t(k,a) \neq F_t(k,a)$)
Probability for a fundamentalist with k links to switch to c:

$$\tilde{g}_k(\theta_t) = \sum_{a=0}^{k} F_t(k,a) \binom{k}{a} \theta_t^a (1-\theta_t)^{k-a}$$

Probability for a chartist with k links to switch to f:

$$\tilde{q}_k(\theta_t) = \sum_{a=0}^{k} R_t(k,a) \binom{k}{a} \theta_t^a (1-\theta_t)^{k-a}$$

- Different selection mechanisms:
  1. Brock and Hommes (1998).

$$F_t(k,a) = 1 - R_t(k,a) = \Delta_t, \ \forall a, k \geq 0.$$

  2. Panchenko et al (2013).

$$F_t(k,a) \ = 1 - R_t(k,a) \ = \begin{cases} 0 & \text{for } a = 0 \\ \Delta_t & \text{for } 0 < a < k \\ 1 & \text{for } a = k \end{cases}$$

  3. Generalization - smooth transition from 0 to 1 depending on a

Dynamics

$$\begin{cases} x_t & = & \frac{g}{R}\left[\sum_k P(k) n_{k,t}\right] \cdot x_{t-1} \\ & \vdots & \\ n_{k,t} & = & n_{k,t-1} - n_{k,t-1}\tilde{q}_k(\theta_t) + (1 - n_{k,t-1})\tilde{g}_k(\theta_t) \\ & \vdots & \\ \theta_t & = & \sum_k \frac{k \cdot P(k)}{\langle k \rangle} n_{k,t-1} \end{cases}$$

- The economy is described by a system of $(k+2)$ equations
- Each $n_{k,t}$ tracks the evolution of c-type traders endowed with k links

GENERALIZATION OF PANCHENKO ET AL. 2013

- With Panchenko et al. diffusion protocol, the LoM is:

$$n_{k,t} = n_{k,t-1}\theta_t^k + \left(n_{k,t-1}\left(1-\theta_t^k\right) + (1-n_{k,t-1})\left(1-(1-\theta_t)^k\right)\right)\cdot\Delta_t$$

- In equilibrium, the system is described by:

$$\begin{cases} x &= \dfrac{g}{R} x \sum_{k>0} P(k) \left[\dfrac{\Delta_x((1-\theta)^k - 1)}{\Delta_x((1-\theta)^k - \theta^k) - (1-\theta^k)}\right] \\ \theta &= \sum_{k>0} k \dfrac{P(k)}{\langle k \rangle} \left[\dfrac{\Delta_x((1-\theta)^k - 1)}{\Delta_x((1-\theta)^k - \theta^k) - (1-\theta^k)}\right] \end{cases}$$

FUNDAMENTAL STEADY STATES: $g < (1+r)$

Consider $E = (x, \theta)$ the stationary states of the system.

1. Fundamental s.s. $E_0 = (0,0)$, $E_1 = (0,1)$ always exist.
2. Fundamental s.s. $E_2 = (0, \bar{\theta})$ exists iff neighboring traders have on average at least one neighbor of either type:

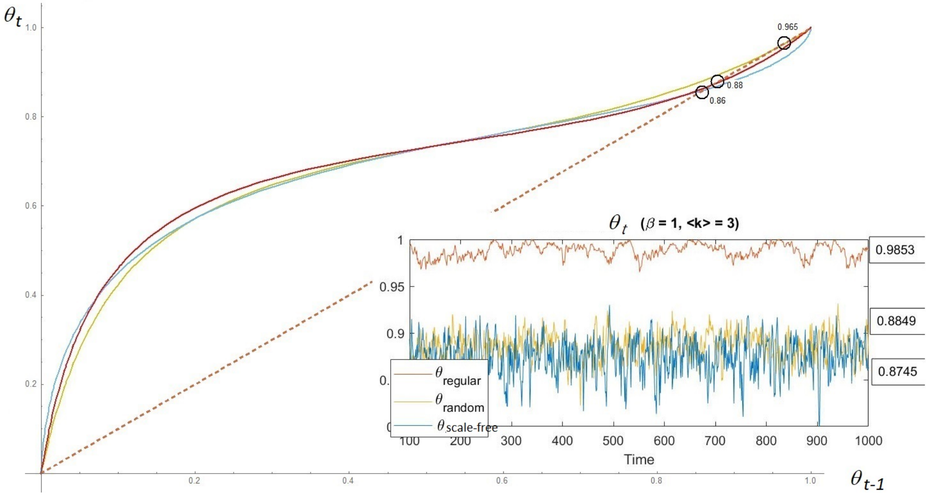
$$\frac{\Delta_0}{1-\Delta_0} \cdot \underbrace{\left(\frac{\langle k^2\rangle}{\langle k\rangle}\right)}_{\text{avg. deg of neighbor}} > 1 \quad \text{and} \quad \frac{1-\Delta_0}{\Delta_0} \cdot \left(\frac{\langle k^2\rangle}{\langle k\rangle}\right) > 1$$

3. For $\beta < \beta^1 = \ln\left(\frac{\langle k^2\rangle}{\langle k\rangle}\right)/c$ s.s. E_2 exists:

Regular	Random	Scale-Free
$\langle k^2\rangle = \langle k\rangle^2$	$\langle k^2\rangle = \langle k\rangle + \langle k\rangle^2$	$\langle k^2\rangle = \infty$

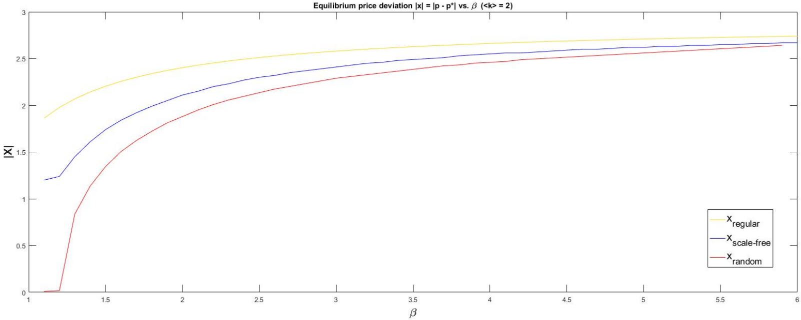
$$\beta^1_{regular} < \beta^1_{random} < \beta^1_{scale-free}$$

Fixed Point Analysis vs Simulations: ($\langle k \rangle = 3, \beta = 1, g < 1 + r$)



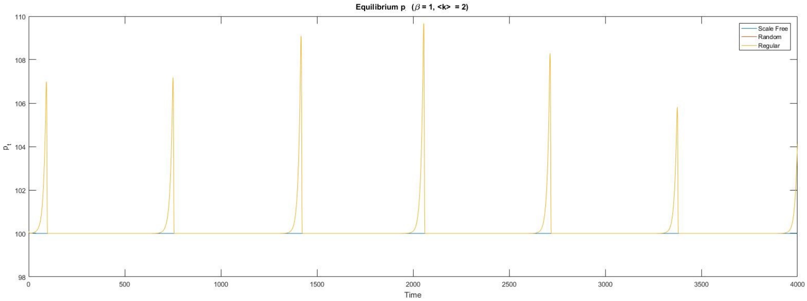
- Simulations match well approximation of analytical mappings.

AVERAGE PRICE DEVIATION $|x|$ FOR $\langle k \rangle = 2, g > 1 + r$



- Ordering of primary and secondary bifurcations seems to depend on the network type:
  Regular $<$ Scale-Free $<$ Random.
- Amplitude of $|x|$: Regular $>$ Scale-Free $>$ Random.

SIMULATIONS: $\langle k \rangle = 2,\ \beta = 1, g > 1 + r$



- Economy is sensitive to network typology for realistic range of $\langle k \rangle$

CONCLUSIONS

- Analytically tractable model for random networks with $P(k)$
  - importance of neighborhood size
- Major features generated by the BH model are preserved under various communication structures
- Importance of the network structures
  - depending on $P(k)$ faster bifurcations - less stability
  - short period between primary and secondary bifurcations
- Future work - other network features, e.g. clustering