Connecting the dots: Econometric methods for uncovering networks with an application to the Australian financial institutions

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ACE 2015

Which dots are we connecting?

- Correlations and principal components from financial econometrics
- Gaussian graphical models from statistics
- Centrality measures from network theory

Overview:

- Modeling framework
  - Correlations from DCC
  - Partial correlations and relations to linear regressions
  - Reconstructed network based on partial correlations
- Network-based measures and their relations
- Empirical Application:
  - Australian banks, economic sectors, and international markets

Motivation for Network Approach

Financial Crisis led to increased interest in the relationship between systemic risk and network effects

- "In the current crisis, we have seen that financial firms that become too interconnected to fail pose serious problems for financial stability and for regulators. Due to the complexity and interconnectivity of todays financial markets, the failure of a major counterparty has the potential to severely disrupt many other financial institutions, their customers, and other markets."
  – Charles Plosser, 03/06/09

Recent Contributions: just to name a few...

- Theoretical network models
  - Acemoglu, Ozdaglar and Tahbaz-Salehi (2014, AER). "Systemic Risk and Stability in Financial Networks".
  - Elliott, Golub and Jackson (2014, AER). Financial networks and contagion.
  - Glasserman and Young (2014, JBF). How likely is contagion in financial networks?
- Empirical networks (Reconstructed networks)
  - Diebold and Yilmaz (2014, JoE) "On the network topology of variance decompositions: Measuring the connectedness of financial firms".
  - Dungey, Luciani and Veredas (2014, JBF). "Googling CIFIs".
  - Barigozzi and Brownlees (2013, WP). "Nets: Network estimation for time series."

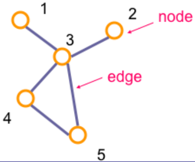
Partial correlations

- Let $X = (X_1, \dots, X_n)'$ be a random variable; $\mathcal{V} = \{X_1, \dots, X_n\}$.
- Partial correlation measures linear dependence between any two components of X after controlling for linear dependence with all other components

$$\rho_{ij|\cdot} \equiv \rho_{X_i,X_j \,|\, \mathcal{V}\setminus\{X_i,X_j\}} = \mathrm{Corr}\Big(X_i \,\Big|\, \mathcal{V}\setminus\{X_i,X_j\},\;\; X_j \,\Big|\, \mathcal{V}\setminus\{X_i,X_j\}\Big)$$

Network of Partial Correlations:

- X_i : node i (bank / economic sector / international market)
- edges connect nodes with non-zero partial correlations



Relation to linear regression

- X can be represented with a system of linear regressions (for all i)

$$X_i - \mu_i = \sum_{j \neq i} \beta_{ij}(X_j - \mu_j) + \varepsilon_i, \quad \varepsilon_i \sim N(0, \Sigma)$$

Orth. cond. $\mathrm{E}(\varepsilon_i X_j) = 0$ is satisfied iff $\quad \beta_{ij} = \rho_{ij|\cdot} \sqrt{\frac{\mathrm{Var}(\varepsilon_i)}{\mathrm{Var}(\varepsilon_j)}}.$

- rescale $x_i = (X_i - \mu_i) \Big/ \sqrt{\mathrm{Var}(\varepsilon_i)}$ and $e_i = \varepsilon_i \Big/ \sqrt{\mathrm{Var}(\varepsilon_i)}$

$$x_i = \sum_{j \neq i} \rho_{ij|\cdot} x_j + e_i.$$

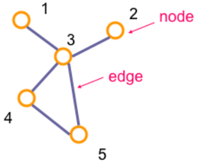
Possible Interpretation

$$x_i = \sum_{j \neq i} \rho_{ij|\cdot} x_j + e_i$$

- in matrix form, where $\mathbf{P}$ has 0 on-diagonal and $\rho_{ij|\cdot}$ off-diagonal

$$x = \mathbf{P}x + e$$

- kth-order effect of shock e is defined as $\mathbf{P}^k e$
- total effect of e is $e + \mathbf{P}e + \mathbf{P}^2 e + \cdots = (\mathbf{I} - \mathbf{P})^{-1} e$
- Note: this works when $\lim_{k\to\infty} \mathbf{P}^k = 0$



Relation to Variance-Covariance matrix

- Define $\Omega = \text{Cov}(X)$
- Set $\mathbf{K} = \{k_{ij}\} \equiv \Omega^{-1}$, concentration matrix
- By inverse of partitioned variance Whittaker (2009) shows

$$\rho_{ij|\cdot} = -\frac{k_{ij}}{\sqrt{k_{ii}k_{jj}}}.$$

- Matrix $\mathbf{P} = \mathbf{I} - \mathbf{D}_{\mathbf{K}}^{-1/2}\mathbf{K}\mathbf{D}_{\mathbf{K}}^{-1/2}$ is adjacency matrix of network
- Note:

$$\mathbf{D}_{\mathbf{K}} = \text{diag}\{k_{11}, \ldots, k_{nn}\} = \text{diag}\left\{\frac{1}{\text{Var}(\varepsilon_1)}, \ldots, \frac{1}{\text{Var}(\varepsilon_n)}\right\} = \mathbf{D}_{\Sigma}^{-1},$$

and so the matrix of the total effect is

$$(\mathbf{I} - \mathbf{P})^{-1} = \mathbf{D}_{\Sigma}^{-1/2}\,\Omega\,\mathbf{D}_{\Sigma}^{-1/2}$$

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Centrality measures (network theory)

- degree centrality - the sum of weights over all adjacent edges
- eigenvector centrality - takes into account centrality of neighbors
  - the eigenvector u_1 corresponding to the largest eigenvalue λ_1 of $\mathbf{P}$
  - Let e_0 be shock, its k-order effect is

$$\mathbf{P}^k e_0 = \mathbf{P}^k \sum_i b_i u_i = \sum_i b_i \mathbf{P}^k u_i = \sum_i b_i \lambda_i^k u_i = \lambda_1^k \sum_i b_i \left(\frac{\lambda_i}{\lambda_1} \right)^k u_i$$

- Thus u_1 gives the asymptotic direction for $\mathbf{P}^k e_0$ when $k \to \infty$
- Intuitively: vector u_1 shows in which node any initial shock will asymptotically accumulate

Centrality measures

- *Bonacich centrality* - accumulation of degree centrality, degree centrality of neighbors (dampened by $\alpha \in [0, 1]$), degree centrality of neighbors of neighbors (dampened by α^2), and so on

$$c^{\mathrm{B}}(\alpha) = \mathbf{P} \cdot \mathbf{1} + \alpha \mathbf{P}^2 \cdot \mathbf{1} + \alpha^2 \mathbf{P}^3 \cdot \mathbf{1} + \cdots = (\mathbf{I} - \alpha \mathbf{P})^{-1} \mathbf{P} \cdot \mathbf{1},$$

- when $\alpha = 1$ Bonacich centrality is equal to

$$c^{B}(\mathbf{1}) = (\mathbf{I} - \mathbf{P})^{-1} \mathbf{P} \cdot \mathbf{1} = (\mathbf{I} - \mathbf{P})^{-1} \cdot \mathbf{1} - \mathbf{1}$$

  sum of indirect effects (i.e., first-orders, second-order, etc.) of unit shock $e = \mathbf{1}$
- compare with total effect of e $\quad e + \mathbf{P}e + \mathbf{P}^2 e + \cdots = (\mathbf{I} - \mathbf{P})^{-1} e$

Conditional Correlations (Bollerslev, 1990, Engle, 2002)

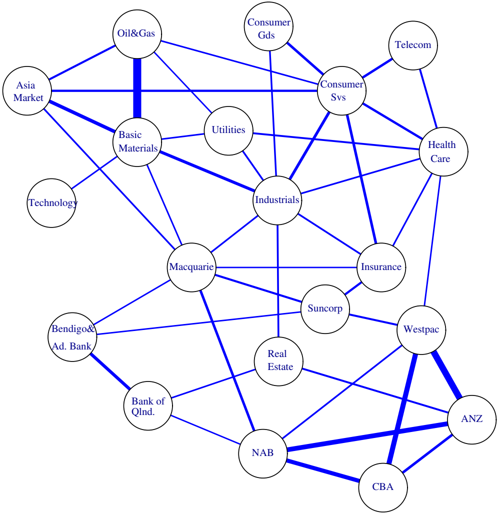
- Start with each entity and filter its returns (Y_t) to remove conditional mean/variance
  - we use AR(1)-GARCH (1,1) model

$$
\begin{aligned}
Y_t &= c + bY_{t-1} + u_t\,, \qquad u_t \sim N(0, \sigma_t^2) \\
\sigma_t^2 &= \gamma + \delta\sigma_{t-1}^2 + \phi u_{t-1}^2
\end{aligned}
$$

- Form matrix X_t by taking standardized innovations u_t/σ_t for all entities; and compute correlation R from X_t
- Reconstruct **P** by inverting R.

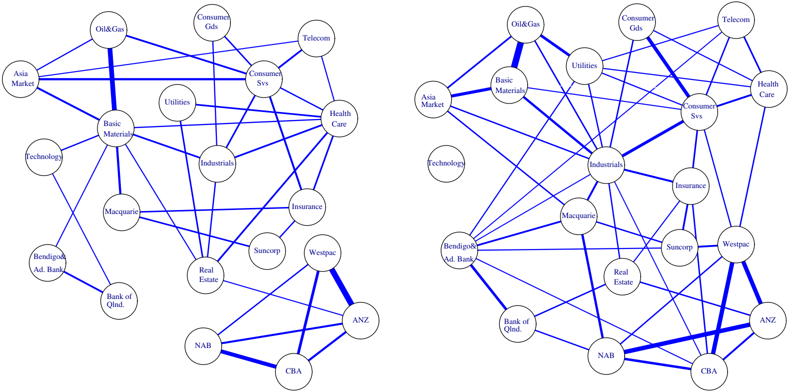
Data

- Source/classification: Datastream
  - 6 publicly traded banks:
    - “Big Four” (ANZ, CBA, NAB, Westpac) + Macquarie and Suncorp
    - 2 regional: ’Bank of Qnd’ and ’Bendigo and Adelaide bank’
  - sectors of the real economy: basic materials, industrials, etc.
  - Asian market.
- Time-span $6/11/2000$ to $22/08/2014$
- $3,600$ daily observations in total
- Network is built for $22/08/2014$
  - pre 2008
  - post 2008



Network for 22-Aug-2014, cut-off level $\kappa = 0.075$

Stability of the networks



(a) pre 2008 (b) post 2008

	R^2 full		Degree full		Eigenvec. full		Bonacich full	
ANZ	0.660	1	1.074	4	1.000	1	23.481	1
Westpac	0.646	2	1.056	6	0.968	2	22.759	3
Industrials	0.621	4	1.363	1	0.967	3	23.478	2
NAB	0.618	5	1.063	5	0.932	4	21.993	4
CBA	0.609	6	0.995	8	0.906	5	21.333	5
Basic Materials	0.623	3	1.121	3	0.877	6	21.189	6
Consumer Svs	0.557	8	1.221	2	0.831	7	20.258	7
Oil & Gas	0.578	7	0.971	9	0.795	8	19.204	8
Insurance	0.490	9	1.010	7	0.743	9	17.975	9
Macquarie	0.469	10	0.851	10	0.713	10	17.031	10
Asia Market	0.451	11	0.793	13	0.665	11	15.993	11
Real Estate	0.395	12	0.813	11	0.615	12	14.818	12
Suncorp	0.377	13	0.740	17	0.591	13	14.135	13
Bank of Qlnd.	0.365	14	0.741	16	0.570	14	13.676	14
Bend&Ad.Bank	0.365	15	0.799	12	0.564	15	13.597	15

Summary

- linked Graphical Gaussian models with theoretic network literature
  - established link between partial correlations and shock propagation
  - discussed links with network-based measures
- reconstructed the network for Australia financial institutions and linked them with other domestic sectors and world economy