Planar Beauty Contests

Mikhail Anufriev[a] John Duffy[b]
Valentyn Panchenko[c]

[a] University of Technology Sydney
[b] University of California Irvine
[c] University of New South Wales, Sydney

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Outline

Planar Beauty Contests

- We present an experiment that extends the Keynesian beauty contest game to the second dimension
  - Beauty Contest is a "forecasting game", convenient to study theories of level-k thinking and of adaptive learning
  - coupled dynamical systems arise naturally as a reduced-form of many macroeconomic models
- We confront behavior of participants in four treatments that differ in feedback coefficients and lead to various stability predictions
- We find that not only the location of eigenvalues w.r.t. unit circle but also the sign of unstable eigenvalue is important for convergence

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Experiment: The guessing game

Game between N players.

Every participant writes down a number from the set

$$S = [0, 100]$$

Let x_i be the number written down by participant i and let

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i,$$

be the average of these numbers. The participant whose guess x_i is closest to the target number

$$x^* = \frac{2}{3}\bar{x} + 30$$

wins the game.

The guessing game: Nash Equilibrium

In the game with target

$$x^* = \frac{2}{3}\bar{x} + 30$$

the Nash Equilibrium is "everyone plays 90".

- it can be computed as the fixed point of the equation
- alternatively, it can be approached via an iterative elimination of the dominated strategies (from both sides)
- Beauty contest game is subject to strategic uncertainty: playing equilibrium strategy hinges on assumption that all other players play equilibrium strategies as well.
- k-level thinking theories assume that players play best response on less sophisticated strategies of other players

Literature Overview

- Guessing game
  - Nagel (AER, 1995); Stahl and Wilson (GEB, 1995); Duffy and Nagel (EJ, 1997); Ho et al (AER, 1998); Bosch-Domenech et al (AER, 2002); Guth et al (EL, 2002); Costa-Gomes and Crawford (AER, 2006); Grosskopf and Nagel (GEB, 2008); Sutan and Willinger (JEDC, 2009); Agranov et al (JESI, 2015)
- Learning-to-Forecast experiments
  - Hommes et al (RFS, 2005); Heemeijer et al (JEDC, 2009); Sonnemans and Tuinstra (JEPs, 2010); Bao et al (JEDC, 2012)
- *k*-Level thinking theories
  - Camerer et al (QJE, 2004); de Giorgi, Reimann (GEB, 2008)
- Adaptive vs. Eductive learning
  - Camerer and Ho (ETCA, 1999); Bao and Duffy (EER, 2016)

Why planar Guessing Game?

- It is interesting to see how k-level reasoning generalizes on the second dimension
- In many macroeconomic models the reduced forms are given by the two-dimensional systems.
- The properties of discovering REE depend on the eigenvalues.
- Learning-to-Forecast experiments in two dimensions focus on adaptive learning, whereas we present and explain the laws of motion to the subjects
    - Assenza et al (WP, 1999)

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Basic Design

- $N = 10$ students play game for $T = 15$ periods
- At each time they have to submit a pair of numbers called in the experiment "A-number" and "B-number", each number within an interval $[0, 100]$.
- With the pairs (x_i, y_i) the target numbers are given by

$$\begin{pmatrix} x^* \\ y^* \end{pmatrix} = \mathbf{M} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} + \boldsymbol{b} = \begin{pmatrix} m_{11} & 0 \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},$$

  where $\bar{x}$ and $\bar{y}$ are the averages of x_i and y_i
- Matrix $\mathbf{M}$ and vector $\boldsymbol{b}$ depend on the treatment and are known to all participants.
- Reward of participant i is

$$\frac{500}{5 + |x_i - x^*| + |y_i - y^*|} \quad \text{points}$$

Equilibrium Prediction

- The game has a unique Nash Equilibrium

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} = (\mathbf{I} - \mathbf{M})^{-1} \boldsymbol{b} \quad \text{for every } i \,,$$

- Alternatively, participants may play best responses on assumed level of sophistication of others (level-k theories)
- The game is played repeatedly, and players may respond to the previous history; each participant can see:
  - past own pairs of numbers for each period
  - past averages of both numbers for each period
  - past target numbers for each period
  - past points earned

Treatments

Sink

$$\mathbf{M} = \begin{pmatrix} 2/3 & 0 \\ -1/2 & -1/2 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} 30 \\ 75 \end{pmatrix} \rightsquigarrow \begin{pmatrix} x^{NE} \\ y^{NE} \end{pmatrix} = \begin{pmatrix} 90 \\ 20 \end{pmatrix}$$

SaddleNeg

$$\mathbf{M} = \begin{pmatrix} 2/3 & 0 \\ -1/2 & -3/2 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} 30 \\ 95 \end{pmatrix} \rightsquigarrow \begin{pmatrix} x^{NE} \\ y^{NE} \end{pmatrix} = \begin{pmatrix} 90 \\ 20 \end{pmatrix}$$

SaddlePos

$$\mathbf{M} = \begin{pmatrix} 2/3 & 0 \\ -1/2 & 3/2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 30 \\ 35 \end{pmatrix} \rightsquigarrow \begin{pmatrix} x^{NE} \\ y^{NE} \end{pmatrix} = \begin{pmatrix} 90 \\ 20 \end{pmatrix}$$

Source

$$\mathbf{M} = \begin{pmatrix} 3/2 & 0 \\ -1/2 & -3/2 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} -45 \\ 95 \end{pmatrix} \rightsquigarrow \begin{pmatrix} x^{NE} \\ y^{NE} \end{pmatrix} = \begin{pmatrix} 90 \\ 20 \end{pmatrix}$$

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First Period A-Choices

- The first ("A") number is motivated by guessing

$$x^* = p\bar{x} + b_1 \, ,$$

- it is identical to the standard (1D) beauty contest game with $p = 2/3$ and $b_1 = 30$ (Sink, SaddleNeg, SaddlePos) and $p = 3/2$ and $b_1 = -45$ (Source)
- levels of rationality (for $p = 2/3$ and $b_1 = 30$):

  0: $x_0 = 50$ (average of the interval)

  1: $x_1 = px_0 + b_0 = 63.33$

  2: $x_2 = px_1 + b_0 = 72.22$

- levels of rationality (for $p = 3/2$ and $b_1 = -45$):

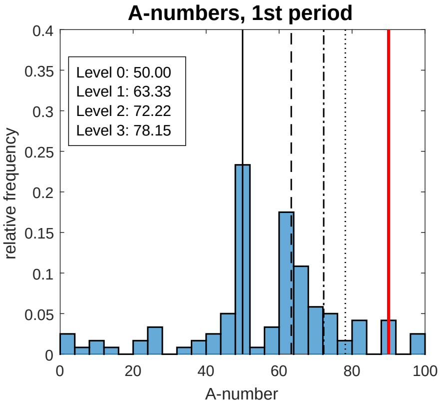
  0: $x_0 = 50$ (average of the interval)

  1: $x_1 = px_0 + b_0 = 30$

  2: $x_2 = px_1 + b_0 = 0$

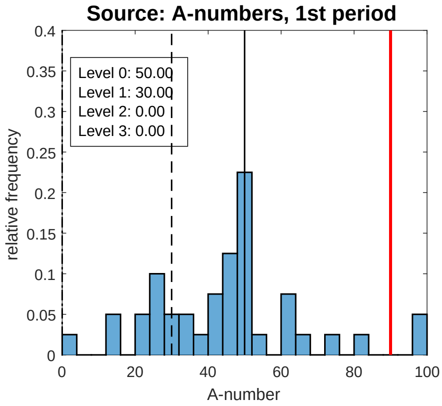
First Period A-Choices

$$x^* = \frac{2}{3}\bar{x} + 30$$



First Period A-Choices: Source Treatment

$$x^* = \frac{3}{2}\bar{x} - 45$$



First Period B-Choices

- Target of the second ("B") number depends on both "A" and "B" numbers

$$y^* = m_{21}\bar{x} + m_{22}\bar{y} + b_2 \,,$$

- For example in Sink we have

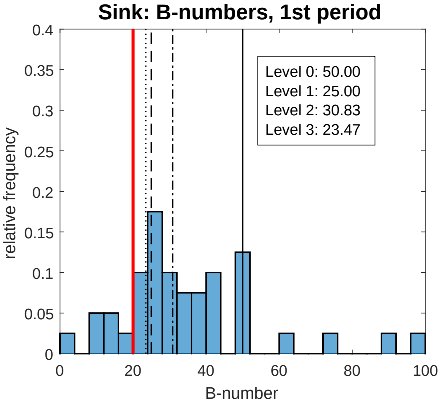
$$y^* = -\frac{1}{2}\bar{x} - \frac{1}{2}\bar{y} + 75, \,,$$

- levels of rationality:

0: $y_0 = 50$ (average of the interval)
1: $y_1 = m_{21}x_0 + m_{22}y_0 + b_2 = 25$
2: $y_2 = m_{21}x_1 + m_{22}y_1 + b_2 = 30.83$
3: $y_3 = m_{21}x_2 + m_{22}y_2 + b_2 = 23.47$

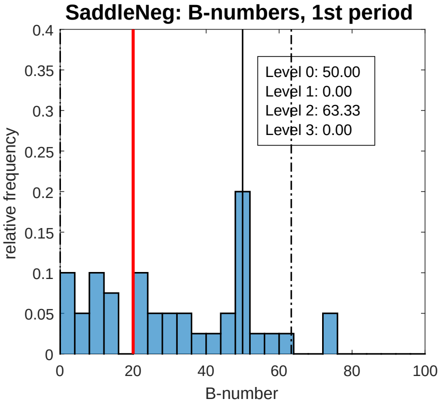
First Period B-Choices: Sink

$$y^* = -\frac{1}{2}\bar{x} - \frac{1}{2}\bar{y} + 75$$



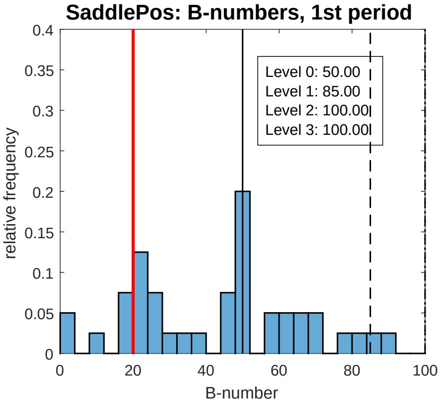
First Period B-Choices: SaddleNeg Treatment

$$y^* = -\frac{1}{2}\bar{x} - \frac{3}{2}\bar{y} + 95$$



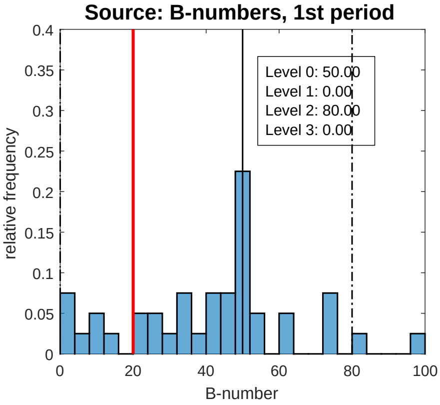
First Period B-Choices: SaddlePos Treatment

$$y^* = -\frac{1}{2}\bar{x} + \frac{3}{2}\bar{y} + 35$$



First Period B-Choices: Source Treatment

$$y^* = -\frac{1}{2}\bar{x} - \frac{3}{2}\bar{y} + 95$$



First Period Choices

- First period choices are heterogeneous. There is a spike around the middle of the interval (0-level of rationality)
- For the A-number there is another spike near the 1st level of rationality and tendency of subjects to move towards the 2nd level of rationality
- For the B-number there is a tendency to move towards the 1st level of rationality

Treatment	***x* (A-number)**			***y* (B-number)**		
	Mean x_1	Median x_1	Std.Dev x_1	Mean y_1	Median y_1	Std.Dev y_1
Sink	57.39	60	17.67	34.98	30	20.01
SaddleNeg	55.01	56.01	22.28	31.06	29	21.46
SaddlePos	56.25	58.80	20.50	42.97	46.75	23.18
Source	44.78	45.75	20.97	40.92	45	23.21

First period choices

- level k theories assume that subjects are boundedly rational stopping at some level of thinking
- in the Beauty Contest game, it is typical to find $k = 1$ and $k = 2$, but rarely higher than 3.

Take Away Messages:

1. for A-number: the Beauty Contest game with the NE located inside the interval seems to be more difficult to subjects and leads to a decrease in the level of rationality
2. for B-number: the presence of a coupled variable in the Beauty Contest game leads to even further decrease in the level of rationality

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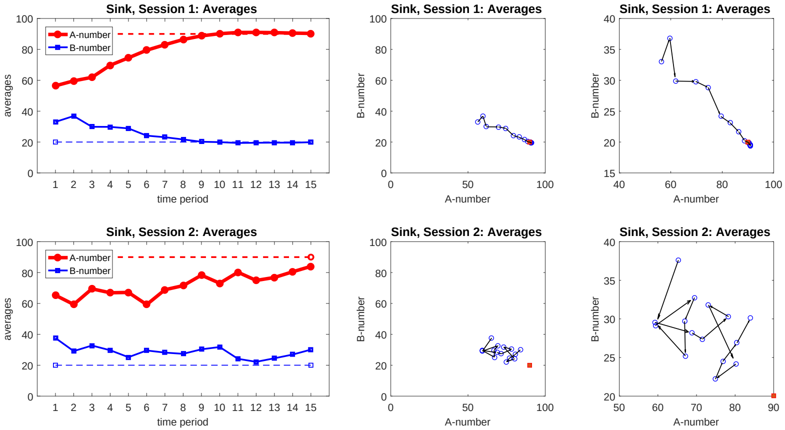
Dynamics: from the first period to the last

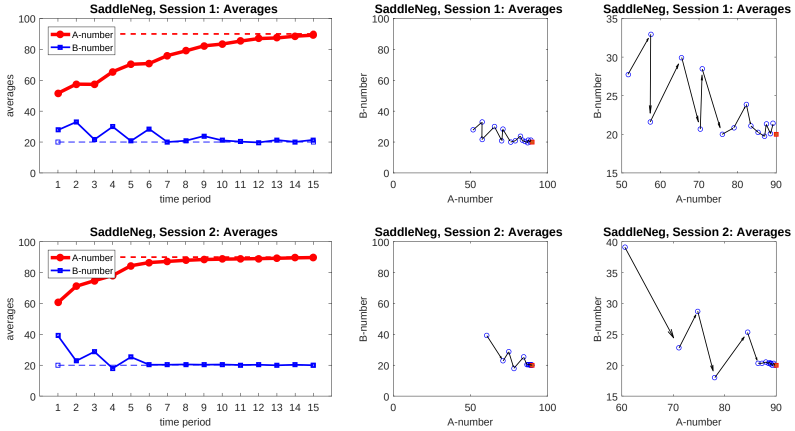
Treatment	x (A-number)			y (B-number)		
	Mean x_1	Median x_1	Std.Dev x_1	Mean y_1	Median y_1	Std.Dev y_1
Sink	57.39	60	17.67	34.98	30	20.01
SaddleNeg	55.01	56.01	22.28	31.06	29	21.46
SaddlePos	56.25	58.80	20.50	42.97	46.75	23.18
Source	44.78	45.75	20.97	40.92	45	23.21

Treatment	x (A-number)			y (B-number)		
	Mean x_{15}	Median x_{15}	Std.Dev x_{15}	Mean y_{15}	Median y_{15}	Std.Dev y_{15}
Sink	84.21	86.86	14.40	25.40	21.48	17.44
SaddleNeg	89.81	89.93	0.55	20.31	20	1.49
SaddlePos	87.33	88	3.06	92.31	100	24.95
Source	7.46	0	13.49	35.56	35	6.03

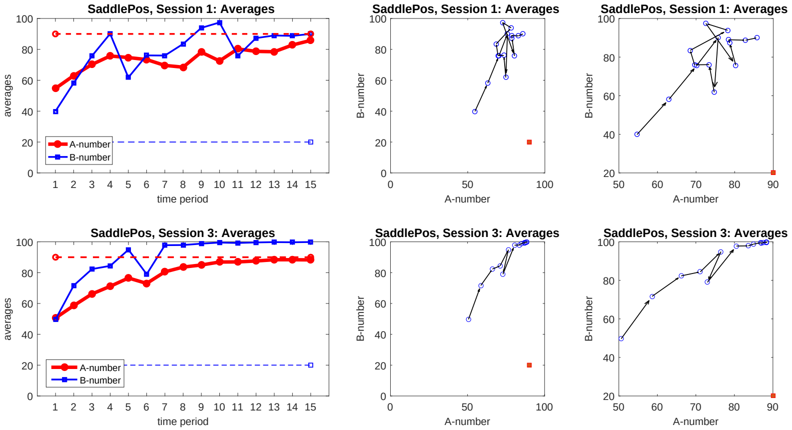
Sink T (Averages, sessions 1 and 2)



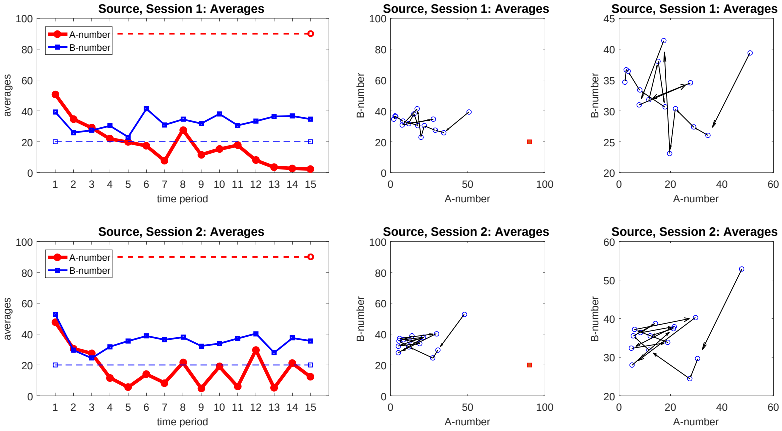
SaddleNeg T (Averages, sessions 1 and 2)



SaddlePos T (Averages, sessions 1 and 3)



Source T (Averages, sessions 1 and 2)



Summary of Results on Dynamics

Dynamics converge only in Sink and SaddleNeg

Dynamics of x ... (identical to the beauty contest game)

- ... converges almost monotonically in Sink, SaddleNeg and SaddlePos.
- ... diverges in the Source treatment to the lower bound 0.

Dynamics of y ...

- ... converges almost monotonically in Sink and converges (through oscillations) in SaddleNeg.
- ... does not converge to the equilibrium value in SaddlePos and Source.
- ... for the Source the dynamics is stable, near a constant level inside the range.

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Model Comparison

We compared the following models

1. NE
2. Naive
3. Adaptive models with estimated λ_x, λ_y
4. Average with 2, 3, 4, 5 and 14 lags
5. Heuristic Switching Model (Anufriev and Hommes, 2012)

using the AIC, i.e., the sum of the MSEs for x and y adjusted for the number of parameters.

For every session, at each step $t = 3, \ldots, 15$ we compute model predictions $x^{e,\text{Mod}}$ and $y^{e,\text{Mod}}$ to find $\text{MSE}_x + \text{MSE}_y$

$$\text{MSE}_x = \frac{1}{13}\left(\bar{x}_t - x^{e,\text{Mod}}\right)^2 \quad \text{and} \quad \text{MSE}_y$$

Winners of the Model Comparison

MSE's by treatments and in total:

Treatment	NE	Average		Adaptive		HSM	
		Naive	MAve(4)	$\lambda_x \neq \lambda_y$, loc	$\lambda_x = \lambda_y$, glob	local	glob
Sink	953	161	128	118	126	**104**	124
SaddleNeg	420	60	79	**22**	42	35	30
SaddlePos	19642	848	723	**321**	338	330	355
Source	24490	1239	958	**432**	476	**432**	499
Total MSE	45506	2309	1888	**893**	983	901	1007
AIC	2231	1611	1569	1477	**1435**	1479	1440

Using AIC to penalize different number of parameters, we find a support to Adaptive model with common $\lambda = 0.44$.

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Conclusion

- the first experiment of planar Beauty Contest game
- the simplest possible structure (variable x is decoupled)
- four treatments with various possibilities for eigenvalues
- more complicated 1D equation and presence of extra variable for another equation results in lower level of rationality
- convergence is achieved in the Sink treatment and in Saddle treatment but only with negative feedback
- in Saddle with positive feedback and Source dynamics diverge
- adaptive model describes the data best